ON FUZZY SUPRA PRE σ -BAIRE SPACESE.Poongothai¹ and G.Thangaraj²¹Department of Mathematics, Shanmuga Industries Arts and Science College Tiruvannamalai – 632601, Tamilnadu, India.²Department of Mathematics, Thiruvalluvar University Vellore – 632 115, Tamilnadu, India

ABSTRACT: In this paper, the concepts of fuzzy supra pre σ -nowhere dense set, fuzzy supra pre σ -first category set and fuzzy supra pre σ -second category set in fuzzy topological spaces are introduced and studied. By means of fuzzy supra pre σ -nowhere dense sets, the concept of fuzzy supra pre σ -Baire space is defined and several characterizations of fuzzy supra pre σ -Baire spaces are studied. Several examples are given to illustrate the concepts introduced in this paper.

KEYWORDS: Fuzzy supra pre dense set, fuzzy supra pre nowhere dense set, fuzzy supra pre F_σ -set, fuzzy supra pre G_δ -set, fuzzy supra pre σ -nowhere dense set, fuzzy supra pre σ -first category, fuzzy supra pre σ -second category, fuzzy supra pre σ -Baire space.

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1. INTRODUCTION

Dealing with the uncertainties in our real life phenomena has been a critical matter in recent years. Classical set theory was not suitable in such cases. Since 1965, Zadeh [1] has introduced a more suitable set theory providing the concept "fuzzy set". A large community of Mathematics has put their relentless efforts to the investigation of various concepts of general topology in fuzzy setting likely, [2-5]. In 2012, Selvi and Dharani, [6] introduced the concept of pre* open (supra – preopen) sets in general topology. In 2016, Hakeem A.Othman [7] introduced the concept of fuzzy supra pre open sets in supra topology. The concept of fuzzy supra σ –Baire spaces were introduced and studied by Poongothai and Thangaraj [8]. In this paper, the concepts of fuzzy supra pre σ –nowhere dense sets and fuzzy supra pre σ –Baire spaces are introduced and studied. Several characterizations of fuzzy supra pre σ –Baire spaces are studied.

2. PRELIMINARIES

Definition 2.1 [8]: A fuzzy topology T on a set X is a family of fuzzy sets in X such that

$$0_X, 1_X \in T;$$

$$A, B \in T \Rightarrow A \wedge B \in T \text{ and}$$

$$A_i \in T \Rightarrow \bigvee A_i \in T$$

The pair (X, T) is called a fuzzy topological space (FTS).

The elements of T are called fuzzy open sets and the complement of fuzzy open set is called fuzzy closed set.

Definition 2.2 [4]: The closure and the interior of a fuzzy set A in FTS (X, T) are denoted and defined respectively as

$$Cl(A) = \bigwedge \{B: A \leq B, B \text{ is a fuzzy closed set in } X\},$$

$$Int(A) = \bigvee \{B: B \leq A, B \text{ is a fuzzy open set in } X\}.$$

Definition 2.3[1]: A collection T^* of fuzzy open sets in a set X is called a fuzzy supra topology on X if the following conditions are satisfied:

$$(1) 0_X, 1_X \in T^* \text{ and } (2) A_i \in T^* \Rightarrow \bigvee A_i \in T^*$$

The pair (X, T^*) is called a fuzzy supra topological space (FSTS). The elements of T^* are called fuzzy supra open sets (FSOS) and the complement of a fuzzy supra open set is called fuzzy supra closed set (FSCS). The collection of all fuzzy supra open sets (resp.fuzzy supra closed sets) of the FSTS (X, T^*) is denoted by FSOS(X) (resp. FSCS(X)).

Remark 2.4 (4):

Every FTS is a FSTS.

If (X, T^*) is an associated FSTS with the FTS (X, T) (i.e. $T \subseteq T^*$), then every fuzzy open (closed) set in the FTS (X, T) is fuzzy supra open (closed) set in the FSTS (X, T^*) .

Definition 2.5 [7]: Let (X, T^*) is a FSTS and A be fuzzy set in X , then the fuzzy supra closure and fuzzy supra interior are denoted and defined respectively as $Cl^*(A) = \bigwedge \{B: A \leq B, B \text{ is a fuzzy supra closed set in } X\}$, $Int^*(A) = \bigvee \{B: B \leq A, B \text{ is a fuzzy supra open set in } X\}$.

Remark 2.6 [7]:

The fuzzy supra closure of a fuzzy set A in a FSTS is the smallest fuzzy supra closed set containing A .

The fuzzy supra interior of a fuzzy set A in a FSTS is the largest fuzzy supra open set containing A .

If (X, T^*) is an associated FSTS with the FTS (X, T) and A is any fuzzy set in X , then

$$Int(A) \leq int^*(A) \leq A \leq Cl^*(A) \leq Cl(A).$$

Properties of fuzzy supra closure and fuzzy supra interior which are needed in the sequel, are summarized in

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the following theorem.

Theorem 2.7 [4]: For any fuzzy sets A and B in a FSTS (X, T^*) .

$$A \in FSCS(X) \Leftrightarrow Cl^*(A) = A, A \in$$

$$FSOS(X) \Leftrightarrow Int^*(A) = A;$$

$$A \leq B, \Rightarrow Cl^*(A) \leq$$

$$Cl^*(B) \text{ and } Int^*(A) \leq Int^*(B);$$

$$Cl^*(Cl^*(A)) =$$

$$Cl^*(A) \text{ and } Int^*(Int^*(A)) =$$

$$Int^*(A);$$

$$Cl^*(A \vee B) \geq Cl^*(A) \vee Cl^*(B);$$

$$Cl^*(A \wedge B) \leq Cl^*(A) \wedge Cl^*(B);$$

$$Int^*(A \vee B) \geq Int^*(A) \vee Int^*(B);$$

$$Int^*(A \wedge B) \leq Int^*(A) \wedge Int^*(B);$$

$$Cl^*(A^c) = (Int^*(A))^c, Int^*(A^c) =$$

$$(Cl^*(A))^c$$

Definition 2.8 [5]: A fuzzy set λ of fuzzy space X is called fuzzy supra preopen (supra preclosed) set if $\lambda \leq int^*cl \lambda$ ($cl^*int \lambda \leq \lambda$). The class of all fuzzy supra-preopen (supra - preclosed) sets in X will as denoted be $SFPO(X)$ ($SFPC(X)$).

Definition 2.9 [5]: The closure and the interior of a fuzzy set A in FTS (X, T) are denoted and defined respectively as

$$spCl(A) =$$

$$\wedge$$

$$\{B: A \leq$$

$$B, B \text{ is a fuzzy supra preclosed set of } X\},$$

$$spInt(A) = \vee \{B: B \leq A, B \text{ is a fuzzy supra pre open set of } X\}.$$

Definition 2.10 [5]: Let A be a fuzzy supra set of a fuzzy supra topological space (X, T) . Then (i)

$$1 - pcl^*(A) = pint^*(1 - A) \text{ (ii) } 1 -$$

$$pint^*(A) = pcl^*(1 - A)$$

Definition 2.11 [9]: A fuzzy set A in a FSTS (X, T^*) is called fuzzy supra F_σ -set in (X, T^*) if $A = \vee_{i=1}^\infty A_i$, where $1 - A_i \in T^*$ for $i \in I$.

Definition 2.12 [9]: A fuzzy set A in a FSTS (X, T^*) is called a fuzzy supra G_δ -set in (X, T^*) if $A = \wedge_{i=1}^\infty A_i$, where $A_i \in T^*$ for $i \in I$.

Definition 2.13 [9]: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set A in (X, T^*) is called a fuzzy supra σ -nowhere dense set if A is a fuzzy supra F_σ -set in (X, T^*) such that $int^*(A) = 0$.

Definition 2.14 [9]: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set A in (X, T^*) is called fuzzy supra σ -first category if $A = \vee_{i=1}^\infty (A_i)$ where $(A_i)'s$ are fuzzy supra σ -nowhere dense sets in (X, T^*) . Any other fuzzy set in (X, T^*) is said to be fuzzy supra σ -second category in (X, T^*) .

3. FUZZY SUPRA PRE σ -NOWHERE DENSE SETS

Definition 3.1: A fuzzy set A in a FSTS (X, T^*) is called fuzzy supra pre dense if there exists no fuzzy supra pre closed set B in (X, T^*) such that $A < B < 1$. That is, $pcl^*(A) = 1$.

Definition 3.2: A fuzzy set A in a FSTS (X, T^*) is called fuzzy supra pre F_σ -set in (X, T^*) if $A = \vee_{i=1}^\infty A_i$, where $(A_i)'s$ are fuzzy supra pre-closed sets in (X, T^*) .

Definition 3.3: A fuzzy set A in a FSTS (X, T^*) is called a fuzzy supra pre G_δ -set in (X, T^*) if $A = \wedge_{i=1}^\infty A_i$, where $(A_i)'s$ are fuzzy supra pre-open sets in (X, T^*) .

Definition 3.4: A fuzzy set A in a FSTS (X, T^*) is called fuzzy supra pre nowhere dense if there exists no non-zero fuzzy supra pre - open set B in (X, T^*) such that $B < pcl^*(A)$. That is, $pint^*pcl^*(A) = 0$.

Definition 3.5: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set A in (X, T^*) is called a fuzzy supra pre σ -nowhere dense set if A is a fuzzy supra pre F_σ -set in (X, T^*) such that $pint^*(A) = 0$.

Example 3.6: Let $X = \{a, b\}$ be a set with a fuzzy supra topology (X, T^*) . Then the fuzzy sets $A = \{a_{.7}, b_{.7}\}, B = \{a_{.7}, b_{.6}\}, C = \{a_{.6}, b_{.8}\}$. Then

$$T^* = \{0, A, B, C, A \vee B, B \vee C, B \wedge C, 1 - A, 1 - B, 1 - C, 1 - A \vee B, 1 - B \vee C, 1 - B \wedge C, 1\}$$

Now, consider the fuzzy set $\alpha = (1 - A \vee B) \vee (1 - B \vee C) \vee (1 - B \wedge C)$ in (X, T^*) . Then α is fuzzy supra pre F_σ -set in (X, T^*) and $\text{pint}^*(\alpha) = 0$ and hence α is a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

Remarks 3.7: If A and B are fuzzy supra pre σ -nowhere dense sets in a fuzzy supra topological space (X, T^*) , then $A \vee B$ need not be a fuzzy supra pre σ -nowhere dense set in (X, T^*) . For, consider the following example:

Example 3.8: Let $X = \{a, b, c\}$ be a set with a fuzzy supra topology (X, T^*) . Then the fuzzy sets $A = \{a_{.7}, b_{.6}, c_{.5}\}$, $B = \{a_{.4}, b_{.4}, c_{.8}\}$, $C = \{a_{.7}, b_{.6}, c_{.6}\}$

$$T^* = \{0, A, B, C, A \vee B, B \vee C, B \wedge C, 1 - A, 1 - B, 1 - C, 1 - A \vee B, 1 - B \vee C, 1 - B \wedge C, 1\}$$

Now, consider the fuzzy set $\alpha = [(1 - B) \vee \{1 - (A \vee B)\}] \vee \{1 - B \vee C\}$ and $\beta = [(1 - A) \vee (1 - C)]$ are fuzzy supra pre F_σ -sets in (X, T^*) . Also $\text{pint}^*(\alpha) = 0$ and $\text{pint}^*(\beta) = 0$. Therefore α and β are fuzzy supra pre σ -nowhere dense sets in (X, T^*) . Clearly $(\alpha \vee \beta)$ is a fuzzy supra pre F_σ -set in (X, T^*) . On computation, $\text{pint}^*(\alpha \vee \beta) = (B \wedge C) \neq 0$, implies that $(\alpha \vee \beta)$ is not a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

Proposition 3.9: In a fuzzy supra topological space (X, T^*) a fuzzy set A is fuzzy supra pre σ -nowhere dense set in (X, T^*) if and only if $1 - A$ is a fuzzy supra pre dense and fuzzy supra pre G_δ -set in (X, T^*) .

Proof: Let A be a fuzzy supra pre σ -nowhere dense set in (X, T^*) . Then $A = \bigvee_{i=1}^{\infty} A_i$, where $1 - A_i \in T^*$ for $i \in I$ and $\text{pint}^*(A) = 0$. Then $1 - \text{pint}^*(A) = 1 - 0 = 1$ implies that $\text{pcl}^*(1 - A) = 1$. Also $(1 - A) = 1 - \bigvee_{i=1}^{\infty} A_i = \bigwedge_{i=1}^{\infty} (1 - A_i)$ where

$(1 - A_i) \in T^*$, for $i \in I$. Hence we have $(1 - A)$ is a fuzzy supra pre dense and fuzzy supra pre G_δ -set in (X, T^*) .

Conversely, let A be a fuzzy supra pre dense and fuzzy supra pre G_δ -set in (X, T^*) . Then $A = \bigwedge_{i=1}^{\infty} (A_i)$ where $(A_i) \in T^*$, for $i \in I$. Now $(1 - A) = 1 - \bigwedge_{i=1}^{\infty} (A_i) = \bigvee_{i=1}^{\infty} (1 - A_i)$. Hence $1 - A$ is a fuzzy supra pre F_σ -set in (X, T^*) and $\text{pint}^*(1 - A) = 1 - \text{pcl}^*(A) = 1 - 1 = 0$. [Since A is a fuzzy supra pre dense]. Therefore $1 - A$ is a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

Proposition 3.10: If A is fuzzy supra pre dense set in (X, T^*) such that $B \leq (1 - A)$, where B is a fuzzy supra pre F_σ -set in (X, T^*) , then B is a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

Proof: Let A be a fuzzy supra pre dense set in (X, T^*) such that $B \leq (1 - A)$. Now $B \leq (1 - A)$ implies that $\text{pint}^*(B) \leq \text{pint}^*(1 - A) = 1 - \text{pcl}^*(A) = 1 - 1 = 0$

and hence $\text{pint}^*(B) = 0$. Therefore B is a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

Proposition 3.11: If A is a fuzzy supra pre F_σ -set and fuzzy supra pre nowhere dense set in (X, T^*) , then A is a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

Proof: Now $A \leq \text{pcl}^*(A)$ for any fuzzy set in (X, T^*) . Then, $\text{pint}^*(A) \leq \text{pint}^* \text{pcl}^*(A)$. Since A is a fuzzy supra pre nowhere dense set in (X, T^*) , $\text{pint}^* \text{pcl}^*(A) = 0$ and hence $\text{pint}^*(A) = 0$ and A is a fuzzy supra pre F_σ -set implies that A is a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

Definition 3.12: Let (X, T^*) be a fuzzy supra topological space. A fuzzy set A in (X, T^*) is called fuzzy supra pre σ -first category if $A = \bigvee_{i=1}^{\infty} (A_i)$ where (A_i) 's are fuzzy supra pre σ -nowhere dense sets in (X, T^*) . Any other fuzzy set in (X, T^*) is said to be fuzzy supra pre σ -second category in (X, T^*) .

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Definition 3.13: Let A be a fuzzy supra pre σ -first category set in (X, T^*) . Then $1 - A$ is called a fuzzy supra pre σ -residual set in (X, T^*) .

Definition 3.14: A fuzzy supra topological space (X, T^*) is called fuzzy supra pre σ -first category if the fuzzy set 1_X is a fuzzy supra pre σ -first category set in (X, T^*) . That is, $1_X = \bigvee_{i=1}^{\infty} (A_i)$ where (A_i) 's are fuzzy supra pre σ -nowhere dense sets in (X, T^*) . Otherwise, (X, T^*) will be called a fuzzy supra pre σ -second category space.

Proposition 3.15: If A is a fuzzy supra pre σ -first category set in (X, T^*) , then there is a fuzzy supra pre F_{σ} -set B in (X, T) such that $A \leq B$.

Proof: Let A be a fuzzy supra pre σ -first category set in (X, T^*) . Then $A = \bigvee_{i=1}^{\infty} (A_i)$, where (A_i) 's are fuzzy supra pre σ -nowhere dense sets in (X, T^*) . Now $[1 - p\,cl^*(A_i)]$'s ($i=1$ to ∞) are fuzzy supra pre open sets in (X, T^*) . Then $C = \bigwedge_{i=1}^{\infty} (1 - p\,cl^*(A_i))$ is a fuzzy supra pre G_{δ} -set in (X, T^*) and $1 - C = 1 - [\bigwedge_{i=1}^{\infty} (1 - p\,cl^*(A_i))] = [\bigvee_{i=1}^{\infty} p\,cl^*(A_i)]$.

Now $(A_i) \leq p\,cl^*(A_i)$, implies that $\bigvee_{i=1}^{\infty} (A_i) \leq [\bigvee_{i=1}^{\infty} p\,cl^*(A_i)]$. Hence $A = \bigvee_{i=1}^{\infty} (A_i) \leq [\bigvee_{i=1}^{\infty} p\,cl^*(A_i)] = [1 - B]$. That is, $A \leq [1 - B]$ and $[1 - B]$ is a fuzzy supra pre F_{σ} -set in (X, T^*) . Let $B = [1 - C]$. Hence, if A is a fuzzy supra pre σ -first category set in (X, T^*) , then there is a fuzzy supra pre F_{σ} -set B in (X, T) such that $A \leq B$.

Proposition 3.16: If A is a fuzzy supra pre σ -first category set in (X, T^*) , then there is a fuzzy supra pre F_{σ} -set B in (X, T^*) such that $A \leq B \leq p\,cl^*(A)$, where B is a fuzzy supra pre F_{σ} -set in (X, T^*) .

Proof: Let A be a fuzzy supra pre σ -first category set in (X, T^*) . Then $A = \bigvee_{i=1}^{\infty} (A_i)$, where (A_i) 's are fuzzy supra pre σ -nowhere dense sets in (X, T^*) . Now $[1 - p\,cl^*(A_i)]$'s ($i=1$ to ∞) are fuzzy supra pre open sets in (X, T^*) . Then

$B = \bigwedge_{i=1}^{\infty} (1 - p\,cl^*(A_i))$ is a fuzzy supra pre G_{δ} -set in (X, T^*) and

$$1 - B = 1 - [\bigwedge_{i=1}^{\infty} (1 - p\,cl^*(A_i))] = [\bigvee_{i=1}^{\infty} p\,cl^*(A_i)].$$

Now

$$A = \bigvee_{i=1}^{\infty} (A_i) \leq [\bigvee_{i=1}^{\infty} p\,cl^*(A_i)] = [1 - B].$$

That is, $A \leq [1 - C] \leq p\,cl^*(A)$ and $[1 - A]$ is a fuzzy supra pre F_{σ} -set in (X, T^*) . Let $B = [1 - C]$. Hence, if A is a fuzzy supra pre σ -first category set in (X, T^*) , then there is a fuzzy supra pre F_{σ} -set B in (X, T^*) such that $A \leq B \leq p\,cl^*(A)$, where B is a fuzzy supra pre F_{σ} -set in (X, T^*) .

Proposition 3.17: If A is a fuzzy supra pre closed set in a fuzzy supra topological space (X, T^*) and if $pint^*(A) = 0$, then A is a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

Proof: Let A be a fuzzy supra pre closed set in (X, T^*) . Then we have $p\,cl^*(A) = A$. Now $pint^*[p\,cl^*(A)] = pint^*(A)$ and $pint^*(A) = 0$,

implies that A is a fuzzy supra pre σ -nowhere dense set in (X, T^*) .

4. FUZZY SUPRA PRE σ -BAIRE SPACE

Definition 4.1: Let (X, T^*) be a fuzzy supra topological space. Then (X, T^*) is called a fuzzy supra pre σ -Baire Space if $pint^*(\bigvee_{i=1}^{\infty} (A_i))$ where (A_i) 's are fuzzy supra pre σ -nowhere dense sets in (X, T^*) .

Example 4.2: Let $X = \{a, b, c\}$ be a set with a fuzzy supra topology (X, T^*) . Then the fuzzy sets $A = \{a_{.7}, b_{.6}, c_{.7}\}$, $B = \{a_{.6}, b_{.8}, c_{.8}\}$, $C = \{a_{.7}, b_{.5}, c_{.9}\}$

Then

$$T^* = \{0, A, B, C, A \vee B, A \vee C, B \vee C, B \wedge C, 1 - A, 1 - B, 1 - C, 1 - A \vee B, 1 - B \vee C, 1 - B \wedge C, 1\}$$

Now, consider the fuzzy set $\alpha = (1 - A \vee B) \vee (1 - B \vee C) \vee (1 - B \wedge C)$ in (X, T^*) . Then α is fuzzy supra pre F_{σ} -set in

(X, T^*) and $\text{pint}^*(\alpha) = 0$ and hence α is a fuzzy supra pre σ – nowhere dense set in (X, T^*) .

$$\beta = (1 - B) \vee (1 - A \vee C) \vee (1 - B \vee (A \wedge C))$$

and $\text{pint}^*(\beta) = 0$ and hence β is a fuzzy supra pre σ – nowhere dense set in (X, T^*) . Then α and β are fuzzy supra pre σ – nowhere dense set in (X, T^*) and also $\text{pint}^*(\alpha \vee \beta) = 0$ and therefore (X, T^*) is a fuzzy supra pre σ – Baire Space.

Proposition 4.3: Let (X, T^*) be a fuzzy supra topological space. Then the following are equivalent:
 (X, T^*) is a fuzzy supra pre σ – Baire Space.

$\text{pint}^*(A) = 0$ for every fuzzy supra pre σ – first category set A in (X, T^*) .

$\text{pcl}^*(B) = 1$ for every fuzzy supra pre σ – residual set B in (X, T^*) .

Proof: (1) $\Rightarrow$ (2). Let A be a fuzzy supra pre σ – first category set A in (X, T^*) . Then $A = (\bigvee_{i=1}^{\infty} (A_i))$ where (A_i) 's are fuzzy supra pre σ – nowhere dense sets in (X, T^*) . Then, we have $\text{pint}^*(A) = \text{pint}^*(\bigvee_{i=1}^{\infty} (A_i))$.

Since (X, T^*) is a fuzzy supra pre σ – Baire Space, $\text{pint}^*(\bigvee_{i=1}^{\infty} (A_i)) = 0$. Hence $\text{pint}^*(A) = 0$ for any fuzzy supra pre σ – first category set A in (X, T^*) .

(2) $\Rightarrow$ (3). Let B be a fuzzy supra pre σ – residual set B in (X, T^*) . Then $(1 - B)$ is a fuzzy supra pre σ – first category set B in (X, T^*) . By hypothesis, $\text{pint}^*(1 - B) = 0$. Then $1 - \text{pcl}^*(B) = 0$. Hence $\text{pcl}^*(B) = 1$ for any fuzzy supra pre σ – residual set B in (X, T^*) .

(3) $\Rightarrow$ (1). Let A be a fuzzy supra pre σ – first category set A in (X, T^*) . Then $A = (\bigvee_{i=1}^{\infty} (A_i))$ where (A_i) 's are fuzzy supra pre σ – nowhere dense sets in (X, T^*) . Now A is a fuzzy supra pre σ – first category set in (X, T^*) implies that $(1 - A)$ is a fuzzy supra pre σ – residual set in (X, T^*) . By hypothesis, we have $\text{pcl}^*(1 - A) = 1$. Then $1 - \text{pint}^*(A) = 1$. Hence $\text{pint}^*(A) = 0$. That is, $\text{pint}^*(\bigvee_{i=1}^{\infty} (A_i)) = 0$

where A_i 's are fuzzy supra pre σ – nowhere dense sets in (X, T^*) . Hence (X, T^*) is a fuzzy supra pre σ – Baire Space.

Proposition 4.4: If $\text{pcl}^*(\bigwedge_{i=1}^{\infty} (A_i)) = 1$, where (A_i) 's are fuzzy supra pre dense and fuzzy supra pre G_δ – sets in (X, T^*) , then (X, T^*) is a fuzzy supra pre σ – Baire Space.

Proof: Now $\text{pcl}^*(\bigwedge_{i=1}^{\infty} (A_i)) = 1$ implies that $1 - \text{pcl}^*(\bigwedge_{i=1}^{\infty} (A_i)) = 0$. Then we have $\text{pint}^*(1 - \bigwedge_{i=1}^{\infty} (A_i)) = 0$, which implies that $\text{pint}^*(\bigvee_{i=1}^{\infty} (1 - A_i)) = 0$. Let $B_i = 1 - A_i$. Then $\text{pint}^*(\bigvee_{i=1}^{\infty} (B_i)) = 0$. Since A_i is a fuzzy supra pre dense and fuzzy supra pre G_δ – set in (X, T^*) , by proposition 3.9, $1 - A_i$ is a fuzzy supra σ – nowhere dense set in (X, T^*) . Hence $\text{pint}^*(\bigvee_{i=1}^{\infty} (B_i)) = 0$, where (B_i) 's are fuzzy supra σ – nowhere dense set in (X, T^*) . Therefore (X, T^*) is a fuzzy supra σ – Baire Space.

Proposition 4.5: If the fuzzy supra topological space (X, T^*) is a fuzzy supra pre σ – Baire Space, then (X, T^*) is a fuzzy supra pre σ – second category space.

Proof: Let (X, T^*) is a fuzzy supra pre σ – Baire Space. Then $\text{pint}^*(\bigvee_{i=1}^{\infty} (A_i)) = 0$ where (A_i) 's are fuzzy supra pre σ – nowhere dense set in (X, T^*) . Then $\bigvee_{i=1}^{\infty} (A_i) \neq 1_X$ [otherwise, $\bigvee_{i=1}^{\infty} (A_i) = 1_X$ implies that $\text{pint}^*(\bigvee_{i=1}^{\infty} (A_i)) = \text{int}^* 1_X = 1_X$, which implies that $0 = 1$, a contradiction]. Hence (X, T^*) is a fuzzy supra pre σ – second category space.

Proposition 4.6: If the fuzzy supra topological space (X, T^*) . If $\bigwedge_{i=1}^{\infty} (A_i) \neq 0$, where (A_i) 's are fuzzy supra pre dense and fuzzy supra pre G_δ – set in (X, T^*) , then (X, T^*) is a fuzzy supra pre σ – second category space.

Proof: Now $\bigwedge_{i=1}^{\infty} (A_i) \neq 0$ implies that $1 - (\bigwedge_{i=1}^{\infty} (A_i)) \neq 1 - 0 = 0$. Then we have $(\bigvee_{i=1}^{\infty} (1 - A_i)) \neq 1$. Since A_i is a fuzzy supra pre dense and fuzzy supra G_δ – set in (X, T^*) , by proposition 3.9, $(1 - A_i)$ is a fuzzy supra pre σ – nowhere dense set in (X, T^*) . Hence

$(\bigvee_{i=1}^{\infty} (1 - A_i)) \neq 1$, where $(1 - A_i)$'s are fuzzy supra pre σ -nowhere dense set in (X, T^*) . Hence (X, T^*) is not a fuzzy supra pre σ -first category space. Therefore (X, T^*) is a fuzzy supra pre σ -second category space.

Proposition 4.7: If the fuzzy supra topological space (X, T^*) is a fuzzy supra pre σ -first category space, then (X, T^*) is not a fuzzy supra pre σ -Baire Space.

Proof: Let the fuzzy supra topological space (X, T^*) is a fuzzy supra pre σ -first category space. Then $\bigvee_{i=1}^{\infty} (A_i) = 1_X$, where (A_i) 's are fuzzy supra pre σ -nowhere dense sets in (X, T^*) . Now $\text{pint}^*(\bigvee_{i=1}^{\infty} (A_i)) = \text{pint}^*(1_X) = 1 \neq 0$. Hence by definition, (X, T^*) is not a fuzzy supra pre σ -Baire Space.

Proposition 4.8: If the fuzzy supra topological space (X, T^*) is a fuzzy supra pre σ -Baire Space and if $\bigvee_{i=1}^{\infty} (A_i) = 1$, then there exists atleast one fuzzy supra pre F_{σ} -set A_i such that $\text{pint}^*(A_i) \neq 0$.

Proof: Suppose that $\text{pint}^*(A_i) = 0$, for $i = 1$ to ∞ , where (A_i) 's are fuzzy supra pre σ -nowhere dense sets in (X, T^*) . Then $\bigvee_{i=1}^{\infty} (A_i) = 1$. Implies that $\text{pint}^*[\bigvee_{i=1}^{\infty} (A_i)] = \text{pint}^*[1] = 1 \neq 0$, a contradiction to (X, T^*) being a fuzzy supra pre σ -Baire Space. Hence $\text{pint}^*(A_i) \neq 0$, for at least one fuzzy supra pre F_{σ} -set A_i in (X, T^*) .

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