A Frequency Domain Methodology for Design and Control of Radiant Floor Systems

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Abstract

Radiant floor heating systems (RFH) offer significant potential for studying and developing energy flexibility strategies for buildings and their interaction with smart grids. Efficient design and operation of RFHs require several critical decisions on design and control variables to maintain comfortable thermal conditions in the space and floor surface temperatures within the recommended range. In this context, frequency domain techniques can assist in finding optimal design parameters prior to the performance of numerical finite difference simulations.

This paper discusses the application of frequency domain techniques to the design and control of hydronic RFHs. Transfer function analysis is used to determine the optimal thickness of a concrete slab (including different types of concrete) that minimises air temperature fluctuations. Then, this paper discusses the use of a transfer function to calculate the delay between the radiant floor heating input and the zone air temperature. An application of this transfer function to a case study is presented.

Introduction

Non-optimal material selection with regards to building type and energy supply system can lead to excessive energy consumption. Building energy models are useful tools to analyse the building designs and materials selections in order to avoid the unnecessary energy consumption in our buildings. The developed building energy models can also be utilized in various building operation and control design applications, including passive solar design and thermal mass control strategies (Candanedo and Athienitis 2010) to optimize the building operation. While nearly all modelling approaches can be used for any of those purposes, certain modelling techniques provide valuable information during early stages of design and can guide us in choosing the optimum material for the building from the thermal performance point of view.

Frequency domain techniques have been shown to be a useful tool for the design analysis on a relative basis (Athienitis and Santamouris 2002). By means of the frequency domain modelling techniques important building transfer functions can be obtained and studied and for this usually no simulation is required. Also, it has been shown to be a practical tool for building

simulation and deriving simplified grey-box building models (Saberi Derakhtenjani et al. 2015, Wang and Xu 2006, Xu et al. 2007). Frequency domain techniques are especially useful for periodic analysis of phenomena inside a building. The weather-related inputs affecting energy consumption inside a building such as solar gains, exterior temperature and heating/cooling sources are cyclic phenomena and can be modelled by means of frequency domain techniques assuming periodic conditions in the calculations. There have been a number of studies on the application of frequency domain techniques to analyse building thermal dynamics and control applications including modelling and design of building-integrated thermal energy storage (BITES) (Chen et al. 2013a, Chen et al. 2013b) and active pipe-embedded building envelope (Xie et al. 2015) as well as designing and implementing control strategies (Athienitis et al. 1990 and Chen et al. 2014). However, the full potential of frequency domain techniques in analysing different designs and control strategies for buildings is far from being reached; this may be partly attributed to the unfamiliarity of building engineers with frequency domain analysis.

This paper focuses on the application of frequency domain techniques for designing and controlling of hydronic radiant floor heating systems. It is discussed how frequency domain techniques can be utilized in analysing different thicknesses considered of the concrete radiant floors and finding the optimum thickness. Also it is demonstrated how building transfer functions can aid in designing the controlling strategies for zones with RFH. Radiant floor heating systems have been receiving considerable attention recently due to the multiple advantages they offer such as improved thermal comfort and lower energy consumption. The use of radiant heating leads to improvement in the efficiency of low-temperature heat sources: condensing oil or gas boilers, heat pumps and solar collectors assuring lower fuel or electric energy consumption (Werner Juszczuk 2018). Moreover, radiant systems operate quietly and save space. However, compared to conventional systems, radiant systems has several added complexities such as the delayed transient heat conduction within the system itself, combined surface convection and radiation from/to the system, and a drastically different resulting thermal environment,

which make them difficult to model and integrate in a comprehensive simulation. By means of frequency domain techniques and studying important transfer function over a certain frequency range, useful information can be obtained to design and control of the radiant floor heating systems.

Frequency Domain Model

This section presents the methodology to develop frequency domain models for a thermal zone. A thermal network for a zone is considered which represents the heat transfer between room surfaces, indoor and outdoor temperatures. Walls are modeled using the two-port Norton-equivalent representation. This representation gives an exact solution assuming 1-D heat conduction inside the wall (Athienitis et al. 1990). The level of detail in the thermal network depends on how radiation and convection heat transfer are modeled in the room interior zone. Combined radiative-convective heat transfer coefficients are often assumed which can be represented by a star network as shown in Figure 1.a. This network is used when there are small differences between the surfaces and room air temperature and when the heating source is mainly convective. However, in zones with high level of solar gains or in rooms with radiant heating systems, using combined heat transfer coefficients can produce significant error in the results. In those cases, convective and radiative heat transfer needs to be modeled separately (Figure 1.b). The convective and radiative coefficients are calculated as:

$$\text{convective } U_{i1} = A_i h_{c,i}, \text{ radiative } U_{ij} = A_i F_{ij}^* (4\sigma T_m^3)$$

where A_i =area of surface i , $h_{c,i}$ =convective heat transfer coefficients of surface i , F_{ij}^* =radiant heat exchange factor between surfaces i and j and T_m =estimated mean temperature.

Figure 1: Thermal network for frequency domain considering a) combined heat transfer coefficients (top), b) separate modelling of convective and radiative exchanges (bottom).

In the thermal networks shown in Figure 1, all the walls and elements with thermal mass are modelled using the two-port Norton-equivalent subnetwork which consists of the wall self-admittance, Y_s , and an equivalent source, $Q_{eq}=Y_s \times T_{ext}$, which represents the effect of an external temperature on the surface temperature. This representation eliminates all the middle nodes of the wall and gives an exact solution for one-dimensional heat conduction inside the wall without spatial discretization. The wall self-admittance (Y_s) and transfer admittance (Y_t) are calculated using the following equations (please see the appendix for more details):

$$Y_{si} = \frac{U_i + A_i k_i \gamma_i \tanh(\gamma_i l_i)}{A_i k_i \gamma_i \tanh(\gamma_i l_i) + 1}, Y_{ti} = \frac{-A_i}{U_i \cosh(\gamma_i l_i) + \frac{\sinh(\gamma_i l_i)}{k_i \gamma_i}} \quad (1)$$

Building transfer functions

In this section derivation of the building transfer functions for the detailed frequency domain model is presented for which the thermal network is shown in Figure 1.b. Node (1) represents the air inside the zone. Energy balance at node (1) gives:

$$C_a \frac{dT_1}{dt} + U_o(T_1 - T_o) + \sum_{J1} U_{1,J1}(T_1 - T_{J1}) = Q_{aux} \quad (2)$$

In which U_o is the infiltration conductance between indoor air and outdoor air, $U_{1,J1}$ are the convective conductances between the air node and other surfaces ($J1=2, 3, \dots, 8$), C_a is the thermal capacitance of the room air, T is temperature and Q_{aux} is the auxiliary source at the air node (which can be equal to zero). Using Laplace transform, equation (2) will be:

$$sC_a T_1(s) + U_o T_1(s) + \sum_{J1} U_{1,J1}(T_1(s) - T_{J1}(s)) = Q_{aux}(s) + \underbrace{U_o T_o(s)}_{Q_{eq}}$$

where s is the Laplace variable. Now, considering the floor (node (2)), the energy balance equation yields:

$$Y_{s,2}(T_2(s) - T_{ref}(s)) + \sum_{J2} U_{2,J2}(T_2(s) - T_{J2}(s)) = \underbrace{Q_{sg,2}}_{\text{solar gain}} + \underbrace{Q_{eq,2}}_{\text{radiant source}} + Q_{rad,2}$$

where $J2=1, 3, 4, \dots, 8$ and $U_{2,J2}$ are the radiative conductances between floor and other surfaces. The energy balance for all the nodes can be written in the form below:

$$\begin{bmatrix} sC_a + \sum U_{1,j} + U_{inf} & -U_{1,2} & -U_{1,3} & -U_{1,4} & -U_{1,5} & -U_{1,6} & -U_{1,7} & -U_{1,8} \\ -U_{1,2} & Y_{s,2} + \sum U_{2,j} & -U_{2,3} & -U_{2,4} & -U_{2,5} & -U_{2,6} & -U_{2,7} & -U_{2,8} \\ -U_{1,3} & -U_{2,3} & Y_{s,3} + \sum U_{3,j} & -U_{3,4} & -U_{3,5} & -U_{3,6} & -U_{3,7} & -U_{3,8} \\ -U_{1,4} & -U_{2,4} & -U_{3,4} & Y_{s,4} + \sum U_{4,j} & -U_{4,5} & -U_{4,6} & -U_{4,7} & -U_{4,8} \\ -U_{1,5} & -U_{2,5} & -U_{3,5} & -U_{4,5} & Y_{s,5} + \sum U_{5,j} & -U_{5,6} & -U_{5,7} & -U_{5,8} \\ -U_{1,6} & -U_{2,6} & -U_{3,6} & -U_{4,6} & -U_{5,6} & Y_{s,6} + \sum U_{6,j} & -U_{6,7} & -U_{6,8} \\ -U_{1,7} & -U_{2,7} & -U_{3,7} & -U_{4,7} & -U_{5,7} & -U_{6,7} & Y_{s,7} + \sum U_{7,j} & -U_{7,8} \\ -U_{1,8} & -U_{2,8} & -U_{3,8} & -U_{4,8} & -U_{5,8} & -U_{6,8} & -U_{7,8} & \sum U_{8,j} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \\ Q_7 \\ Q_8 \end{bmatrix}$$

or :

$$\mathbf{Y}_{8 \times 8} \mathbf{T}_{8 \times 1} = \mathbf{Q}_{8 \times 1} \quad (3)$$

Where $\mathbf{Y}$ is the admittance matrix, $\mathbf{T}$ is the temperature vector and $\mathbf{Q}$ is the source vector. Elements of equation (3) are in term of the Laplace variable s and can be

calculated at different frequencies ($s = j\omega_n$, $j = \sqrt{-1}$, $\omega_n = 2\pi n/p$, $p = 24$ h, $n =$ harmonic number). An important parameter is the impedance matrix $\mathbf{Z} = \mathbf{Y}^{-1}$ which contains building transfer functions at different frequencies:

$$Z_{i,j} = \frac{T_i}{Q_j} \rightarrow Z_{i,1} = \frac{T_i}{Q_1}, Z_{i,2} = \frac{T_i}{Q_2}, \dots$$

The impedance transfer function represents change in temperature of node i resulting from the effect of the source Q_j at node j . Thus, considering equation (3), temperature of node i for each frequency will be calculated as:

$$T_i = \sum_{j=1}^8 Z_{i,j} Q_j \quad (4)$$

Studying important building transfer function provides much insight into building thermal dynamics and we can evaluate design alternatives on a relative basis without detailed simulations.

In the next section, application of this methodology for a thermal zone with radiant floor heating system is demonstrated. The thermal zone considered in this study is a 3m×3m×3m office room. The ceiling and walls (except from the front wall) have 10cm of insulation giving thermal resistance of R32 (RSI 5.64 m²K/W) for each wall. The front facing wall has an approximate thermal resistance of R5 (RSI 0.88 m²K/W). The floor is made of concrete and represents the main thermal mass in the zone. A detailed frequency domain model (Figure 1.b.) is developed for the room

Transfer Function for Design Analysis

Frequency domain modeling approach is a useful tool for the design analysis on a relative basis. In this section the utilization of this tool is demonstrated.

Effect of floor concrete thickness on air temperature fluctuations for different types of concrete

Thermal properties of the concrete vary with age, temperature and humidity (Marshall 1972). Therefore, a wide range of thermal properties can be obtained depending on the conditions. However, the approach that is presented here can be applied to the relative analysis of any types of concrete (and thermal mass in general) once thermal properties are known. Here thermal performance of two types of concrete based on ASHRAE values are evaluated: light-weight and normal-weight concrete. Thermal properties of those concretes are shown in Table 1.

TABLE 1: THERMAL PROPERTIES OF TWO TYPES OF CONCRETE.

Thermal Property	Normal-weight concrete	Light-weight concrete
Conductivity (W/mK)	1.7	0.51
Specific heat (J/kgK)	800	800

Density (kg/m ³)	2200	1400
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Transfer function $Z_{1,2}$ ($=T_1/Q_2$) shows the effect of combined heat sources acting on the node 2 (floor surface) on the node 1 (inside air temperature). The magnitudes of $Z_{1,2}$ versus different frequencies for the light-weight concrete of different thicknesses are shown in Figure 2:

Figure 2: Magnitudes of transfer function $Z_{1,2}$ (i.e. air to heat input at the floor surface) for different thicknesses of light-weight concrete.

The magnitude corresponding to 5-cm shows a significant difference in comparison with the other thicknesses for the first three harmonics, confirming a significant impact for the time scales of 24, 12 and 8 hours. The fundamental (first) harmonic is particularly useful. The magnitude of the first harmonic is plotted in as a function of floor thickness.

Figure 3: Magnitude of the first harmonic for $Z_{1,2}$ (air temperature to heat input at the floor surface) as a function of concrete thickness (light-weight concrete).

It is interesting to observe that a 15-cm thickness yields the minimum room air temperature fluctuation for a given heat source on the floor (floor solar gain or radiant floor heating).

Now, if we assume a specific solar gain profile (for example a simple half sinusoidal profile with peak of 1100 W at noon) for the floor as shown in Figure 4, then the approximate daily air temperature swing due to the floor solar gain can be calculated by multiplying the magnitude of the first harmonic of the transfer function $Z_{1,2}$ and first harmonic of floor solar gain profile, Q_2 as:

$$\Delta T_{air} \cong |Z_{1,2}(1)| \times |Q_2(1)| \quad (5)$$

By means of Fourier transform, the magnitude of the first harmonic for solar gain profile (Figure 4) can be calculated and is equal to 270 W.

Figure 4: Floor solar gain profile.

Therefore, the temperature swing for each thickness is calculated from equation (5) and shown in Figure 5:

Figure 5: Approximate air temperature swing due to floor solar gain for different lightweight concrete thicknesses.

For the normal-weight concrete the magnitudes of different thicknesses at each frequency are shown in Figure 6.

Figure 6: Comparison of transfer function $Z_{1,2}$ magnitudes for different thicknesses of normal-weight concrete.

Figure 7 shows that for the normal weight concrete the optimum thickness is 20cm. Also, compared to the light-weight concrete, much smaller air temperature swings due to floor solar gain is observed as shown in Figure 8. Therefore, it is demonstrated how air temperature swing due to a specific floor heat gain source can be calculated by analysing the transfer function $Z_{1,2}$ and the floor heat gain profile in frequency domain.

It should be noted that the optimum thickness is calculated to minimize air temperature swing in the zone. However, in real design other criteria and parameters should be considered and evaluated as well including the building usage type, practicality of considering certain thicknesses for the concrete slab, etc.

Figure 7: Magnitude of the first harmonic for transfer function $Z_{1,2}$ for different thicknesses of normal weight concrete.

Figure 8: Room air temperature swing due to floor solar gain.

Derivation of the transfer function $Z_{1,aux}$

Another useful transfer function to study is the one that shows the effect of the radiant floor heating, inserted under the concrete slab, directly on the zone air temperature. Here derivation of this transfer function for the case when we have floor-heating pipes at the bottom of the slab is described.

As mentioned in the previous section, $Z_{1,2}$ is the transfer function between the zone air temperature and the source at node 2 (floor surface), $Z_{1,2} = T_1/Q_2$. The source Q_2 represent the summation of all the heating sources

that are present at node 2 including solar radiation and the effect of the radiant floor heating when transferred to the surface of the floor (Q_{rad}). Therefore, it can be written as:

$$Q_2 = Q_{sg,2} + Q_{rad} \quad (6)$$

where $Q_{sg,2}$ represent the floor solar gain and Q_{rad} is the transferred floor heating (Q_{aux}) to the surface. Figure 9 shows the schematic of the floor.

Figure 9: Schematic of floor heating.

In the frequency domain modelling methodology explained earlier, by using the Norton-equivalent representation all the intermediate nodes are eliminated and the effect of the external temperature, which can be the basement temperature for example, is represented by the equivalent source $Q_{eq} = -Y_t \times T_b$ (please see the appendix for more details). Here since the pipes are located at the bottom and auxiliary load is inserted at the bottom of the floor slab, using the energy balance equation, the temperature of the node T_{aux} can be represented as an equivalent temperature (similar to sol-air temperature):

$$T_{aux} = T_b + Q_{aux} \frac{R_{ins}}{A_2}, \quad A_2 = \text{floor area} \quad (7)$$

Therefore, by means of equation (7) we can consider T_{aux} as the external and equivalent temperature and by using the transfer admittance of the floor, the equivalent source representing the effect of the basement temperature and the auxiliary load on the floor surface temperature is defined as:

$$Q_{rad} = Q_{eq} = -Y_t(T_{aux}) = -Y_t(T_b + Q_{aux} \frac{R_{ins}}{A_2}) \quad (8)$$

Then, we know from the definition that:

$$Z_{1,2} = \frac{T_1}{Q_{sg,2} + Q_{rad}} = \frac{T_1}{Q_{sg,2} - Y_{t,2}(T_b + Q_{aux} \frac{R_{ins}}{A_2})} \quad (9)$$

$$T_1 = Z_{1,2}(Q_{sg,2}) + Z_{1,2}(-Y_{t,2}T_b) + Z_{1,2}(-Y_{t,2}Q_{aux} \frac{R_{ins}}{A_2}) \quad (10)$$

Now in the above equation let us put all the sources to zero except Q_{aux} . Then, we will have:

$$Z_{1,aux} = \frac{T_1}{Q_{aux}} = Z_{1,2} \left[\frac{-Y_{t,2}R_{ins}}{A_2} \right] \quad (11)$$

Equation (11) calculates transfer function $Z_{1,aux}$ at different frequencies with which the effect of radiant floor heating source on the room air temperature can be studied.

As expected, as the floor thickness increases the magnitude of the transfer function $Z_{1,aux}$ decreases (Figure 10).

Figure 10: Comparison of transfer function $Z_{1,aux}$ magnitudes for different thicknesses of normal-weight concrete.

An important information that can be obtained from this transfer function is the delay between the heat input from the radiant floor and its effect on the zone air temperature. This delay can be estimated by looking at the phase angle of the fundamental harmonic of the transfer function $Z_{1,aux}$. This information is very useful when designing the control strategies for a zone with radiant floor heating system. This is validated for a case study described in the next section.

RFH Experiment

The Solar Simulator-Environmental Chamber (SSEC) laboratory is an experimental facility located at Concordia University in downtown Montreal, Canada. This facility allows accurate and repeatable testing of solar systems and advanced building envelopes under standard test conditions with well-simulated solar radiation and indoor plus outdoor conditions. The temperature test range of the environmental chamber is -40°C to $+50^\circ\text{C}$.

The perimeter zone test cell (PZTC) is (very similar to the thermal zone presented in the previous sections) a $3\text{m} \times 3\text{m} \times 3\text{m}$ office placed inside the environmental chamber and it has a radiant floor heating system. The side and back walls and ceiling consists of 10cm of insulation with thermal resistance of R32 (RSI 5.64 $\text{m}^2\text{K/W}$). The front wall is a BIPV/T façade with approximately R5 (RSI 0.88 $\text{m}^2\text{K/W}$) thermal resistance. The floor is made of an 8cm thick concrete with R10 (RSI 1.76 $\text{m}^2\text{K/W}$) insulation under the hydronic system pipes in addition to a R32 (RSI 5.64 $\text{m}^2\text{K/W}$) layer at the bottom. Therefore the total thermal resistance under the concrete slab and hydronic pipes is R42 (RSI 7.4 $\text{m}^2\text{K/W}$). The pipes of the radiant floor system are made of conventional cross-linked polyethylene (PEX), and have an external diameter of 1.75cm. The pipes are installed in a “foam matrix” of

insulating material that also facilitates keeping them in place. The pipes have an approximate separation of 15 cm between them. A mechanical room provides controlled flow rate of fluid (propylene-glycol and water mixture) for the radiant floor. The schematic of the SSEC and PZTC is shown in Figure 11.

Figure 11: Schematic of SSEC with PZTC and radiant floor.

Figure 12 shows the piping configuration of the radiant floor before the concrete was poured (left) and final look of the radiant floor slab (right).

Figure 12: Radiant floor before (left) and after (right) pouring the concrete.

Considering the physical properties of the PZTC, the magnitude and phase angle of the transfer function $Z_{1,aux}$ was calculated at different frequencies using equation (11). Figure 13 shows the phase angles of $Z_{1,aux}$ at different frequencies.

Figure 13: Phase angles of transfer function $Z_{1,aux}$.

As it can be observed from Figure 13, the phase angle of the fundamental harmonics is -73.02° with negative sign indicating the delay. Therefore, considering the period of one day for the first harmonic, the magnitude of the delay in unit of time is calculated as:

$$73.02 \times 24h / 360^\circ \cong 4.9 \text{ h} \quad (12)$$

which means that there will be about 4.9 hours of delay between the heat input from the hydronic system of the

concrete slab and the PZTC air temperature. Using the experimental measurements, we can observe this delay between the hydronic floor heat input and the zone air temperature. As shown in Figure 14, when the PZTC air temperature setpoint is suddenly raised at $t = 17.1$, the thermostat commands the controller to immediately increase the radiant heat input. However, as calculated from the transfer function $Z_{1,aux}$ (i.e., the transfer function between room air and heat input) there is a delay of approximately 4.9 hours between the peak heating power and its full effect on the room air temperature peak. It should be noted that the zone thermostat used pulse-width modulation to control the air temperature; this fact accounts for the smaller “heating peaks”.

Figure 14: Delay between the radiant floor heat input and PZTC air temperature.

The result shows that in a zone that is mainly heated by a radiant floor system, when it is desired to reach a specific zone air temperature at a specific time, the controller of the heating system needs to be designed in such a way that considers this delay.

The same procedure can be applied to determine $Z_{2,aux}$, the transfer function between the floor surface temperature (T_2) response and the heat source (Q_{aux}):

$$Z_{2,aux} = \frac{T_2}{Q_{aux}} = Z_{2,2} \left[\frac{-Y_{t,2} R_{ins}}{A_2} \right] \quad (13)$$

It was observed that the phase angle of the first harmonic for the $Z_{2,aux}$ is -62° meaning $62^\circ \times 24h / 360^\circ \approx 4.1h$ of delay between the radiant floor heating and floor surface temperature. This is confirmed by the experimental measurements (Figure 15). This information can be quite useful to design the appropriate control strategy.

Figure 15: Delay between the radiant floor heat input and floor surface temperature.

Conclusion

This paper presented an application of frequency domain techniques for the design and control of zones with RFHs. It was shown that the analysis of critical transfer functions provides useful information on the thermal dynamics of RFHs, without needing to perform time-domain simulations. This information is particularly useful in the early design stages and in the development of control strategies.

It was observed that there exists an optimum floor slab thickness that minimises air temperature fluctuations. Interestingly, slabs thicker than necessary may be counterproductive, as they may result in large room temperature swings (e.g., when high solar gains are present). When the properties of the concrete are known, frequency domain analysis can be efficiently applied to find the optimum thickness for that specific concrete.

Then, derivation of the transfer function between the room air temperature and the radiant floor heating source ($Z_{1,aux}$ in this paper) is carried out for the case when the pipes are located at the bottom of the slab. The analysis of $Z_{1,aux}$ is useful to calculate the delay between the heat injection and its effect on the air temperature. This delay was calculated using the geometry and material properties of a case study, and later confirmed with experimental measurements. The paper also discussed how this information would be useful for designing the control strategies of zones with radiant floor heating systems.

Future studies will include the design and application of predictive control strategies for zones with radiant floor heating systems by means of information obtained from the introduced transfer functions. Also, when radiant floor control is done through surface temperature instead of the air temperature, reduced cycling of heating system and improved energy flexibility is expected which will be examined in the future tests.

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Appendix

The procedure to obtain the transmission matrix (also known as the cascade matrix) of heat transfer for one dimensional homogenous wall in Laplace domain is described here. Consider a slab as shown in figure below with two surfaces (exterior and interior) named surface 1 and 2:

The 1-D Fourier conduction law is applied as follows:

$$\alpha \frac{\partial^2 T(x,t)}{\partial x^2} = \frac{\partial T(x,t)}{\partial t} \quad (14)$$

where α is thermal diffusivity and equals to: $k/\rho C_p$. Applying the Laplace transform to the equation above assuming the initial condition $T(x, t=0) = 0$, transforms the partial differential equation to an ordinary differential equation:

$$\alpha \frac{\partial^2 T(x,s)}{\partial x^2} = sT(x,s) \quad (15)$$

for which the solution is obtained as:

$$T(x,s) = C_1 e^{\gamma x} + C_2 e^{-\gamma x} = M \cosh(\gamma x) + N \sinh(\gamma x) \quad (16)$$

Where $\gamma = \sqrt{s/\alpha}$. Then, for the heat flux we will have:

$$q = -k \frac{dT}{dx} \Rightarrow q(x,s) = -Mk\gamma \sinh(\gamma x) - Nk\gamma \cosh(\gamma x) \quad (17)$$

Applying last two equations to each surface gives:

$$\begin{cases} T_1 = T(x=0, s) = M \\ q_1 = q(x=0, s) = -Nk\gamma \\ T_2 = T(x=L, s) = M \cosh(\gamma L) + N \sinh(\gamma L) \\ q_2 = q(x=L, s) = Mk\gamma \sinh(\gamma L) + Nk\gamma \cosh(\gamma L) \end{cases}$$

or in the matrix form it can be written as:

$$\begin{bmatrix} T_2 \\ q_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cosh(\gamma L) & -\sinh(\gamma L)/k\gamma \\ k\gamma \sinh(\gamma L) & -\cosh(\gamma L) \end{bmatrix}}_{\text{TR}} \begin{bmatrix} T_1 \\ q_1 \end{bmatrix} \quad (18)$$

in equation (18), the matrix TR is the transmission matrix for the slab. Equation (18) is the exact theoretical solution for the one-dimensional heat transfer that relates the temperatures and heat fluxes on the two surfaces of the slab to each other.

Now by rearranging equation (18) we will have:

$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} k\gamma / \tanh(\gamma L) & -k\gamma / \sinh(\gamma L) \\ -k\gamma / \sinh(\gamma L) & k\gamma / \tanh(\gamma L) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \quad (19)$$

and the respective admittance transfer functions are obtained as:

$$(\text{self-admittance}) \quad Y_{1,1} = \frac{q_1}{T_1} = \frac{k\gamma}{\tanh(\gamma L)} \quad (T_2 = 0) \quad (20)$$

$$(\text{Transfer-admittance}) \quad Y_{1,2} = \frac{q_1}{T_2} = \frac{-k\gamma}{\sinh(\gamma L)} \quad (T_1 = 0) \quad (21)$$

Actually if we consider the transmission matrix in equation (18) as:

$$\text{TR} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Then it can be shown that:

$$Y_{1,1} = D/B, \quad Y_{1,2} = (BC - AD)/B \quad (22)$$

In the case when there is negligible thermal mass (an insulation layer) the TR matrix for the layer will be equal to:

$$\begin{bmatrix} 1 & 1/u \\ 0 & 1 \end{bmatrix}$$

where u is the insulation u-value. Therefore, in the case where we have an insulation layer on the exterior surface of the slab, the transmission matrix will be:

$$\begin{bmatrix} \cosh(\gamma L) & \sinh(\gamma L)/k\gamma \\ k\gamma \sinh(\gamma L) & \cosh(\gamma L) \end{bmatrix} \begin{bmatrix} 1 & 1/u \\ 0 & 1 \end{bmatrix} \quad (23)$$

Then, the admittance transfer functions will be obtained as:

$$Y_{1,1} = \frac{u + k\gamma \tanh(\gamma L)}{\frac{u}{k\gamma} \tanh(\gamma L) + 1}, \quad \text{Self admittance } (Y_s)$$

$$Y_{1,2} = \frac{-1}{\frac{\cosh(\gamma L)}{u} + \frac{\sinh(\gamma L)}{k\gamma}}, \quad \text{Transfer admittance } (Y_t)$$

Then using the above transfer functions we can find the Norton-equivalent thermal network for a multilayered wall as shown below:

Figure 16: Multi-layered wall (left) and equivalent thermal network (right).