Multi-Objective Optimization on Daylight Control System's Qualitative and Quantitative Performance

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Abstract

In recent years, the parametric design has become a great interest to architects. Especially in a building envelope, daylight control systems (e.g. shades, reflectors) take a method of continuous deformation until the architect finds interesting patterns or shapes that satisfy the architect's favour. However, those new design approaches bring a controversial question with a reasonable sense of priority. In some cases, aesthetic variables control the true purpose of the daylight system; contrarily, the function-oriented daylight system could mar the visual interest of architects and building users.

This paper would provide a suggestion to this issue by demonstrating how aesthetic preference and daylighting performance could be well balanced by means of Multi-Objective Optimization (MOO). This paper aims to demonstrate how aesthetic preference and visual comfort can be woven together into a daylight control system. In detail, how using Multi-Objective Optimization can find a solution that fulfils both aesthetic satisfaction as a qualitative performance and visual comfort as a quantitative performance.

Introduction

The architectural design process has undergone many changes since the assistance of computer science has been introduced. The first trial had been studied for hospital planning by Souder and Clark (1963). After initial footage of the innovation, the Computer Aided Drafting (CAD) system has been making further progress in computerization (Jabi, 2013; Woodbury, 2010).

One of the first pioneers that used computers to generate parametric architecture was Greg Lynn. The early examples of computer-generated parametric architecture are his blob and fold architecture. Italian architect Massimiliano Fuksas' work is another example of parametric design with the support of the engineering firm, Knippers Helbig, to create a large-scale building (Helbig, T., et al., 2014; Lynn 1998).

Advances in computer science also encourage optimization in the field of architecture. There are several approaches developed for optimization in architectural design including space, programming, and building cost. It should be acknowledged that Radford and Gero (1980) had organized the building optimization field as a whole and laid a basic foundation for the architecture field in

1980. Since then, the building optimization has been increasingly used.

Recently, new technologies have been able to capture the unknown natural phenomena. However, the solution to older problems has opened the door to more complex ones in nature. Single objective optimization methods were not enough to solve these complex questions, requiring the development of multi-optimization methods to emerge as a solution to these challenges (Nguyen, et al., 2014; Kheiri, 2018).

Multi-Objective Optimization methods can be identified as two different types. One simple solution is "scalarization," which was produced with simple weight factors to each criterion based on the situations of simulation (Yi and Malkawi, 2009). Another way is using "Pareto front optimization," which originated from the concept of Pareto Optimality. The Pareto Optimal set usually considers a large number of Pareto points in any case. In these days, the MOO with the Genetic Algorithm (GA) seems the most popular set in building simulation technology (Nguyen, et al., 2014).

In recent years, building envelopes and daylight control systems (e.g. shades, reflectors) use a method of continuous deformation until the architect finds interesting patterns or shapes that satisfy the architect's favour with visual comfort. However, those new design methods pose questions in regards to overall satisfaction. Sometimes aesthetic interests trump the function of the daylight system that contrasts its true function. Contrarily, the function-oriented daylight system could mar the visual interest of architects and building users.

In a similar vein, this paper is a continuation of work (Yi, 2019) that covers how to satisfy the aesthetic preference and visual comfort of the shading system in the MOO method. The main issue from the previous paper is the limitation of its application because of the way it measures qualitative performance. This paper aims to improve the measurement of aesthetic preference and apply it to a more practical case that can weave both qualitative and quantitative performance together into one system. The following covers in detail the use of the MOO method to find a solution that fulfils both aesthetic satisfaction as a qualitative performance and visual comfort as a quantitative performance.

Figure 1: Overall process.

Method

Figure 1 shows the overall process of this paper. The process was divided into three main sections, wherein the first step of this approach is created with the use of the Parametric Building Layout (PBL) model and Agent-based Geometry Control System (AGCS). This approach allows the management of geometry points with fewer control points, which means that shifting geometry could be smoother and more efficient compared to changing whole geometry points.

After the geometry stage is constructed, the second step is to evaluate the model to obtain both qualitative and quantitative performance. Figure 1 shows how the qualitative and quantitative performance can be measured through the simulation. This paper uses the daylight measurement as the objective function for the quantitative measure.

The qualitative performance, which is the aesthetic preference of each geometrical variations, were measured by using an expert system. Questions were asked to users to find their preference of the shade system's geometry configuration. Responses from the users were converted to numerical measurements for aesthetical preferences on the shade design. Those converted numerical values were used for qualitative objective value in the optimization stage.

The final phase is the optimization stage by using obtained performance values from the previous step. The MOO method is used for this paper. The collected

performance data from previous steps was plugged into MOO to find the best rankings among the given populations. After passing the evaluation process, if the result is not satisfied, reproduction will proceed to produce the new population of digital models. This new population will then be passed to step one to regenerate the next generation. The new generation will process the same steps until the target is fulfilled. This process will be continued until the goal is satisfied or the maximum number of generations is met.

In summary, the initial iterations of AGCS in a PBL model are generated and transferred to the next step to revise the shade system. Once the shade geometry was constructed, simulation tools were used to determine the performance and whether they satisfy the objective functions. If the outcome of the objective function did not meet the goals, the next population is generated based on the variation process and transferred to AGCS to generate a new shade layout to evaluate the next generation's performance. This loop continues until it reaches the goal. The following sections discuss each step in detail.

Agent-based Geometry Control System (AGCS)

Creating a PBL model is the first step of generating the shade geometry to determine the performance. In this paper, a PBL model was created based on user-defined variables and constraints that include each shade's hanging direction (horizontal or vertical), position (inside or outside), shaft distribution (even or exponential), width (w), shaft density (low, medium, and high), and angle (a). Once a PBL was generated, the variables of geometry

could be defined through the AGCS. The defining agent points are the core of the AGCS and, by determining the hierarchical communication between the agent points and the variables, control the numerous variables of the PBL model efficiently. A more detailed definition of PBL and AGCS can be found in the author's previous paper (Mardaljevic, 2012).

Quantitative Performance Evaluation

Once the geometry is defined and built, the performances can be evaluated. This study considers the average illuminance level (Lux) and Daylight Glare Probability (DGP) for the quantitative objectives. The illuminance is mostly used to ensure a person's observation and accomplish the visual task. According to the European lighting standard (EN12464-1), the office working area generally requires an illuminance level of 500 lux (2002) and historical recommended illuminance for mid-range of office task is 500-1,000 lux (Osterhaus, 1993).

DGP is the most recent indicator used to classify glare discomfort from daylight (Konstantzos, and Tzempelikos, 2014). The DGP showed a very strong interaction with the user's response regarding glare perception. A recent study, by Wienold, J, suggests that the recommended DGP limit is less than or equal to 0.35 with a 0.38 of average DGP limit within 5% band (Mardaljevic, et al, 2012; Wienold, and Christoffersen, 2006).

This paper sets an objective goal of DGP less than 0.34, and an average illuminance value above 500 lux as two quantitative performance goals. The quantitative optimization goal performed in this paper is only a general guideline that requires a more sophisticated evaluation. Comfort illuminance level can be varied and its nonlinear effect on humans. The reason for the use of this simple measure is to focus more on the optimized methodology with respecting aesthetic preference rather than attempting to investigate only for practical visual comfort.

Qualitative Performance Evaluation

To measure qualitative performance, the paper applied an expert system. An expert system is a computer system that imitates the decision-making ability of a human expert (Jackson, 1998). Expert systems are a type of an inference engine to solve complex problems based on logical reasoning.

Figure 2: Process of expert system (Yi, 2019).

To communicate with the user, the system inquires questions to determine answers from a stored knowledge base by an expert. As seen on Figure 2, the inference engine calculates appropriate answers from input data of the user interface. The following section 3.4 demonstrates how the expert system was modelled and set up for the test. More detail information can be found in the author's previous paper (Yi, 2019).

Multi-Objective Optimization (MOO)

The MOO process starts with the selection of the parents for the next generation using the selection function on the current parent population (k). The next generation (o) is created from the selected parents by mutation and crossover. The next process combining the current population (k) and the children (o) into one cluster, the extended population ($k_{(ex)}$). Extended population ($k_{(ex)}$) computed the rank and crowding distance for all individuals. Based on the ranking, this extended population will be trimmed to retain the appropriate number of individuals of each rank. The process will stop if it meets the goal that, if the generation average of the relative change in the value of the spread over value is less than tolerance, the final spread is less than the mean spread over the past generations or the maximum number of generations is exceeded. If the process does not satisfy the stop condition, the next step is to repeat the first step of the loop to continue the process until it satisfies the goal (Figure 3).

Figure 3: Evolution algorithm selection process (Yi, 2019).

Application to Design Process

Test Model Setup

Figure 4: Schematic Test Building Dimensions.

This study chose a simple building geometry to focus more on the solar shade design optimization. The dimension of this building is 9 x 7 x 3.8 (L x W x H, m) with a 2 by 5 (m) southern facing window. Figure 4 shows

the tested building geometry. The window is illustrated with a grey colour.

Solar Shade PBL Model Setup

As the test building geometry was set, a solar shade was built with the AGCS. The first step was to develop a PBL model based on the variables to create a solar shade geometry. As shown in Figure 5.1, solar shade slats could be arrayed in a horizontal or vertical way based on the user's choice. The second variable is shown in Figure 5.2. This is a simple variable where a user wants to place the solar shade itself. There are two options: inside and outside.

The next variable of the AGCS is a slat distribution strategy. This variable is defined in two ways: exponential distribution or even distribution (Figure 5.3). Thick lines of each variable represent where the slats could be deployed based on the position value. The fourth variable is called the density variable which defines the number of slats in the designated size of the window (Figure 5.4). The thicker lines represent the number of slats.

The next two variables are the changing of each slats' width (Figure 5.5) and slats' angle (Figure 5.6). All those variables defined in this paper are selected examples that showcase how shades can be designed. The definition of geometry variables can be different and can be changed based on the project's location and situations.

Figure 5: Variables and Constraints in a Conceptual PBL Model.

Based on the solar shade geometry variables, a PBL model was used to build the AGCS. Grasshopper is used to build the AGCS model. This algorithm allows for the modification of a solar shade shape based on the given six variables above.

Quantitative Simulation Setup

The quantitative simulation is accomplished with DIVA tool (DIVA, 2019) of Grasshopper which uses Radiance as the light simulation engine. The basic parameter of the daylighting simulation is location, sky condition, date, time, and material property of geometry. The location of this test was set for Chicago. The test selected two different date and time sets for the simulation. These dates are December 21st, and June 21st at noon to make sure the illuminance level and DGP were within the desired range while the sun is located in the worst case during the year. Even though sDA or DA can give an annual performance of the daylight condition. The main reason for using illuminance level and DGP is the limitation of sDA or DA that can make it difficult to find the impact of reflected or diffused daylight through the shading system and it is a significant factor to find an optimal solution from various geometric variables of shading for this paper. The sky condition for the daylight simulation is a clear sky with the sun. Table 1 shows the daylight model boundary conditions and simulation parameters used for the daylight simulation.

Table 1: Radiance setup.

Boundary Condition		
Name	Set up	
Ceiling	void plastic GenericCeiling_70 0 0 5 0.7 0.7 0.7 0 0	standard ceiling reflectance of 70%
Wall	void plastic GenericInteriorWall_50 0 0 5 0.5 0.5 0.5 0 0	standard grey wall reflectivity of 50%
Floor	void plastic GenericFloor_20 0 0 5 0.2 0.2 0.2 0 0	standard floor reflectivity of 20%
Window	Void glass Glazing_DoublePane_L owE_Argon_65 0 0 3 0.71 0.71 0.71	Tau_vis = 0.65 SHGC= 0.27 # visual transmittance: 65% # visual transmissivity: 71%
Solar Shade Shaft	void metal SheetMetal 0 0 5 .9 .9 .9 .8 0	
Simulation Setup		
Illuminc e	-aa .15 -ab 2 -ad 512 -ar 256 -as 128 -dr 2 -ds .2 -lr 6 -lw .004 -dc .75 -dp 2048 -dt .05 -ms 0.063 -st .01	
Glare	-ps 8 -pt .15 -pj .6 -dj 0 -ds .5 -dt .5 -dc .25 -dr 0 -dp 64 -st .85 -ab 2 -aa .25 -ar 16 -ad 512 -as 128 -lr 4 -lw .05 -av 0 0 0	

Expert System Interface

The expert system in this paper asked for a preference look of the test shade. The shade configuration was governed by 6 variables. The query was composed based on six questions to find the user preference regarding parameters that decide the shape of the shade. These six questions can be divided into two groups. The first group is composed of parameters that should be fixed upon the decision of the user. The second group is parameters that can be changed, such as the shaft's width, density, and angle.

Figure 6 shows the interface responding to the query. The query comes with a 3D view that allows the user to explore the façade shape of their choice. This interface is built using a grasshopper plugin called Human UI.

Figure 6: The user interface shows the questions and 3D view of shades on a user's choice.

Once the user selects the preferred look, the expert system proceeds to the inference engine to find the target values for different responses by following a decision tree used in the inference engine. The detail on how to develop the decision tree can be found in previous work (Nguyen, 2014). In this instance, multiplying all possible selections (2X2X2X3X3X3), there can be 216 possible cases for the user's selection. Once the expert system is able to find the user's qualitative preference, its preferred look must be converted into numerical values for optimization.

A new approach is needed to convert 216 cases into an identifiable numeric value. In previous work (Nguyen, 2014), the conventional method of summing each variable's value was used and the result shows limitation on identifying sensibility of variables. For that reason, the method used in this paper entails assigning each parameter a different digit. For instance, as in Figure 7, this test has six questions that will have six place holders for each variable. Address 1a is assigned to hanging direction and 1b is for hanging position. In each address, values are assigned to the possible choices, if the hanging direction is vertical the value for the address 1a is 9 and if it is horizontal the assigned value is 0.

For instance, if the user's choice is Vertical/Inside/Evenly/Narrow/Low/+45 then the assigned value is 000000, if it is Horizontal/Outside/Exponentially/Wide/High/-45 then the value is 999999, and if it is Horizontal/inside/Exponentially/Medium/High/0 then the value is 909595. Based on this logic, all

	a	b	c	d	e	f
1	Hanging direction? Vertical/ Horizontal	Hanging position? Inside/ Outside	Shaft distribution? Evenly/ Exponentially distribute	Shaft width? Narrow/ Medium/ Wide	Shaft density? Low/ Medium/ High	Shaft angle? +45 / 0 / -45
1	Vertical = 0 Horizontal = 9	Inside = 0 Outside = 9	Evenly = 0 Exponentially = 9	Narrow = 0 Medium = 5 Wide = 9	Low = 0 Medium = 5 High = 9	+45 = 0 0 = 5 -45 = 9

Figure 7: Variable assignments.

216 different cases can be defined by numeric values, which were used to define the preferred objective functions in the next section.

Multi-Objective Optimization Function

Six test cases were conducted by following the workflow as shown in Figure 1. The same PBL building model was used to find better design options regarding the daylight and look preference decided by the user. All test cases used the same MOO functions to determine the ideal shade design used to optimize illuminance, GDP, and the user's preferred shade look. The objective function for all case studies can be written as follows:

$$\min_{x \in X} f(x) = [f_1(x), f_2(x), f_3(x)] \quad (1)$$

Where,

$$f_1(x) = - \left(\frac{\sum_{i=1}^n (r_{Lux})_n}{n} \right) \quad (2)$$

$$f_2(x) = \frac{\sum_{i=1}^n (r_{DGP})_n}{n} \quad (3)$$

The goal of the case building was to find a solution that minimizes all three objectives. The $f_1(x)$ is the average illuminance value (Lux) of all measuring grid cells in the building. For illuminance, to maintain consistency with the other two objectives, the outcome of the simulation was multiplied by -1 to convert the function from maximization to minimization. $f_2(x)$, which is the average DGP value of summer and winter at noon.

An objective function for the preferred looks of a shade ($f_3(x)$) can be defined as below, the objective function for this test is considering 6 variables (a, b, c, d, e, and f) that sum all variables with a designated multitude (10^i) and subtract by the objective value (Obj). Each different case has different objective values as discussed in the above section. Objective value is based on the response from the user's preference of the shade appearance. If the user's look preference is Horizontal/inside/Exponentially/Medium/High/0 then the objective value is 909595. The minimum of the difference between the objective value and the calculated value is closer to the goal. The

following table shows the objective value (**Obj**) that was tested in this paper.

$$f_3(x) = |\mathbf{Obj} - (a \times 10^5 + b \times 10^4 + c \times 10^3 + d \times 10^2 + e \times 10 + f)| \quad (4)$$

Table 2: Shade preference objective value for six test case.

Test case	Objective value (Obj)
H-I-EV-N-L+45	900000
H-I-EV-W-H-45	900999
V-O-EV-N-L-0	090005
V-I-EX-N-L+45	009000
V-O-EV-M-M-0	090555
H-O-EV-M-L-0	990505

H: Horizontal, V: Vertical, I: inside, O: outside, EV: Evenly, EX: Exponentially, W: Wide, N: Narrow, H: High, L: Low, M: Middle, +45: 45degree up, -45:45 degree down

Test Results

Among 216 cases, this paper selects six test cases to show the efficacy of the proposed method. The six cases were selected that can represent all possible cases. However, selecting more cases would reveal more findings.

Table 3: Comparison between base case and optimized solution.

TEST CASE		BASE CASE	DIFFERENCE
		POINT 1	
ILLUMINANCE (LUX)			
CASE 1	H-I-EV-N-L+45	284.910	201.37
		486.284	
CASE 2	H-I-EV-W-H-45	44.188	259.76
		303.946	
CASE 3	V-O-EV-N-L-0	481.906	4.88
		486.788	
CASE 4	V-I-EX-N-L+45	305.333	180.11
		485.443	
CASE 5	V-O-EV-M-M-0	333.428	153.55
		486.983	
CASE 6	H-O-EV-M-L-0	469.775	15.71
		485.486	
DGP			
CASE 1	H-I-EV-N-L+45	0.283	0.006
		0.289	
CASE 2	H-I-EV-W-H-45	0.279	0.009
		0.288	

CASE 3	V-O-EV-N-L-0	0.289	0.000
		0.289	
CASE 4	V-I-EX-N-L+45	0.287	0.002
		0.289	
CASE 5	V-O-EV-M-M-0	0.288	0.001
		0.289	
CASE 6	H-O-EV-M-L-0	0.289	0.000
		0.289	

The selected cases were conducted with the same boundary conditions. Appendix 1 shows each test case's scattered plot of the columns of one matrix against another measurement. As discussed above the goal of the optimization was to minimize all three objectives, for that reason illuminance value shows in appendix 1 is negative to make maximization to minimization. Comparing different cases, plots clearly shows which test case performs better than other cases. For example, the plots of case 2 show a higher range of Look value (1600-1500) than other cases (400-500). This could be further developed into methods that it can suggest a better performance configuration to the user and able to guide them to find a better design.

Table 3 shows the base case where illuminance and DGP values were calculated based on the user's initial chosen shade look. The results of this base case were compared with one point of Pareto front result. Overall, the solution shows an increase of average illuminance from receiving daylight and able to keep a similar DGP.

The preference value (Table 4) shows the magnitude in how close it is to the initially selected shape based on the objective function $f_3(x)$. A value closer to zero means more similarities to the initially selected shape. The results indicate that test case 2 (H-I-EV-W-H-45), which has a wider shade in higher density, is the most different from the initially selected shape. Shades like those in test case 2 block daylight and reduce the illuminance. So the solution shows a different shape than initially choose by a user.

Table 4: Preference look value of test cases.

TEST CASE	CONFIGURATION	LOOK
CASE 1	H-I-EV-N-L+45	391.10
CASE 2	H-I-EV-W-H-45	1,501.70
CASE 3	V-O-EV-N-L-0	396.50
CASE 4	V-I-EX-N-L+45	390.80
CASE 5	V-O-EV-M-M+0	946.40
CASE 6	H-O-EV-M-L-0	916.20

Figure 8 shows three test cases. Each case shows one point of Pareto front's shade looks (1) compared with the initial shade look selected by user (Base Case). The test case 1 and 3 show a similarity between the base case and the result from optimization. The test case 2 show less success compared to test cases 1 and 3. The reason can be found from the preference objective function($f_3(x)$), as discussed above, the shade the user prefers mostly blocks daylight that is different from the other two functions' ($f_1(x), f_2(x)$) goal. For that reason, the resulting shape shows more difference than the other two cases.

Discussion

This paper proposes a design process that can satisfy two different types of performance, that is, the qualitative and quantitative performances of the building. To satisfy both performances, MOO was set to satisfy the user's visual favour but also to effectively filter the sunlight from entering the building by adjusting the building shade geometry in response to varying outdoor and sky conditions. To measure these design objectives, Illuminance and DGP were used to measure quantitative performance.

For the user's visual preference of the shade design, this paper proposes utilizing an expert system to evaluate the appearance of the shade as a qualitative performance. First, the test building design was analyzed, and the design principles identified. These design principles were programmed to the NURBS system that allowed the manipulation and control of the geometry. Based on the six design principles, the building shade can be configured into 216 cases from an expert system. These cases were converted into a value that was used as the objective function for the test.

The proposed methodology then tested the six different design cases to see how solutions were similar to the user's initial choice. In terms of two quantitative objectives, Illuminance and DGP, the test result shows improvement in Illuminance level compared to the base case. For DGP, the results show little change in comparison to the base case. Overall, the test shows promising results, the Pareto-front solutions show a similar shape to the initial choices. Some cases like the wider slates with a higher density shade (CASE 2) showed less agreeable shapes compared to the other five cases. However, the main reason for the significant difference in case 2 was the initial shape selected by a user, it was not a good choice for allowing daylight in.

Conclusion

The paper was able to demonstrate that the proposed method can be applied to more practical and complex test cases by refining the qualitative measurement method to improve its accuracy. The limitation of the previous paper was that the objective function for the qualitative measure requires several equations for each case, however, by proposing a new method, the qualitative objective function can be written in one equation. This allows the method to be utilized with more complex and practical variables that can be used to calculate both qualitative and

quantitative performances which can then be applied at the early stages of the practical building design process with support from MOO.

It is also important to note the scope and several limitations of this work that offers opportunities for future research:

- This study utilizes an expert system for testing the quantitative aspect of performance. However, the result shows limitations in the expert systems that could be improved further with algorithms like unsupervised learning, such as k-mean clustering.
- The test case was difficult to clearly show improvement from the base case. Therefore, it is suggested to test with a more significant test case or actual building to understand the proposed method's limitations and possibility.
- This work focused on daylight conditions. Other environmental factors should be included in future research, such as natural ventilation, thermal comfort, and building energy use.

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Figure 8: Three test cases of a Pareto optimal front with a selected solution appearance

Appendix 1: Six Test Case Results in 2d