A methodology to model the performance of a dynamic mirror light-shelf based on solar radiant flux impinging on the window

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Abstract

Building energy simulation codes treat light-shelves as perfectly diffuse, while examples in bibliography with dynamically controlled light-shelves are really limited. The paper examines two control strategies of an exterior specular light-shelf, proposing a fast computational methodology for the estimation of solar radiant flux impinging on the window. This methodology is used to develop a selection indicator based on sunlight redirection ability together with the solar radiation in comparison with the “No light-shelf” case. Results show that the best performing dynamic light-shelf is the one controlled according to the maximization of average daylight levels in areas away from the perimeter zone.

Introduction

Light shelves represent one of the most popular –and simplest– design choices in contemporary buildings and are often suggested in the literature as effective devices that can improve the lighting quality of a space and offer energy savings especially when daylight controls are used. Light-shelf’s placement relative to the window makes it possible to act as a shading device (by blocking the excessive sunlight from entering the interior space if installed externally reducing solar gains) and as a daylighting device (by forwarding the reflected sunlight, usually onto the ceiling plane advancing daylight deeper into the space). Till today many light shelf designs have been proposed (Kontadakis et al., 2018) from flat or curved static or sun-tracking, using specular reflection or refraction. Their performance is mainly examined in relation to the indoor lighting levels which were estimated using measurements and/or simulations without taking into account solar gains. A rare case in the bibliography is a work by Franco I.M. (2007) who examined the difference between static and dynamic light shelves (tilt angle adjustment) who performed Illuminance and radiant energy measurements. Two test cells were used in parallel with one of them having a static light shelf and the other a dynamic one using polished aluminum on the upper surface. In comparison with standard horizontal shading devices the heat gain using light shelves increased, but this does not seem to affect overheating conditions. Raphael (2011) examined a light shelf with adaptive geometry. Using simulation together with a global optimization algorithm, estimated

the hourly variation of the light shelf tilt angle in an effort to increase light levels in areas away from the window, achieving about 12% additional lighting energy savings compared to the savings achieved using static light-shelf. Warrier and Raphael (2017) using both experiments is scale models and simulations concluded that about 21% increase in illuminance can be achieved using horizontal light shelves made of aluminum or glass mirror. Illuminance can be increased even more (threefold) if a rotating external light shelf is used. Building energy simulation algorithms such as EnergyPlus (2018) considers static horizontal light shelves by adding the extra luminous flux due to reflectance on the light-shelf’s upper surface to the flux that the window receives from the sky and ground. This methodology is not capable of taking into account either specular reflections or light shelf’s variable tilt angles. It is therefore evident that although there are design recommendations for light-shelves in relation to their daylighting behaviour, the solar gains are omitted. In this paper a simple method (combination of the flux transfer approach together with ray tracing) is proposed to assess the performance of light shelves in relation to the solar radiant flux impinging on the window. It is computationally fast and considers the effects of specular reflection and the dynamic operation of the light-shelf according a) to the sun’s elevation and b) the maximization of daylight levels in the area away from the perimeter zone, on an hourly basis for a whole year.

Methodology

To examine the solar flux on the window surface, a 7m depth space has been simulated with south oriented façade. Window is divided into two parts by an external specular dynamic light-shelf. Examined space characteristics are presented in the following table.

Table : Test room characteristics

Dimensions (internal)	4 x 7 x 3 m
View Window	1.6 x 1 m (window sill 1m)
Clerestory window	1.6 x 0.5 m
Floor reflectance	0.3
Walls reflectance	0.6
Ceiling reflectance	0.8
Light-shelf’s upper surface reflectance	Total reflectance 91% Diffuse reflectance 12 %
Light-shelf’s down surface	0.5 (diffuse)

reflectance	
Glazing transmittance	0.6 (normal-normal)
Ground reflectance	0.2
Light-shelf length	0.5 and 0.7 m
Working surface height	0.8 m
Grid size	0.5 x 0.5 m

The tilt angle of the light-shelf is controlled using two strategies. In the first one (CS #1) light-shelf tilt angle changes according to the projected sun's elevation angle on the facade and an aiming point on the ceiling. The aiming point is located at the approximately at the middle of the non-daylight zone in an effort to increase illuminance levels in that area. Daylight zone depth (DZ) as defined in EN 15193-1:2017 (2017) can be estimated according to the following equation:

$$DZ = 2.5 * (WHH - WSH) \quad (1)$$

Where WHH and WSH are the window head height-working surface height accordingly.

Figure 1 below presents the relation between the sun's position and aiming point.

Figure 1: Definition of various angles used to calculate light-shelf tilt angle (θ) when CS#1 control strategy is used.

Using the above figure, the tilting angle of the lightshelf is defined as follows (Kostantoglou M. & Tsangrassoulis A., 2012) :

$$\theta = (\theta_{sun} - \omega) / 2 \quad (2)$$

where θ_{sun} is the projected sun's elevation angle and ω is a constant angle defined by the aiming point.

The second control strategy (CS#2) is based on achieving the maximum illuminance value in the non-daylight zone. In order to estimate the light-shelf's tilt angle, hourly Radiance (Ward G, 1994) simulations were performed using the sun as the only light source (direct and reflected on ground) without any diffuse light from the sky. For each hour 19 different tilt angles were simulated from 0° to 45° with 2.5° step and the angle achieved the highest average illuminance in the non-daylight zone was selected. The two figures below present the hourly distribution of average illuminance in the non-daylight zone for both control strategies and the

light-shelf tilt angle. The days presented are the 21st day for all months of the year.

Figure 2: Hourly distribution during the 21st day of each month of average illuminance in the non-daylight area using sunlight only as light source. (Light-shelf depth 0.5 m)

Figure 3: Hourly distribution during the 21st day of each month of light-shelf tilt angle for both control strategies using sunlight only as light source. (Light-shelf depth 0.5 m)

As expected CS#2 provide more illuminance in the non-daylight zone while the light-shelf's tilt angle throughout the day differs substantially from those achieved with control strategy CS#1. This increase in illuminance is more profound during summer months. During winter, due to the lower elevation of the sun, direct sunlight enters the room, diminishing the effect of the sunlight redirection resulting in smaller relative illuminance differences. Although increasing the illuminance in the non-daylight zone can reduce lighting energy consumption, cooling consumption can be increased due to increased solar gains. These gains are directly associated to the solar flux balance on the window. In our theoretical model, the calculation of the solar and the sky radiant flux impinging on the window is based on the following assumptions:

- The vertical window is of limited dimensions and it is located on an infinite perfectly absorbing wall,

- The upper surface of the light shelf is treated as specular for the calculation of the reflected solar beam radiation and as a diffuse with the same reflectance for sky diffuse radiation,
- An isotropic sky model is used,
- The width of the light-shelf is equal to the width of the window.

The geometry used to support the calculation procedure is based on the use of two different coordinate systems, a Global Coordinate System (GCS) and a Local Coordinate System (LCS). GCS can be placed on any arbitrary point with X-axis-axis and Y-axis-axis facing East and North respectively. LCS is placed on the lower left corner of the window with x-axis and y-axis axes parallel to the width and height of the window. The coordinate systems together with the window examined is presented depicted in Figure 3.

Figure 3: Window-light shelf geometrical configuration.

The window is separated into two parts an upper (clerestory) and a lower (view) one. For the light shelf's surfaces, we adopt the terms front and back. Front is defined as the upper mirror surface while back is the opposite. The total radiant flux arriving on the upper window can be calculated by adding a number of luminous fluxes either directly from sun and sky or indirectly through reflection. The calculation of these fluxes is presented in the following equations for the upper and lower window respectively.

Radiant flux on the clerestory window:

- Radiant flux due to the solar beam radiation:

$$\Phi_1 = E_{dir_norm} * A_{dir_upper} * \cos \theta_1 \quad (3)$$

Where E_{dir_norm} is direct normal irradiance, A_{dir_upper} is the area of the upper window that is radiated by solar beam radiation and θ_1 is the angle of incidence of the beam radiation on the window plane. It is possible that the A_{dir_upper} can be smaller than the total area of the upper window because a tilted light shelf can shade a part of the upper window. The calculation of this shaded area is based on the same methodology used for the calculation of the reflected radiation as this is presented below.

- Radiant flux due to beam radiation that is reflected from the front surface of the light shelf:

$$\Phi_2 = E_{dir_norm} * \rho_{front} * A_{reflected} * \cos \theta_2 \quad (4)$$

Where ρ_{front} is the reflectance of the upper surface of the light shelf, $A_{reflected}$ is the area of the upper window surface radiated by reflected sunlight and θ_2 is the angle of incidence of the reflected beam with the window plane. Although there are more sophisticated techniques for the calculation of $A_{reflected}$ (Sutherland G. & Hodgman W., 1974), (Weiler K & Atherton A. 1977) our method implements a simpler approach due to the simple geometry used. By using a corner point on the light-shelf and considering specular reflection, the reflected ray ($\vec{R}$) is calculated from the incident ray ($\vec{I}$) and the normal ($\vec{N}_L$) to the light-shelf plane (normalized) according to the law of reflection:

$$\vec{R} = \vec{I} - 2\vec{N}_L(\vec{N}_L \cdot \vec{I}) \quad (5)$$

The right corner point (P_0), facing the window, is used when sun's azimuth is smaller than that of the window as presented in Figure 4 while the left corner point is used in any other case.

Figure 4: Schematic representation of the calculation methodology used to estimate the area of the upper window radiated by reflected sunlight.

Intersecting the reflected ray with the window the position of point P can be calculated. Depending on the position of this point, the area lighted by the reflected sunlight is calculated. Figure 5 below presents all parameters involved in the calculation of $A_{reflected}$, when the sun's azimuth is smaller than that of the window.

Figure 5: Parameters used for calculation of the clerestory area radiated by reflected sunlight.

In Table 2 all alternative scenarios are presented in relation to the position of the intersection point together with the calculation of $A_{\text{reflected}}$.

Table 2: Calculation of $A_{\text{reflected}}$ according to the intersection point local coordinates when the sun's azimuth is smaller than that of the windows

If $0 < P'_x \leq w$ and $P'_z \leq h_1$ then $A_{\text{reflected}} = 0.5 \cdot (P'_x + w) \cdot P'_z$
If $0 < P'_x \leq w$ and $P'_z > h_1$ then $A_{\text{reflected}} = 0.5 \cdot (w + P'_x + (P'_z - h_1) \cdot (w - P'_x) / P'_z) \cdot h_1$
If $0 < P'_x \leq w$ and $P'_z \leq h_1$ then $A_{\text{reflected}} = 0.5 \cdot w \cdot (P'_z + (P'_z \cdot P'_x) / (w - P'_x))$
If $P'_x \leq 0$ and $\zeta \geq \arctan(h_1/w)$ then $A_{\text{reflected}} = 0.5 \cdot (w - (h_1 \cdot (w - P'_x) / P'_z)) \cdot w \cdot h_1$
If $P'_x \leq 0$ and $\zeta > \arctan(h_1/w)$ then $A_{\text{reflected}} = 0.5 \cdot w \cdot P'_z \cdot w / (w - P'_x)$

For estimating the shaded area in the lower window, a similar procedure is used. The proposed method is extremely fast. Hourly calculations of $A_{\text{reflected}}$ for a whole year on an Intel i7-3520M 2.9Ghz, take 2 secs.

- Radiant flux due to the sky:

$$\Phi_3 = E_{\text{dif_hor}} * F_{uws} * A_{uw} \quad (6)$$

Where $E_{\text{dif_hor}}$ is the horizontal diffuse irradiance, F_{uws} is the upper window sky view factor and A_{uw} is the area of the upper window.

The Sky view factor (SVF) is the ratio of the illuminance received by a surface to the illuminance received from the entire sky hemisphere. Several approaches have been presented for estimating SVFs, using either analytic or numerical methods and photographic techniques with fisheye images (Wu Jie et al., 2013). Unfortunately, the analytical calculation of the sky factor between an arbitrary oriented surface and the sky can be quite complicated. In this work, a ray tracing technique was used to estimate the illuminance on the window from a uniform sky of arbitrary luminance L w/o considering the contribution of interreflections. This was achieved by dividing the sky hemisphere into patches of altitude and azimuth and the illuminance on a point on the window is found for each patch. The number of elevation and azimuth angles used for the subdivision can affect considerably the calculated illuminance from the sky and the computation time as well. Inherently, this approach results in an improvement of the accuracy towards the zenith. To overcome the

problem, a fine resolution of 1 degree is used with the disadvantage of longer processing time. Consequently, after the separation of the window area into small patches, the sky illuminance on a point (E_i) located on the center of each patch is given by:

$$E_i = \sum_{n=1}^{\omega} L * \sin i_n * SA_n \quad (7)$$

Where, ω is the total number of patches, i_n is the angle of incidence and SA_n is the subtended size of the sky-patch. The solid angle is calculated by the following formula:

$$SA_n = \frac{2 * \pi * (\sin \theta_{\text{upper}} - \sin \theta_{\text{lower}})}{NPA} \quad (8)$$

Where, NPA is the number of patches in an azimuth band, θ_{upper} and θ_{lower} are the upper and lower altitude limits of the sky patch. Thus, the Sky View Factor is calculated as follows (Compagnon R., 2004):

$$F_{uws} = \frac{\sum_i E_i}{\pi * L} \quad (9)$$

The same approach can be applied to the lower window as well. To verify its results a simple case was selected. One rectangle window and an overhang of the same length, having one common edge, and at an angle of 90° to each other as presented in Figure 6.

Figure 6: Test case used to calculate sky view factor.

The view factor between the window and the overhang F_{w-o} is 0.14618 according to Howell et al. (2010) while the view factor between the window and ground F_{w-g} is 0.5. Thus, the sky view factor F_{w-s} is equal to:

$$F_{w-s} = 1 - F_{w-o} - F_{w-g} = 0.35382 \quad (10)$$

Two sky resolutions have been tested (1° and 10°) together with eight cases concerning the number of patches ($n_x * n_z$) that the window is divided into. Figure 7 presents the results.

Figure 7: View Factor calculation using raytracing algorithm against analytic solution for case of Figure 6.

Thus using 1^0 sky resolution, it seems that after a 10×10 window subdivision the accuracy is not further improved, making ray tracing reliable enough to calculate the *SVF* of the window for the examined geometric configuration. Processing time is 56 secs on an Intel i7 – 3520M 2.9Ghz.

- Radiant flux due to the reflection of diffuse sky radiation on the light-shelf:

$$\Phi_5 = \rho_{front} * F_{uwl} * E_{dif\ hor} * F_{fslsky} * A_{uw} \quad (11)$$

Where, F_{uwl} and F_{fslsky} are the view factors of the upper window to the upper surface of the light-shelf and the sky factor of the upper surface of the light shelf to the sky respectively. The F_{uwl} is quite complicated to be estimated analytically especially in cases where the light-shelf is tilted. To overcome this, *VIEW3D* algorithm was used (Walton G., 1986). *VIEW3D* uses an integration technique to calculate the view factors between planar surfaces which consist of complicated 3D scenes. F_{fslsky} is calculated as follows:

$$F_{fsl/sky}=0.5*(1+\cos(90+\theta)) \quad (12)$$

Where θ is the light-shelf tilt-angle with respect to horizontal.

- Radiant flux due to the ground reflected radiation:

$$\Phi_6 = E_{tot\ hor} * \rho_{ground} * F_{uwg} * A_{uw} \quad (13)$$

Where, $E_{tot,ho}$ is the global horizontal irradiance, ρ_{ground} is the ground reflectance and F_{uwg} is the upper window view factor to the ground. The latter can be calculated indirectly as:

$$F_{uwg} = 1 - F_{uws} * \rho_{ground} + F_{uwi} \quad (14)$$

Radiant flux on the view window:

- Radiant flux due to the solar beam radiation:

$$\Phi'_1 = E_{dir\ norm} * A_{dir\ lower} * \cos(\theta_{11}) \quad (15)$$

Where, $A_{dir.lower}$ is the area of the window that is not shaded and θ_{11} is the angle of incidence of beam sunlight on the lower window.

- Radiant flux due to the sky diffuse radiation:

$$\Phi'_2 = E_{dif_hor} * F_{lws} * A_{lw} \quad (16)$$

Where, F_{lws} is the lower window *Sky View Factor* and A_{lw} is its area. The method used for the calculation of sky view factor is the same with the one applied in the upper window.

- Radiant flux due the ground reflected radiation:

$$\Phi'_3 = E_{tot\ hor} * \rho_{ground} * F_{lwq} * A_{lw} \quad (17)$$

Since the tilt of the light-shelf is always positive the lower window view factor to the ground, F_{lwg} is 0.5.

- Radiant flux due to the reflection of diffuse sky and ground radiation on the light-shelf:

$$\Phi'_4 = \rho_{back} * F_{lwl} * [E_{dif_{hor}} * F_{bls} + E_{tot_{hor}} * \rho_{ground} * F_{blg}] * A_{lw} \quad (18)$$

Where ρ_{back} is the reflectance of the back side surface of the light shelf, while F_{lwl} , F_{bbs} and F_{bbs} are the view factors of the lower window to the back surface of the light-shelf, of the back face and to the sky and ground respectively. F_{bbs} can be easily calculated using the formula:

$$F_{bls}=0.5*(1+\cos(180-\theta)) \quad (19)$$

The calculation of $F_{back_surface,lightshelf-gr}$ is more complicated. Ground view factor is the ratio of the illuminance on the light shelf's back face to the horizontal illuminance on the ground. Figure 8 shows a representation of the sky-ground dome of radius R .

Figure 8: Geometry used for the calculation of the view factor of the back side of the light-shelf to the ground.

The irradiance at a point on the back side of the light-shelf due to an element of the ground of unit radiance L is given by:

$$\delta E = \frac{L \cdot R^2 \cdot d\alpha \cdot \cos(\gamma) \cdot d\gamma}{R^2} \cdot (\sin(\gamma) \cdot \cos(90 - \theta) - \cos(\gamma) \cdot \sin(\theta) \cdot \cos(\alpha)) \quad (20)$$

The last part of the above equation represents the cosine of the incidence angle, α is the azimuth angle, γ is the elevation angle and θ is the angle formed between the light-shelf and the horizontal axis. Integrating the above equation, the illuminance on the back surface can be calculated:

$$E = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \int_0^{\pi} L \cdot da \cdot \cos(\gamma) \cdot d(\gamma) \cdot (\sin(\gamma) \cdot \cos(90 - \theta) - \cos(\gamma) \cdot \sin(\theta) \cdot \cos(\alpha)) \quad (21)$$

$$E = L \cdot \cos(90 - \theta) \cdot \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} da \int_0^{\pi} \cos(\gamma) \cdot d(\gamma) \cdot \sin(\gamma) - L \cdot \sin(\theta) \cdot \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos(\alpha) \cdot da \int_0^{\pi} d\gamma \cdot \cos(\gamma)^2 \quad (22)$$

$$E = L \cdot \frac{\cos(90 - \theta) + \sin(90 - \theta)}{2} \cdot \pi \quad (23)$$

Thus, the view factor of the back surface of the shelf to the ground is:

$$F_{bg} = 0.5 \cdot (\cos(90 - \theta) + \sin(90 - \theta)) \quad (24)$$

- Radiant flux due to the reflection of direct beam radiation on the back surface of the light-shelf:
If the angle of incidence between the beam sunlight and light shelf's back surface (θ_3) > 0, then

$$\Phi'_5 = \rho_{back} \cdot F_{lwl} \cdot E_{di \text{ norm}} \cdot A_{lw} \cdot \cos \theta_3 \quad (25)$$

Else,

$$\Phi'_5 = 0 \quad (26)$$

Results

To investigate the impact of a dynamic light-shelf on the radiant flux balance on the window, an analysis was performed for a South oriented window located in Athens, Greece (Lat. 37.90, Lon. 23.730). The analysis was carried out on hourly basis (8:00-18:00) for a whole year using climatic data of the area. As mentioned previously the analysis focuses on exploring the impact of light-shelf's operation on the total radiant flux impinging on the window.

There are two issues that need to be addressed when a specular light-shelf is used. The first one is related to whether the radiant flux on the window due to direct

sunlight is modified by the presence of the light shelf while the second one to its sunlight redirection ability. To express this modification we introduce a new parameter for the latter called Redirection Ratio (RR) which is defined as the ratio between the reflected to total sunlight flux on the window. A higher value of RR together with a reduction of solar radiation in comparison with the "No light-shelf" case, can be a reliable indicator for selecting the proper control strategy of a dynamic light-shelf, during the early stage of design. Figures 9 and 10 show the monthly average daily profile of redirection ratio (RR) and radiant flux (W). Summer months (June/July/Aug) show higher RR values in comparison to that during winter with control strategy CS#1 result in higher values of RR against those of CS#2 during noon. Average RR values for the summer period are 0.52, 0.49 and 0.31 for CS#1, CS#2 and static respectively while during winter months the aforementioned values change to 0.07, 0.05 and 0.1. Increasing light-shelf depth from 0.5 to 0.7 m causes an increase in the RR values by 11%, 11.36% and 0.05% during summer and 35.3%, 29.5% and 35.1% for winter months.

Figure 9: Monthly average daily profile of Redirection ratio for all control strategies (light-shelf depth 0.5m)

Figure 10: Monthly average daily profile of Redirection ratio for all control strategies (light-shelf depth 0.7m)

Since the calculation of radiant flux is performed on hourly basis for the whole year, global monthly daily radiation on the window can be easily calculated. Figure 11 shows the monthly average daily profile of the radiant flux while figure 12 the relative change of monthly average daily global radiation on the window between the all cases with static or dynamic light-shelf and the "no light-shelf" case.

Figure 11: Monthly average daily profile of radiant flux for all control strategies and light-shelf depths.

Figure 12: Relative change of monthly average daily global radiation on the window between the all cases with static or dynamic light-shelf and the “no light-shelf” case.

Installing mirror horizontal light-shelf on a window result in a reduction in radiation. This reduction is higher as the depth of the light-shelf increases especially during summer months. On the other hand, dynamic light-shelves show a completely different behaviour, increasing daily radiation on the window during summer months and reducing it in all other months. Judging from the results it seems that the best control strategy for the case examined is CS#2 (0.5 m) since daily global radiation on the window increases only by 0.88% and 0.02% for June and July and at the same time, daylighting levels, increase considerably in the non-daylit zone of the space.

Conclusions

In this work, a simplified methodology for calculating the radiant flux on a window equipped with a mirror light-shelf was proposed. Using certain assumptions this methodology can be easily implemented in a computer software and give building professionals a way to predict the performance of these systems and explore a series of design possibilities including dynamic control. As mentioned in the introduction, current building energy simulations codes do not treat dynamically moving specular light-shelves while Radiance software due to the change of sun's position and the light-shelf's tilt in each time step requires an increased processing time. For example the hourly calculations of the reflected radiation on the clerestory window for a whole year on an Intel i7-3520M 2.9Ghz, takes ~2 secs. The results indicate that static horizontal mirrored light shelves reduce radiant flux on the window in comparison to the “No light-shelf case”. This reduction is larger during the summer months due to the shading of the lower part of the window and it is accompanied by an increase in the ratio of the reflected to total luminous flux. Daylight in the non-daylit area due to this redirection is not affected much since the maximum change during summer is 5.6% for a light-shelf with 0.5 m depth. This is accompanied by a reduction of global daily radiation between 18.7 and 20.3 %.

On the other hand, dynamic light shelves increase significantly daylight levels due to sunlight redirection in the non-daylight zone by more than 1500% during summer months. It seems that the best performing dynamic light-shelf is the one controlled according to the maximization of average daylight levels in areas away from the perimeter zone (CS#2) with depth equal to the clerestory window height. This arrangement manages to maintain radiation values on the window similar to that of the “no-light-shelf” case with a slight increase (less than 1%) during June and July. The same calculation procedure can be applied in various locations, different working time schedules and different window geometric configurations while its integration to a building simulation energy code can improve the results obtained.

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