Will Mathematics Ultimately Describe Nature?

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It has been almost eighty years since Paul Dirac delivered a lecture on the relationship between mathematics and physics and since 1960 that Eugene Wigner wrote about the unreasonable effectiveness of mathematics in the natural sciences. The field of cosmology and efforts to define a more comprehensive theory (String Theory) have changed significantly since the 1960s; thus, it is time to refocus on the issue. This paper expands on ideas addressed by these two great physicists, specifically, the ultimate effectiveness of mathematics to describe nature.

After illustrating how theoretical physics predicted discoveries, the concepts of established theories are summarized. Then, the “world equation” which combines all key physics theories is conceptually described. As the possible last piece of the puzzle, String Theory is briefly defined. And last, a cursory overview of an extreme proposal is presented — the world as a mathematical object. However, there is no fundamental reason to believe that math will allow mankind to completely comprehend nature. If math does not provide a comprehensive theory, nature will retain her secrets.

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Introduction

Paul Dirac in 1939, said: “The physicist, in his study of natural phenomena, has two methods of making progress: The method of experiment and observation, and the method of mathematical reasoning. The former is just the collection of selected data; the later enables one to infer results about experiments that have not been performed. There is no logical reason why the second method should be possible at all, but one has found in practice that it does work and meets with reasonable success. This must be ascribed to some mathematical quality in Nature, a quality which the casual observer of nature would not suspect, but which nevertheless plays an important role in Nature’s scheme” [Dirac, 1939: 122-129].

In a subsequent, more frequently referenced article, Eugene Wigner reinforced this idea, “The enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it” [Wigner, 1960].

Why does mathematics, based on abstract reasoning, explain the real world? We may not be able to explain *why* (a philosophical issue) but we can elaborate on *how*. The fundamental physics theories explaining nature are: Electromagnetism (EM), Special Relativity (SR), General Relativity (GR), Cosmology (COSM), Quantum Mechanics (QM), Quantum Field Theory (QFT), and String Theory (ST).

After documenting twenty noteworthy predictions based on theoretical physics, our task is to analyze, in a conceptual context, future possibilities. Explaining the “world equation”, provides insight to the complexities involved. Next, we explore ST as a plausible solution. And last, the absolute limit of math’s effectiveness — the world as a mathematical object.

Predictions

In 2012 the Higgs particle (and Higgs field), predicted by Peter Higgs in 1964, was discovered by the Large Hadron Collider (LHC). The math, based on symmetry, predicted massless force carriers (bosons) for the weak force identical to the electromagnetic and strong force carriers; but the force carriers (W and Z particles) did have mass so symmetry was lacking. The solution was to propose the Higgs particle which gave the weak force carriers mass. Forty-eight years after the prediction, the Higgs particle was verified.

This discovery was one of many predicted by theoretical calculations; note nineteen others listed in Table One (selected discoveries, not a complete list). One-half of the predictions are new particles and surprisingly, the average time between prediction and discovery is twenty-two years. The first on the list is the prediction of radio waves (unsuspected forms of radiation) based on Maxwell’s equations describing electromagnetism. As Brian Clegg states: “Maxwell was the first in the field of science who put forward a theory where the mathematics operated in true abstraction from reality His results are incomprehensible to the casual observer” [Clegg, 2016: 146].

In 1919, one of the predictions of general relativity was confirmed when an eclipse showed how star light is bent by the gravitational force of the sun. Einstein confidently noted that had the results been different, “he would have been sorry for the dear lord, since the theory is correct” [Greene, 2011: 320]. If still alive, Einstein might have made a similar statement in 2016 when gravitational waves were discovered by Interferometer Gravitational-Wave Observatory (LIGO), one-hundred years after their prediction. Also, the “observation” evidence of black holes around 1970 verified theoretical calculations of General Relativity.

Both the discovery of the Hubble constant in 1929 and Cosmic Microwave Background (CMB) in 1967 substantiate the expanding universe theory, a central premise of cosmology. The discoveries changed the way scientists envisioned the universe.

Quantum Field Theory (QFT), which includes Quantum Electrodynamics (QED) and Quantum Chromodynamics (QCD), encompass the remaining predictions listed. Most predictions of unknown fields were based on theoretical mathematics and symmetry. As Frank Wilczek states, “There are many other recent examples where new forms of matter appeared in our equations before they appeared in our laboratories. In fact, it’s become the usual case. Quarks, color gluons W and Z bosons, and all three kinds of neutrinos were first seen as solutions of equations, and only later as physical realities” [Wilczek, 2008: 187].

One of the more surprising predictions is non-locality/entanglement. Quoting Brian Greene, “According to quantum theory and the many experiments that bear out its predictions, the quantum connection between two particles can persist even if they are on opposite sides of the universe While intuitively baffling, this phenomenon fully conforms to the laws of quantum mechanics, and was predicted using quantum mechanics long before the technology

existed to do the experiment and observe, remarkably that the prediction is correct” [Greene, 2004: 80].

The last entry is the prediction of supersymmetry, which has not been confirmed. As background, “Supersymmetry is a particular kind of symmetry. The transformations of supersymmetry involve displacement, or translation, in a quantum dimension. When a force particle (boson) moves into a quantum dimension, it becomes a substance particle (fermion), and vice versa” [Wilczek, 2015: 390]. Verification via the LHC would quell continued speculation about supersymmetry.

This list of discoveries reinforces Dirac’s comment that mathematical reasoning has met with reasonable success and also reinforces Wigner’s statement that the usefulness of math borders on the mysterious.

Theories Impacted by Predictions

The predictions in Table One impacted various fundamental theories as shown in the last column. Brief definitions of the theories follow.

Special Relativity (SR) limits the velocity of light and establishes a maximum speed limit for matter and energy. An equivalence between energy and mass ($E = mc^2$), a key equation in physics defined by Einstein in 1905, would not be possible without this speed limit.

General Relativity (GR) incorporates the equivalence principle which equates the force of gravity and the force of acceleration, a surprising relationship. Einstein’s equations unified space, time, energy, and gravity. As John Wheeler once said, “Spacetime tells matter how to move, matter tells spacetime how to curve” [Wheeler, 2000: 235]. There is no way to hide from gravity.

Electromagnetism (EM) is exemplified by the four laws of electromagnetism: Gauss’s Law — electric charges act as sources for generating electric fields and electric fields exert forces that accelerate electric charges; Ampere’s Law — moving electric charges constitute electric currents which act as sources for generating magnetic fields; Faraday’s law and Maxwell’s law — time varying electric fields induce magnetic fields and, conversely, time varying magnetic fields induce electric fields; and last — light consists of time-varying electric and magnetic fields that propagate as a wave and interact with matter by accelerating charged particles and, in turn, accelerating charged particles emit electromagnetic radiation. The first three laws are defined by Maxwell’s equations.

Quantum mechanics (QM) encompasses a framework for explaining phenomena at the microscopic level. It was mostly in place by the late 1930s. Quantum mechanics is a completely non-intuitive concept, entertains bizarre principles, specifically: the uncertainty principle (based on Planck’s constant); Pauli Exclusion Principle; probability waves (wave particle duality); and, entanglement or non-locality. Quantum mechanics is required to construct atoms, “little particles that move around in perpetual motion” (Feynman description). These clever concepts allow elements, molecules, and subsequently biological life. Generating or absorbing photons as electrons move from one energy state to another is an innovative and also required notion.

Quantum field theory (QFT) is relativistic quantum mechanics. The two QFT theories, Quantum Electrodynamics (QED), developed in the 1925-30 period, and Quantum Chromodynamics (QCD), defined in the early 1970s, both successfully predict particle and force interactions to extreme precision. Later, when linking math to physics theories and when explaining equations, *no distinction* is made between quantum mechanics and quantum field theory since the latter supersedes the former. For simplicity, *both are referred to as QM in the following text.*

Cosmology (COSM) is the study of the origin, evolution, and eventual fate of the universe. It follows from general relativity and quantum mechanics. The more speculative area of cosmology overlaps with philosophy considering how initial conditions were established and how energy came into existence, for example via inflation, cyclic/bounce process, or just always existed [Johnson, 2016: 97-98]. Thus, it is separate from GR because it contains initial conditions.

String Theory (ST) is the next possible fundamental theory encompassing GR, COSM, and QM. ST solves the mathematical and conceptual incompatibilities between GR and QM. It also has options for cosmology (initial conditions).

All Physics “World Equation”

This section defines the terms for one equation which embodies virtually all the fundamental theories. In pursuit of super unification, physicists have produced a one-line formula summarizing all known physics — the world in an equation, an equation without equal in all of science [Turok, 2012: 167]. Although the equation is contained on one line, the sophistication of math is significant; for our purpose consider the equation a conceptual expression:

$$\text{All physics world equation: } \Psi = \int e \left(\text{exponent} \right) \frac{i}{\hbar} \int \left(R/(16\pi G) - \frac{1}{4} F^2 + \bar{\Psi} i \not{D} \Psi - \lambda \phi \bar{\Psi} \Psi + 1/2 D\phi^2 - V(\phi) \right)$$

This is what is called the path-integral formulation of quantum mechanics, pioneered by Richard Feynman. The wave function describes a superposition of every possible configuration of the system ... Feynman’s version of quantum evolution (which is equivalent to Schrodinger’s, just written differently) tells you how the system will end up in a particular configuration” [Carroll, 2016: 437].

The world equation is viewed as graduate-level physics incorporating highly sophisticated mathematics. Physicist seldom use the complete formula [Turok, 2012]. The names of the scientists primarily responsible for pieces of the equation are: Schrodinger (Ψ = wave function); Feynman ($\int$ = sum of all histories); Euler (e); Planck ($\hbar$ = *Planck* constant); Hamilton ($\int$ = sum of six terms) number representing all the known physical laws: Einstein, Newton, Maxwell-Yang-Mills, Dirac, Higgs, and Yukawa-Kobayashi-Maskawa [Turok, 2012: 167-176].

Even presented in this condensed version, it appears mysterious, so what can we conclude? On one hand, the equation underlying our everyday lives is precise, ridged, and well defined with no ambiguity — an accurate description of nature and one of the greatest triumphs of science [Carroll, 2016: 441]. “The laws of physics underlying everyday life are completely known” [Carroll, 2016: 177]. But, beyond everyday life, mysteries exist; for example, the equation is *not accurate at all energy levels, requires 19 adjustable parameters, and, excludes strong gravitational effects, dark matter, neutrinos, and possibly yet to be discovered particles*. Will physicists “fix” these deficiencies by modifying existing theories or will a new theory replace them? “As science continues to learn more about the universe, we will keep adding to it [QM or Standard Model of Particle Physics], and perhaps we will even find a more comprehensive theory underlying it that doesn’t refer to quantum field theory at all” [Carroll, 2016: 442].

Last Piece of the Puzzle 0— String Theory, Reconciling GR and QM

Obviously since scientists have been working over 90 years to reconcile GR and QM (in Einstein's time called a unified theory), there are serious issues to overcome. What are the differences between GR and QM that need resolution? The crux of the problem is summarized by Gene Wigner: "Quantum mechanics and general relativity operate in different math concepts: four dimensional Riemann space and infinite dimensional Hilbert space" [Wigner, 1960]. In addition to dimensions, there are other inherent differences between GR and QM. Probabilities rule in QM and are absent in GR. Space and time are continuous in the world of GR, but in the world of QM, energy and mass are quantized using Planck's constant. Also, "the uncertainty principle ensures that even the vacuum of empty space is a teeming, roiling frenzy of virtual particles momentarily erupting into existence and subsequently annihilating one another" [Greene, 1999: 337]. As Feynman once jested, "Created and annihilated, created and annihilated, what a waste of time."

A second motivation for ST, is expressed by Brian Greene: "Because the gravitational field is woven within the very fabric of spacetime, its quantum jitters shake the entire structure through and through. When used to analyze such pervasive quantum jitters, the mathematical methods collapsed. For years, physics turned a blind eye to this problem because it surfaces only under the most extreme conditions ... when gravity and quantum mechanics are together brought to bear on either the big bang or black hole, realms that *do* involve extremes of enormous mass squeezed to a small size, the math falls apart at a critical point in the analysis" [Greene, 2011: 77].

Also another goal of ST is to explain why particles have their particular masses and electric charges, values that are entered into existing theories based on experimentally measured values.

Many physicists believe ST is the answer. In ST, tiny strings or vibrating filaments replace electrons and quarks as nature's building blocks. Strings are so minute they may never be observed (size on order of Planck distance of 10^{-33} cm). The string vibration pattern dictates intrinsic features that may represent an electron or a quark or more importantly a graviton (massless, charge less, and having a spin-2 quantum property). Thus, without contradicting previous theories, ST bridges the gap between general relativity and QM. However, the mathematics, as defined in five unique theories, require nine (ten for M-Theory) rather than three dimensions for space. The extra dimensions are curled up into Calabi-Yau shapes, shapes that dictate particle properties.

"But most researchers feel that our current formulation of ST still lacks the kind of core principle we find at the heart of other major advances. SR has the consistency of the speed of light. GR has the equivalence principle. QM has the uncertainty principle. String theorists continue to grope for an analogous principle that would capture the theory's essence as completely" [Greene, 2004: 376]. However, if the LHC confirms supersymmetry (SUSY), an essential part of ST, more physicists might agree ST is the correct theory.

Others express a more negative view by asking if we have gone too far: "After piling layer on layer of abstract mathematics, theoreticians managed to get string theory working" [Clegg, 2016: 236]. And in a similar vein, "The mathematics used in these studies [ST] is becoming more and more advanced. Not only are ordinary numbers replaced by an extended class of numbers known as Grassmann numbers [they satisfy a different multiplication law, $xy = -yx$] ..., ordinary geometry is also superseded by a special branch known as

noncommutative geometry” [Livio, 2005: 231]. In addition, ST uses abstract math, such as Lie groups and knot theory. However, the ST community view the new math challenges as a positive development. Advanced mathematical topics studied by ST physicists include: differential forms, homology, cohomology, fiber bundles, characteristic classes, index theorems, and K-theory [Schwarz, 2018].

Extreme Effectiveness, the World as a Mathematical Object

Both Dirac and Wigner marveled at the effectiveness of how math explains nature. Math is fascinatingly unique because, although there are about 6,500 languages designed to communicate, there exists only *one* language for mathematics [Livio, 2009: 239]. However, the extent of how completely math reflects nature is disputed. From an overall perspective, there are two divergent positions on how effectively math explains the world. One emphasizes its limits: “Mathematics is indeed extraordinarily effective for some descriptions, especially those dealing with fundamental science, but it cannot describe our universe in all its dimensions” [Livio, 2005: 252]. Thus, inanimate theories like biology and psychology may lie outside its scope. Another more basic but not necessarily obvious problem, how does math compute with real numbers that have decimals which extend indefinitely? Wilczek states his opinion on this issue: “It seems deeply mysterious, at least to me” [Wilczek, 2015: 384]. Greene also addresses this question: “For the laws of physics to be computable ... the traditional reliance on real numbers would have to be abandoned” [Greene, 2011: 352]. Thus, values would have to be discrete or a new type of math invented.

The other perspective, an extreme view, is proposed by Max Tegmark “Our physical world is one giant mathematical object” [Tegmark, 2014: 246]. Tegmark continues (paraphrasing): Clues suggesting that nature isn’t just described by mathematics, but that some aspects *are* mathematical. Space itself is a purely mathematical object in the sense that its only intrinsic properties are numbers, such as, dimensionality, curvature, and topology. Elementary particles are also mathematical, described by charge, spin, and lepton number. The wave function (Schrodinger) and Hilbert space (an infinite-dimensional place) are purely mathematical objects [Tegmark, 2014: 253]. Proving physical reality is a mathematical object or that math will eventually explain all of nature is a difficult and likely impossible task.

Frank Wilczek provides a supporting perspective: “Atoms are the solutions of beautiful equations.” Using these equations, a computer can predict any property of an atom that can be measured. “Nothing else is required. In a precise sense, the atoms *embody* the equations” [Wilczek, 2015: 191]. Reinforcing this idea, Frank Wilczek tells a story: “My laptop computer, which to me is an appendage to my brain, was stolen. I was devastated. But then a miracle occurred. I had all my data backed up, and within a few days I had a new laptop with everything restored — pictures, words, calculations, music, and so on. All those things had been encoded in numbers — strings of 0s and 1s — so faithfully that they could be made to reappear without noticeable modification. It occurred to me that one could hardly ask for a more tangible, direct, or impressive demonstration of the truth of Pythagoras’s vision: All Things Are Number” [Wilczek, 2015: 324].

Paul Dirac encapsulates a similar philosophy: “It seems to be one of the fundamental features of nature that fundamental physical laws are described in terms of mathematical theory of great beauty and power, needing quite a high standard of mathematics for one to understand it. ... One could perhaps describe the situation by saying that God is a mathematician of a very high order and he used very advanced mathematics in constructing the universe. Our feeble attempts at mathematics enable us to understand a bit of the universe,

and as we proceed to develop higher and higher mathematics we can hope to understand the universe better” [Dirac, 1963].

To summarize this section, math may be more than just a sophisticated tool; it might explain all of reality or might be reality (Max Tegmark proposal), a philosophical concept to ponder?

Conclusion

Mankind’s journey to decipher nature had a slow start, requiring 2,000 years to define and explain our observed world with classical mechanics (1800’s). Another 100 years of science was required to discover the macro world of relativity, and subsequently, the micro world of QM (1926). Now over 90 years later, as we refocus on the issue many questions remain. For example, the “world equation” in Sean Carroll’s words is “one of the greatest triumphs of human intellectual history” [Carroll, 2016: 441], but it still has limitations: not accurate at all energy levels, requires 19 adjustable parameters, etc.

Frank Wilczek validates this concern, although answering the question “Does the world embody beautiful ideas?” in the affirmative, he goes on to say: “Nor are all the truths of deep reality beautiful. The core [Standard model] has many loose ends, and there is little prospect of tying them all up. Even if my dreams of axions, supersymmetry, and unification are fulfilled, the messy (non-) pattern of quark and lepton masses and the conceptually opaque dark energy will remain problematic for the foreseeable future” [Wilczek, 2015: 320]. What is the probability of scientists discovering a comprehensive theory overcoming these shortcomings in say 50 years?

Are we in a position where significant advances in theory development is over? The science world in the 1920s thought that there was nothing new to discover, but they were shocked by QM and a host of observational discoveries in cosmology. However, a reconciling QM and GR is an elusive, problematic topic, and ST, as the primary theory proposed, is an unverified theory with sophisticated math (non-commutative geometry, knot theory, Grassmann numbers, and others).

Although we have shown math is effective in describing nature, *there is no fundamental reason* to believe that mathematical tools will be found to lead the way forward and allow mankind to comprehend nature beyond GR, COSM, and QM. Are GR and QM inherently incompatible? Will the magic of math expire? ST may be the answer, but if it is not, will math enable an alternative theory? If nature wants to retain her secrets, they will remain a mystery.

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