Preliminary Numerical Evaluation of the BIPV's Potential in Urban Areas: Which Method to Use for Solar Radiation Calculation?

Blaise Raybaud^{1,2}, Philippe Thony¹, Etienne Vergnault², Lucie Merlier², Jean-Jacques Roux²

¹Université Grenoble Alpes, Grenoble, France

²Université de Lyon, INSA-Lyon, CETHIL UMR5008, Villeurbanne, France

Abstract

In order to locally produce electricity in urban areas, it is important to properly assess the potential of the Buildings Integrated PhotoVoltaic (BIPV) technologies. Therefore, the evaluation of the incident solar radiative flux in dense urban areas, taking into account multiple reflections over all urban elements, is a current subject of interest. Although different models exist to evaluate the intensity of the power density on each zone of each part of an urban scene, the reflective properties of the different urban materials are generally simplified, and they are commonly supposed to reflect uniformly wavelength through the space. Henceforth, this brings up questions about the relevance of these assumptions and their influence in the numerical evaluation of the PhotoVoltaic (PV) potential. Thus, this paper reviews the different alternatives to proceed with a preliminary evaluation of the solar potential in dense urban areas, with the aim of evaluating the influence of the material reflection properties on the power density incident on a façade. Two models are discussed: one based on the radiosity method and the other based on the ray-tracing method. The construction of both methods is detailed and results for both models under similar hypothesis are compared. Moreover, the influence of different reflection properties is also studied.

Introduction

The state of the art shows difficulties in simulating solar incidence taking into account both direct and diffuse solar radiations in urban areas (Simón-Martín et al., 2017). In addition to that, each urban surface has specific reflectance properties. The influence of these properties on the repartition of the power density resulting from solar radiations remains badly evaluated. Moreover, the spectrum issued from the multiple reflections is even less known, although this incident spectrum on the BIPV module should be considered for a better building integration as shown in Ghosh et al. (2019). We can therefore ask questions considering the incident spectrum on a module located on a façade. Or, simply, search for the reality

of the incident power density on a façade taking into account the real reflection properties on all urban surfaces. Usually, it is assumed that all reflections on the buildings and the ground are uniform or lambertian. These assumptions are done using the current models such as Solene, Simplified Radiosity Algorithm (SRA, for CitySim) or DIVA (Robinson and Stone, 2014; Lobaccaro et al., 2017; Imenes and Kanters, 2016). However, some models consider specular reflections under strong assumptions such as Waibel et al. (2017). On the contrary, the French reglementation RT2012 imposes a minimum rate of 17% of glazing on buildings for which the uniform reflection assumption seems to be unsuitable. In this case, a specular reflection assumption appears more appropriated. Figure 1 illustrates the difference between specular and uniform reflections. It clearly appears that assumptions on the reflection model can change the evaluation of the power density incident on façades.

Figure 1: Difference between uniform and specular reflection.

In order to evaluate properly the BIPV performances, and because the aesthetic of the BIPV integration is becoming a crucial criterion (Sánchez-Pantoja et al., 2018; Sánchez-Pantoja, Vidal, and Pastor, Sánchez-Pantoja et al.), this article aims at integrating and comparing different sunlight models where the different reflection properties are integrated. Models used are radiosity and ray tracing, both developed on Matlab[®], and adapted for urban areas. These models simulate the power density resulting from solar radiations with reflections on all surfaces considering different reflection properties. Moreover, models are developed to be accurate at the module scale.

The article is divided into four sections. The first one presents both radiosity and ray-tracing methods. The

methods used to mesh the urban scene are also introduced. Theoretical advantages and disadvantages are tackled and proofs of converging are introduced. In section two, results with both methods are compared in order to validate the models under the assumption of uniform reflections only. Then, the influence of the reflection models are evaluated with the ray-tracing model. In section three, a discussion is made on the pros and cons offered by the two methods. Finally, conclusions are presented.

Method

Urban areas present plenty of different surfaces with several physical properties of reflectance. To evaluate their influences, the state of the art has shown that two main methods are considered : radiosity and ray-tracing. Radiosity allows only simulating uniform properties, whereas the ray tracing model allows simulating both uniform and specular properties with a more complex modelling process, especially for the diffuse radiations from the sky vault. From a computing performance point of view, radiosity requires a large computing time and memory space to calculate and store the form factor matrix. But once this matrix is computed, the resolution of the problem is fast for all sun positions. Conversely, the ray tracing method is faster for a predefined sun position but requires a completely new computation for a new sun position. This section details the construction of these two methods. The air is supposed to be totally transparent for short wave lengths ($\leq 3\mu m$) in both models.

Construction of the scene

The scene, the sky vault and the meshes are drawn using GMSH[®]. GMSH[®] offers the possibility to number the faces composing the scene and assign them a physical name. This is employed for identifying each surface and therefore attribute them physical properties¹.

Meshes are then exported into Matlab[®] where all calculations are performed. Matlab[®] has been chosen in order to use a flexible language where the physical equations can be easily implemented.

Radiosity

Many current models are based on the radiosity principle. Among them, we can quote Solene for Solene-Microclimat (Miguet and Groleau, 2002; Groleau, 2000) or the Simplified Radiosity Algorithm (SRA) for CitySim (Robinson et al., 2009). The radiosity approach is based on the hypothesis that all surfaces are uniform. By this way, all the energy received by a surface are redistributed uniformly in the space. Under this assumption, the problem can be written as a

¹Note that because of the computing time required by the radiosity model, two different meshes are considered in this study, one for the radiosity and the other for the ray-tracing method.

linear system of Equations (1), called the "radiosity method" and is detailed in Fernández and Besuievsky (2014).

$$(I - \rho F)B = E. \quad (1)$$

where B is the radiosity and represents the power density on the mesh, I is the identity matrix, ρ the reflection vector, F the form factor matrix and E the initial density power incident on the scene.

The radiosity method implies the calculation of the form factor matrix F , which is the main difficulty in using this method. Different strategies exist. Some of them use analytical functions whereas others use ray-tracing methods to evaluate the form factors. In this article, analytical functions are used.

Mesh for radiosity

A triangulation is used to mesh the urban scene with the desired size since calculations are more practical with triangles. To avoid getting unrealistic elongated triangles, a Delaunay triangulation is used. Normal orientations are then corrected in Matlab[®] to point outward of the buildings for an accurate calculation of masks between two triangles.

Regarding the sky vault, the Tregenza mesh is not used (Freitas et al., 2015). The energy of each sky triangle (in $[W/m^2]$) is calculated from the horizontal diffused irradiance D_h , using the formulas presented by Robinson and Stone (2004). The Direct Normal Incidence (DNI) and the Diffused Horizontal Irradiance (DHI) required in the previous formulas are obtained using the Perez model (Perez et al., 1987, 1990). Since the sky vault is centred on each triangle of the urban scene to evaluate the masks, the radius of the sky vault does not matter and is fixed to 1. However, the normal orientations of each triangle composing the sky vault is crucial, and must be oriented toward the center of the sky vault.

Form Factors

The radiosity equation (1) introduces the form factor matrix F . Form factors are illustrated in Figure 2. $F_{i,j}$ can be estimate as follows.

$$F_{i,j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\phi_i \cos\phi_j}{\pi r_{i,j}^2} dA_i dA_j. \quad (2)$$

where $F_{i,j}$ is the form factor between the i^{th} and the j^{th} triangle, A_i the area of the i^{th} triangle, $r_{i,j}$ the distance between the two barycentres and ϕ_i are the angles shown in Figure 2.

Using the Green theorem, this calculation can be simplified into a contour integration.

$$F_{i,j} = \frac{1}{2\pi A_i} \oint_{C_i} \oint_{C_j} \log(r_{i,j}) dC_i dC_j. \quad (3)$$

where C_i is the contour of the i^{th} triangle and $r_{i,j}$ is the distance between the triangles.

To improve the computational time, F is calculated from the f matrix described in Equation (4) which is symmetric. Only the upper part of the matrix is then calculated.

$$f_{i,j} = \frac{1}{2\pi} \oint_{C_i} \oint_{C_j} \log(r_{i,j}) dC_i dC_j. \quad (4)$$

Then, the complete form factor matrix F is obtained by dividing the rows of f by the triangle areas:

$$F_{i,:} = \frac{f_{i,:}}{A_i}. \quad (5)$$

Figure 2: Illustration of the form factors.

Since Equation (4) does not take into account inter-visibility between triangles, the form factor matrix is multiplied by an inter-visibility matrix V . In order to improve the time calculation, V is calculated during a preliminary step and is used in the construction of F . By this way, form factors are directly calculated between visible triangles and we do not have to multiply the form factor by V .

Resolution

The problem presented in Equation (6) is linear.

$$B = (I - \rho F)^{-1} E. \quad (6)$$

If the dimension of the problem is small enough, it can be solved directly using the inversion of the $(I - \rho F)$ matrix, where I is the identity matrix. Otherwise, different solutions are possible. In the case where the scene contains a large number of masks, an iterative method for sparse linear model can be used to solve the equation. Model resolutions are detailed in Saad (2003). The use of the progressive radiosity method is also possible (Peyré, 2001).

Before solving the problem, a pre-calculation step is done. It consists of the construction of the E vector: all triangles collecting direct and diffused sunlight are initialized. Taking into account their orientations, the power density in W/m^2 is determined on each triangle resulting from solar radiations without considering any reflections. To decrease the computational time, the sun position is approximated to

the closest barycentre of the sky vault triangles.

Complexity

Since we need to build the visibility matrix V and the form factor matrix F , the complexity is $O(N_p^2)$. This is witnessed when we need to compute solar radiance for large urban scene with a lot of surfaces with different orientations and a detailed mesh. As we said, the solution of the linear numerical problem can be direct if the dimension of the form-factor matrix F is small enough. Otherwise, the complexity of the equation resolution is equal to the one of the chosen method.

Ray Tracing

The ray-tracing methods allow avoiding the hypothesis of uniform reflection for all surfaces. The construction of the method can be deterministic with perfect specular reflections, or stochastic to simulate uniform reflections. Ray-tracing methods are used in Radiance. This software is often used in radiative models in order to obtain radiative maps, as is done in DaySim (Oh and Park, 2018) or DIVA (such as described in the Solemma website (Solemma, 2019)).

Theoretical methodology

The idea is to scan a large number of rays launched from the sun position into the direction of the scene. We do not have to calculate masks nor form factors, but looping over all rays is required. However, to improve the computational time we still calculate the inter-visibility matrix V used in the identification of the impacted triangles.

The floor area of the scene is projected in the sun direction above the highest point of the scene. The obtained surface will be the sky of our model. Initial positions of each ray are randomly selected in this sky, according to a uniform law. The amount of energy carried by a ray E_{ray} is calculated as presented in Equation (7).

$$E_{ray} = \frac{P_h S_{sky}}{N_r}. \quad (7)$$

where P_h is the beam horizontal irradiance, S_{sky} the surface of the sky and N_r the number of rays. For each ray, we are looking for the first intersection with this mesh. The coordinates of the intersection are saved, and a new ray with a new energy is launched from this point. The direction of the new ray depends on the selected reflection model.

Mesh for ray-tracing

To decrease the computational time, the identification of the intersections between the rays and the scene has to be done efficiently. Usually, ray tracing is implemented using octrees (Samet, 1989). For the urban scene, different meshes with different tri-

angle sizes are built (Figure 3). This strategy allows to build a tree structure (Figure 4) between meshes. Until the finest mesh is achieved, each mesh has four children and one mother. New triangles are built from an initial coarse mesh in old triangles. Still in order to avoid elongated triangles, the coarse mesh is defined using a Delaunay triangulation. The refinement to create the $(N+1)^{th}$ mesh from the N^{th} mesh is done by dividing by four each triangle until they reach the desired size. When a triangle reaches the desired size, it is not refined any more. The refinement stops when all triangles reach the desired size.

Ray intersections are firstly looked for in the coarse mesh, and the intersected triangles are identified. To identify the intersected triangles on the $(N+1)^{th}$ mesh from the N^{th} mesh, the ray intersection is searched over the four children. This is illustrated by the red path in Figure 4. Results are given on the finest mesh.

Figure 3: 3 different levels of meshes gradually refined, for a square scene.

Reflections in ray-tracing

The proposed ray-tracing model allows to model both uniform and specular surfaces. The uniform reflection is generated selecting a random direction in the demi-space defined by the orientation of the normal of the impacted mesh. Then the model becomes stochastic and the travel of the ray is random. The specular reflection is generated using the Snell-Descarte law. In this case, the model is deterministic and all the paths of the rays are only determined by their initial position in the sky.

Figure 4: Illustration of the searching procedure for the triangle containing the ray intersection.

In both cases, only one ray is launched from the coordinates of the intersection. The path of the ray is stopped when the carried energy becomes smaller than a chosen percentage of the initial energy.

Complexity

Due to the strategy with tree structure between

meshes, the complexity of the ray-tracing method is $O(N_r \log(N_p))$. This is a huge advantage compared to the radiosity method. Moreover, and in order to reduce the computational time, the code is parallelized using the Matlab® Parallel Computation Toolbox on the loop over all rays. The complexity C_{RT} can be written as in Equation (8).

$$C_{RT} = O(N_r(N_G + 4M)). \quad (8)$$

where N_G is the number of triangles in the coarse mesh and M the number of meshes.

The mesh refinement strategy is translated into Equation (9).

$$4^M = \frac{N_p}{N_G} \Leftrightarrow M = \log\left(\frac{N_p}{N_G}\right). \quad (9)$$

Using Equations (9) and (8), the complexity can be written as in Equation (10).

$$\begin{aligned} C_{RT} &= O\left(N_r(N_G + 4\log\left(\frac{N_p}{N_G}\right))\right) \\ \Leftrightarrow C_{RT} &= O(N_r \log(N_p)). \end{aligned} \quad (10)$$

Convergence criteria

The ray-tracing algorithm has to be stopped if:

- The energy carried by one ray becomes smaller than a global percentage ϵ_{global} of the total incident energy on the scene.
- In average, the variation of the energy received by a mesh becomes smaller than a local criterion defined as the percentage ϵ_{local} .

These two criteria imply a condition on the number of required rays.

Global convergence criterion

$$E_{ray} = \frac{1}{N_l} E_{inc,tot}. \quad (11)$$

where N_l is the number of launched rays.

Then, we can deduce Equation (12) from Equation (11):

$$\frac{E_{ray}}{E_{inc,tot}} \leq \epsilon_{global} \Leftrightarrow \frac{1}{N_l} \leq \epsilon_{global}. \quad (12)$$

Local convergence criterion

Equation (13) gives the local condition.

$$\begin{aligned} E_r &= N_r E_{ray} \\ \Leftrightarrow \Delta E_r &= E_{ray} \\ \Leftrightarrow \frac{\Delta E_r}{E_r} &= \frac{E_{inc}/N_l}{N_r E_{inc}/N_l}. \end{aligned} \quad (13)$$

Then, we get the Equation (14).

$$\frac{\Delta E_r}{E_r} \leq \epsilon_{local} \Leftrightarrow \frac{1}{N_r} \leq \epsilon_{local}. \quad (14)$$

If all triangles have the same sun exposure and the same size, by using the Law of Large Numbers we deduce Equation (15).

$$N_r = N_l / N_p. \quad (15)$$

Equation (15) allows determining the final criterion on the number of rays needed:

$$N_l \geq \max \left(\frac{1}{\epsilon_{global}}, \frac{N_p}{\epsilon_{local}} \right). \quad (16)$$

We can deduce from Equation (16) that if the scene is small enough, the number of launched rays will be driven by the global criterion ϵ_{global} whereas if the scene is big enough with a complex geometry, the number of rays will be driven by the local criterion ϵ_{local} .

Models comparison

Case study

The selected case study is a theoretical street canyon with a North-South orientation, located at Le Bourget du Lac (France) (Longitude : 6° , Latitude : 45.8°). All simulations are performed in the 21st of June at 1.00 pm UTC. The street has a length L of 120 meters and a width W of 20 meters. On each part, buildings are 40 meters high (H). Therefore, the street has an aspect ratio (H/W) of 2.

Three different materials compose the urban scene presented in this article. All the floor areas are named "Ground". Walls of the buildings are named "Concrete" and roofs are named "Roof". Reflective properties are introduced in Table 1, taken from the on-line webtool Tabula (Energy Europe Programme, 2019). Although our model allows to consider transparent surfaces, all surfaces are assumed to be opaque.

Table 1: Reflective coefficients of the materials composing the case study.

Materials	Ground	Concrete	Roof
Reflection coef.	0.3	0.5	0.3
Absorption coef.	0.7	0.5	0.7

Because only short wavelength exchanges are modelled, only surface properties matter and we do not detail the composition of the walls and the ground. Figure 5 illustrates the orientation of the inside façade of the urban canyon as well as an approximative position of the sun. For all the following results, only the direct part P_h of the incident solar radiations is considered, with the purpose to evaluate the influence of the reflective properties of the materials on the power density.

Three cases are studied, with different configurations of surface reflectance :

Figure 5: Sectional drawing of the orientations of the urban canyon's façade.

- The Uniform Configurations (UC) where all surfaces are supposed to generate uniform reflections.
- The Specular Configurations (SC) where all surfaces are supposed to be specular.
- The Mixed Configurations (MC) where all surfaces are supposed to give uniform reflections except the West oriented façade of the urban canyon, where the concrete is supposed to be specular only this façade.

Because our ray-tracing model is not able to simulate the diffused solar radiations from the sky vault, we do not consider this energy source in the following results.

Model comparison

In this section, both model are compared under similar reflective properties for all the materials of the urban scene.

Radiosity

Figure 6 shows the power density repartition calculated using the radiosity method on the urban canyon. The highest power density is obtained near the bottom of the West and East façade of the two buildings with 900 W/m^2 . This corresponds to the surfaces where multi reflections are generated because of the proximity between the ground and the façades. This phenomenon is more important inside the urban canyon because of the extra proximity between the East and the West oriented façades.

Ray-tracing

We present two different types of results obtained with the ray-tracing model. First results are obtained assuming all surfaces uniform. In a second time, results are obtained assuming all surfaces specular. Figure 7 presents the repartition of the incident solar flux on the urban scene with only uniform reflections. The overall incident energy appears homogeneous on horizontal surfaces. Similarly to the radiosity method, the solar potential is higher near basis the basis of the buildings where reflections are concentrated, with a difference of almost 100 W/m^2 between the bottom and the top of the

Figure 6: Power density (in $[W/m^2]$) resulting from direct solar radiations and diffused reflections (resulting from direct radiations only) for the 21th of June at 1:00pm utc using the radiosity model.

West façades. This phenomenon is less important on the ground with $50 W/m^2$ of difference between the near and the far (25m) triangles from the façades.

Figure 7: Power density (in $[W/m^2]$) with diffused reflections only (resulting from direct radiations only) for the 21st of June at 1:00pm UTC estimated using the ray-tracing model. Computation time: 48.1 minutes

Results are similar between both models. The repartition of the power density is similar on both meshes, with similar intensity on roofs ($850 W/m^2$), ground ($850 W/m^2$ far from the bottom of the buildings, and near to $1100 W/m^2$ at the bottom) and walls. For both models, highest intensity are observed on the base of west oriented façade where reflections are multiplied. We do not observe any particular difference between both models in this study case which validates the simulation in the case of uniform reflections only.

Comparison between different case study with different reflective properties

This section presents a comparison between different configurations of the previous case study where only the reflective properties of the material are modified. Three different configurations are studied, which are detailed in the "case study" section.

Figure 8 presents the repartition of the incident solar flux on the urban scene with only specular reflections. Due to the nature of the reflections, distinct areas

are identified. The highest solar potential is located on the East side of the buildings. These areas collect direct and reflected solar fluxes, with $1100 W/m^2$ in average, which is $100 W/m^2$ more than with uniform reflections. Urban areas collecting reflections are clearly identified with homogeneous energy potential.

Figure 8: Power density (in $[W/m^2]$) considering specular reflections only (resulting from direct radiations) for the 21st of June at 1:00pm UTC using the ray-tracing model. Computation time: 48.5 minutes

Figure 9 illustrates the differences between the two previous results. As expected, main differences are located in the areas where specular reflections are concentrated, with differences of $+350 W/m^2$ on the ground for the specular model. The extreme bottom of the West building façades presents also significant differences with $+200 W/m^2$ for the uniform model.

Figure 9: Difference (in $[W/m^2]$) between both ray-tracing models (uniform and specular reflections) : $E_{inc,Specular} - E_{inc,Uniform}$ (estimated using the ray-tracing model).

In order to evaluate more precisely the difference between the models, we examine the energy distribution on specific surfaces of the urban scene. We only consider the power density on the East oriented façade and the ground of the urban canyon, therefore we can evaluate the influence of the first reflection on the West façade. In this manner, we can evaluate the solar potential of a photovoltaic panel located on the West oriented façade inside a street canyon.

Figure 10 reveals three different repartitions according to the configuration implemented. With the

Specular Configuration, we can clearly identify two peaks (at 0 W/m^2 and 28 W/m^2) which correspond to the two "zones" on the façade : the one collecting the energy from the reflection on the west oriented façade, and the one which does not collect any direct or reflected photon. In the case of the Uniform Configuration, the profile of the power density distribution is totally different. Finally, in the case of the Mixed Configuration, the profile of the power density repartition is again different than the two previous profiles. Half of the meshes receive between 6.5 and 22.5 W/m^2 . Between 22.5 and 45 W/m^2 , the power density is almost equally distributed on triangles whereas only few triangles (26, which is $1/10$ of the total number of meshes on the façade) receive more than 45 W/m^2 . Moreover, the average power density obtained with the Mixed Configuration (25 W/m^2) is closer to the average power density reached with the Specular Configuration (14 W/m^2) than that obtained with the Uniform Configuration (67 W/m^2).

Figure 10: Differences on the power density (in W/m^2) repartitions on the west oriented façade between the Uniform Configuration, the Specular Configuration and the Mixed Configuration using ray-tracing models.

Figure 11: Power density $[\text{W/m}^2]$ on the east oriented façade of the urban canyon resulting from the beam solar radiations with the radiosity model.

Considering the radiosity method, the repartition of the power density is similar to the uniform model, with an average power density higher than the average power density obtained with the ray-tracing model (in the case of the Diffused Configuration) of 83 W/m^2 . We also notice numerous meshes with more than 100 W/m^2 . It corresponds to the meshes

located near the bottom of the façade, where the proximity between the ground and the façades generates multiple reflections. However, the radiosity model overestimate the power density on the façades inside the urban canyon, compared to the ray-tracing method.

Discussion

This study develops two radiative models and compares their outputs with respect to three different configurations. Although the models presented in this article are still under development to improve their accuracy, and convergence criteria can be adjusted, first results highlight substantial deviation between power density distribution depending on the surface reflectance properties. The final purpose of the study is to suggest a model providing the possibility to link each surface with its Bidirectional Reflectance Distribution Function (BRDF). Moreover, and as much as possible for studying potential and the aesthetic integration of BIPV, the model has to be fast enough to offer the possibility of modelling all spectrum over a broad band of wavelengths.

In addition the model should be validated against experimental data to ensure its accuracy. Therefore, a real canyon model located on the INCAS platform of INES is currently built. Solar incident fluxes will be measured at different elevations on the façade of the urban canyon and on the street surface. Different surfaces will also be tested in order to experimentally evaluate the correlation between the reflective surface properties and the repartition of the solar incident flux. Moreover, measures will be compared to the simulations to validate numerical results.

Conclusion

The influence of the reflectance properties of the urban façades has been analysed using two different models. Three configurations have been tested : a full uniform reflection configuration, a full specular reflections configuration and a uniform reflection configuration with only one façade having specular properties. These three configurations reveal differences linked to the urban reflectance properties. In average, the power density resulting on the East oriented façade can differed from 25 W/m^2 to 68 W/m^2 with only one optic modification of the material property on the opposite façade. Although these models are still under development, this first study points out the importance of considering the reflectance properties of urban surfaces in dense urban areas. Moreover, it is shown that the ray-tracing and radiosity models present similar results for uniform reflections. Such results support the final suppose of the importance to accurately consider reflectance properties. The ray-tracing model presents the advantage of implementing all surface reflection properties, but not diffused solar radiations from the sky vault. On the other

hand, it is the contrary for the radiosity model. Both models are therefore complementary.

Nomenclature

ρ : Vector of the reflective coefficients.
 α : Vector of the absorption coefficients.
 F : Form factor matrix.
 f : Incomplete form factor matrix.
 A : Matrix of the mesh surfaces [m^2].
 B : Vector of radiosities.
 E : Vector of elementary radiosities.
 I : Identity matrix.
 V : Visibility matrix.
 r : Distance between two meshes [m].
 M : Number of meshes.
 N_p : Number of triangles on the finest mesh.
 N_G : Number of triangles on the coarsest mesh.
 N_r : Number of rays.
 N_l : Number of launched rays.
 E_{ray} : Energy associated to a ray [W].
 E_r : Power density on a triangle.
 E_{inc} : Incident solar radiation energy [W/m^2].
 P_h : Beam horizontal irradiances [W/m^2].
 D_h : Diffuse horizontal irradiances [W/m^2].
 G_h : Global horizontal irradiances [W/m^2].
 C_{RT} : Complexity of the Ray-Tracing algorithm.
 ϵ_{local} : Local convergence criterion.
 ϵ_{global} : Global convergence criterion.
 S_{sky} : Sky surface [m^2].

References

- Energy Europe Programme (2019). TABULA WebTool.
- Fernández, E. and G. Besuievsky (2014). A sample-based method for computing the radiosity inverse matrix. *41*, 1–12.
- Freitas, S., C. Catita, P. Redweik, and M. Brito (2015). Modelling solar potential in the urban environment: State-of-the-art review. *41*, 915–931.
- Ghosh, A., S. Sundaram, and T. K. Mallick (2019). Colour properties and glazing factors evaluation of multicrystalline based semi-transparent photovoltaic-vacuum glazing for BIPV application. *131*, 730–736.
- Groleau, D. (2000). Solene un outil de simulation des éclairagements solaires et lumineux dans les projets architecturaux et urbains. pp. 8.
- Imenes, A. and J. Kanters (2016). 3d solar maps for the evaluation of building integrated photovoltaics in future city districts: A norwegian case study. Volume 2016-November, pp. 3141–3146.
- Lobaccaro, G., S. Carlucci, S. Croce, R. Paparella, and L. Finocchiario (2017). Boosting solar accessibility and potential of urban districts in the nordic climate: A case study in trondheim. *149*, 347–369.
- Miguet, F. and D. Groleau (2002). A daylight simulation tool for urban and architectural spaces—application to transmitted direct and di use light through glazing. pp. 11.
- Oh, M. and H.-D. Park (2018). A new algorithm using a pyramid dataset for calculating shadowing in solar potential mapping. *126*, 465–474.
- Perez, R., P. Ineichen, R. Seals, J. Michalsky, and R. Stewart (1990). Modeling daylight availability and irradiance components from direct and global irradiance. *44*(5), 271–289.
- Perez, R., R. Seals, P. Ineichen, R. Stewart, and D. Menicucci (1987). A new simplified version of the perez diffuse irradiance model for tilted surfaces. *39*(3), 221–231.
- Peyré, G. (2001). Résolution numérique d'équations intégrales : Exemple de la radiosité.
- Robinson, D., F. Haldi, J. Kämpf, P. Leroux, D. Perez, A. Rasheed, and U. Wilke (2009). Citysim: comprehensive micro-simulation of resource flows for sustainable urban planning. pp. 8.
- Robinson, D. and A. Stone (2004). Irradiation modelling made simple: the cumulative sky approach and its applications. In *PLEA conference*, pp. 19–22.
- Robinson, D. and A. Stone (2014). Solar radiation modelling in the urban context. *77*(3), 295–309.
- Saad, Y. (2003). *Iterative Methods for Sparse Linear Systems - Second Edition* (Seconde ed.). SIAM.
- Samet, H. (1989). Implementing ray tracing with oc-trees and neighbor finding. *13*(4), 445–460.
- Simón-Martín, M., C. Alonso-Tristán, and M. Díez-Mediavilla (2017). Diffuse solar irradiance estimation on building's façades: Review, classification and benchmarking of 30 models under all sky conditions. *77*, 783–802.
- Sánchez-Pantoja, N., R. Vidal, and M. C. Pastor. Aesthetic perception of photovoltaic integration within new proposals for ecological architecture. *39*, 203–214.
- Sánchez-Pantoja, N., R. Vidal, and M. C. Pastor (2018). Aesthetic impact of solar energy systems. *98*, 227–238.
- Solemma (2019). Solemma LLC.
- Waibel, C., R. Evins, and J. Carmeliet (2017). Efficient time-resolved 3d solar potential modelling. *158*, 960–976.