Several Types of Stronger and Weaker of α rs- Continuous Functions

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Abstract

This work led us to present a new class of stronger and weaker of α rs-- continuous functions called (completely α rs-- continuous functions, almost α rs-- continuous functions, α rs-- totally continuous and totally α rs-- continuous functions). Furthermore, some of their characteristics and the relationship between them are studied and discussed.

Keywords: α rs--closed sets, α rs--continuous functions, strongly α rs--continuous. Completely continuous, almost continuous and totally continuous.

1. Introduction

Several researchers have introduced and studied some stronger and weaker of -- continuous functions. Arya. S.P and Gupta . R [2], Singal . M .K and Singal . A.R [10] , Jain .R.C [4] and Dunya .M .H [3] , they have presented and studied completely continuous , almost continuous , totally continuous , α rs--continuous functions and strongly α rs--continuous respectively .

This paper aim to study and give new types of strong and weak of α rs--continuous functions namely (completely α rs-- continuous, almost α rs--continuous, α rs-- totally continuous and totally α rs-- continuous) functions and proved some properties of these types of functions .

2. Preliminaries

Some definitions and theories that we need in this paper will be presented.

Definition (2-1) :- A subset $\mathcal{M}$ of space $(X, \mathfrak{t})$ is called :

- 1- α —closed set [8] if $\text{cl}(\text{int}(\text{cl}(\mathcal{M}))) \subseteq \mathcal{M}$.
- 2- semi— pre closed [1] if $\text{int}(\text{cl}(\text{in } \mathfrak{t}(\mathcal{M}))) \subseteq \mathcal{M}$.
- 3- regular open [11] if $\mathcal{M} = \text{int}(\text{cl}(\mathcal{M}))$ and regular closed if $\mathcal{M} = \text{cl}(\text{int}(\mathcal{M}))$.

The α — closure (resp. semi- pre closure) of sub sets $\mathcal{M}$ of X is the intersection of all α —closed (resp. semi- pre closed) sets containing $\mathcal{M}$ and denoted by $\alpha\text{cl}(\mathcal{M})$ (resp. $\text{spcl}(\mathcal{M})$) .

Definition (2-2) : A sub set $\mathcal{M}$ of topological space X is said to be :

- 1- regular generalized closed set (briefly , rg—closed) [9] if $\text{cl}(\mathcal{M}) \subseteq N$ whenever $\mathcal{M} \subseteq N$ and N is a regular open set X .
- 2- regular presemi closed (briefly , rps—closed) [7] if $\text{spcl}(\mathcal{M}) \subseteq N$ whenever $\mathcal{M} \subseteq N$ and N is an rg- open set in X .
- 3- α rps—closed set [3] if $\alpha\text{cl}(N) \subseteq N$ whenever $\mathcal{M} \subseteq N$ and N is rps-open in X .

The complement α rps — closed) sets is called α rps- open sets . **The class** of all regular closed [resp. α rps —closed] subsets of X is denoted by $\mathcal{RC}(X, \mathfrak{t})$ [resp . $\alpha\mathcal{RPS}(X, \mathfrak{t})$] .

Remark (2-3) : In [3] , [5] , [11] the following is proven :

- 1- Every regular open (resp. regular closed) set is open(resp. closed) .
- 2- Every open (resp . closed) set is α rps—open (resp . α rps— closed) set.
- 3- Every clopen set is regular open and regular closed .

Definition (2-4) , [6] :- A space X is called Locally Indiscrete space if every open set is closed .

Theorem(2-5) , [6] :- A space X is called Locally Indiscrete space if every open set is regular open.

Theorem (2-6), [5] :- A space X is Extremely disconnected space if regular open (regular closed) set are both open and closed set (which is mean clopen set).

Theorem (2-7), [3] :- A space X is $\mathcal{T}^{*}_{1/2}$ — spaces if every α rps open (resp. α rps—closed) sets is open (resp. closed) set.

Definition (2-8): A function $h: (X, \mathfrak{t}) \rightarrow (Y, q)$ is :

- 1- Completely continuous [2] if the inverse image of each open(closed) set in Y is an regular open(closed regular) in X
- 2- Almost continuous [10] if the inverse image of each regular open(regular closed) set in Y is an open(closed) set in X
- 3- Totally continuous [4] if the inverse image of each open(closed) set in Y is clopen set in X .
- 4- α rps--continuous [3] if the inverse image of each open(closed) set in Y is α rps—open(is α rps—closed) in X .
- 5- Strongly α rps -- continuous [3] the inverse image of each α rps—open(is α rps—closed) set in Y is open(closed) in X .

3 Several Types of Stronger and Weaker of α rps- Continuous Functions:

In this section, we give a new class of stronger and weaker of α rps -- continuous functions in topological spaces and we discuss the relation among and proved some of its properties . Now , we will start with the first type

Definition (3—1):- A function $h : (X, \tau) \rightarrow (Y, \eta)$ is called **completely α prps –continuous** if $h^{-1}(\mathcal{M})$ is regular open in X for all α prps—open set $\mathcal{M}$ in Y .

Example(3-2):- Let $X = \{i, j, \ell\}$ and $\tau = \{X, \{i\}, \{j, \ell\}, \emptyset\}$, where $\alpha\mathcal{RPSO}(X, \tau) = \mathcal{RO}(X, \tau) = \{X, \{i\}, \{j, \ell\}, \emptyset\}$, define $h : (X, \tau) \rightarrow (X, \tau)$ by $h(i)=i$, $h(j)=j$ and $h(\ell)=\ell$, obviously that h is completely α prps -- continuous.

Proposition(3-3):-

A function $h : (X, \tau) \rightarrow (Y, \eta)$ is completely α prps -- continuous iff $h^{-1}(\mathcal{M})$ is regular closed in X for all α prps—closed set $\mathcal{M}$ in Y .

Proof :-

If a function h is completely α prps -- continuous and $\mathcal{M}$ is α prps—closed set in Y , then $\mathcal{M}^c$ is α prps—open in Y . Since, h is completely α prps -- continuous. Thus $h^{-1}(\mathcal{M}^c)$ is regular open in X , but $h^{-1}(\mathcal{M}^c) = X - h^{-1}(\mathcal{M}) = (h^{-1}(\mathcal{M}))^c$. Hence, $h^{-1}(\mathcal{M})$ is regular closed in X . Reciprocally, let $\mathcal{M}$ be α prps -open in Y . Then, $\mathcal{M}^c$ is α prps—closed in Y . According to the assumption $h^{-1}(\mathcal{M}^c)$ is regular closed set in X . But, $h^{-1}(\mathcal{M}^c) = X - h^{-1}(\mathcal{M}) = (h^{-1}(\mathcal{M}))^c$. Hence, $h^{-1}(\mathcal{M})$ is a regular open in X . Therefore, h is completely α prps—continuous.

Proposition(3-4):- Each completely α prps -- continuous function is

- 1- Completely continuous
- 2- Almost continuous
- 3- α prps—continuous
- 4- Strongly α prps—continuous

Proof (1):-

Let a function $h : (X, \tau) \rightarrow (Y, \eta)$ is completely α prps -- continuous and let a set $\mathcal{M}$ be an open in Y , since each open set is α prps—open. Then, $\mathcal{M}$ is α prps—open in Y . According to the assumption h is completely α prps -- continuous. Hence, $h^{-1}(\mathcal{M})$ is regular open in X . Therefore, a function h is completely continuous.

Likewise, we proof the points 2,3, and 4.

The reverse of proposition (3-4) is not right. The following examples illustrates this

Example (3-5):-

Let $X=Y=\{i, j, \ell\}$, $\tau=\{X, \{i\}, \{j\}, \{i, j\}, \emptyset\}$ and $\lambda=\{Y, \{i\}, \emptyset\}$. where $\alpha\mathcal{RPSO}(Y, \lambda) = \{Y, \{i\}, \{i, \ell\}, \{i, j\}, \emptyset\}$ and $\mathcal{RO}(X, \tau) = \{X, \{i\}, \{j\}, \emptyset\}$, define $h : (X, \tau) \rightarrow (Y, \lambda)$ by $h(i)=i$, $h(j)=\ell$ and $h(\ell)=j$, we can see a function h is completely continuous, but is not

completely α rps -- continuous . since for the open set $\mathcal{M} = \{i, j\}$ in $\mathbb{Y}$, $h^{-1}(\mathcal{M}) = h^{-1}(\{i, j\}) = \{i, \ell\}$ is regular open in $\mathbb{X}$.

Example (3-6):-

Let $\mathbb{X} = \{i, j, \ell\}$, $\mathbf{t} = \{\mathbb{X}, \{i\}, \{j\}, \{i, j\}, \phi\}$. where $\alpha\mathcal{RPSO}(\mathbb{X}, \mathbf{t}) = \{\mathbb{X}, \{i\}, \{j\}, \{i, j\}, \phi\}$ and $\mathcal{RO}(\mathbb{X}, \mathbf{t}) = \{\mathbb{X}, \{i\}, \{j\}, \phi\}$, $h : (\mathbb{X}, \mathbf{t}) \rightarrow (\mathbb{X}, \mathbf{t})$ by $h(i)=i$ by , $h(j)=j$ and $h(\ell)=\ell$, clear that h is (almost continuous , strongly α rps -- continuous and α rps -- continuous) , but is not completely α rps -- continuous . since for the α rps -- open set $\mathcal{M} = \{i, j\}$ in $\mathbb{Y}$, $h^{-1}(\mathcal{M}) = h^{-1}(\{i, j\}) = \{i, j\}$ is regular open in $\mathbb{X}$.

The proposition in down present the condition make the proposition(3-4) true:

Proposition (3-7):- A function $h : (\mathbb{X}, \mathbf{t}) \rightarrow (\mathbb{Y}, \lambda)$ is completely - - α rps continuous If

- 1- $\mathbb{Y}$ is $\mathcal{T}_{*1/2}$ —space and h is completely continuous.
- 2- $\mathbb{Y}$ is discrete space , $\mathbb{X}$ is Locally Indiscrete and h is almost continuous .
- 3- $\mathbb{X}$ is Locally Indiscrete and h is strongly α rps -- continuous .
- 4- $\mathbb{Y}$ is $\mathcal{T}_{*1/2}$ —space , $\mathbb{X}$ is Locally Indiscrete and h is α rps -- continuous.

Proof(1):-

let a set $\mathcal{M}$ be α rps - open in $\mathbb{Y}$, since $\mathbb{Y}$ is $\mathcal{T}_{*1/2}$ —space. Thus , $\mathcal{M}$ is open in $\mathbb{Y}$. by assumption h is completely continuous . Hence , $h^{-1}(\mathcal{M})$ is regular open in $\mathbb{X}$. Therefore , h is completely-- α rps continuous .

Proof(2):-

Let $\mathcal{M}$ be α rps - open in $\mathbb{Y}$, since $\mathbb{Y}$ is discrete space. Thus , $\mathcal{M}$ is both open and closed set in $\mathbb{Y}$ (which is mean $\mathcal{M}$ is clopen) . Since (each clopen set is regular open) . This lead $\mathcal{M}$ is regular open in $\mathbb{Y}$. Also , since h is almost continuous . Hence , $h^{-1}(\mathcal{M})$ is open in $\mathbb{X}$. By hypotheses $\mathbb{X}$ is Locally Indiscrete , then $h^{-1}(\mathcal{M})$ is regular open in $\mathbb{X}$. Therefore , h is completely α rps--continuous .

Proof(3):-

Let $\mathcal{M}$ be α rps - open in $\mathbb{Y}$, his strongly α rps -- continuous. This lead $h^{-1}(\mathcal{M})$ is open in $\mathbb{X}$. By hypotheses $\mathbb{X}$ is Locally Indiscrete , then $h^{-1}(\mathcal{M})$ is regular open in $\mathbb{X}$. Therefore , h is completely α rps--continuous .

Proof(4):-

let a set $\mathcal{M}$ be α rps - open in $\mathbb{Y}$, since $\mathbb{Y}$ is $\mathcal{T}_{*1/2}$ —space. Thus , $\mathcal{M}$ is open in $\mathbb{Y}$. by assumption h is α rps--continuous. Hence , $h^{-1}(\mathcal{M})$ is open in $\mathbb{Y}$, since $\mathbb{Y}$ is discrete space. Thus , $\mathcal{M}$ is both open and closed set in $\mathbb{Y}$ (which is mean $\mathcal{M}$ is clopen) . Since (each clopen set is regular open) . This lead $\mathcal{M}$ is regular open in $\mathbb{Y}$. Hence, completely α rps--continuous .

Now, we will give a new kind of weaker α rps-- continuous functions.

Definition (3—8):-

A function $h : (X, \mathbf{t}) \rightarrow (Y, q)$ is called **almost α rps -- continuous** if $h^{-1}(\mathcal{M})$ is α rps—open set in X for all regular open $\mathcal{M}$ in Y .

Example(3-9):-

Let $X = \{i, j, \ell\}$ and $\mathbf{t} = \{X, \{j\}, \phi\}$, where $\alpha\mathcal{RPSO}(X, \mathbf{t}) = \{X, \{j\}, \{i, j\}, \{j, \ell\}, \phi\}$ and $\mathcal{RO}(X, \mathbf{t}) = \{X, \phi\}$, define $h : (X, \mathbf{t}) \rightarrow (X, \mathbf{t})$ by $h(i) = i$, $h(j) = j$ and $h(\ell) = \ell$, obviously that h is almost α rps -- continuous.

Proposition(3-10):-

$h : (X, \mathbf{t}) \rightarrow (Y, q)$ is almost α rps--continuous function iff $h^{-1}(\mathcal{M})$ is α rps—closed set in X for each regular closed set $\mathcal{M}$ in Y .

Proof :-

Let h is almost α rps -- continuous and $\mathcal{M}$ is regular closed set in Y , then $\mathcal{M}^c$ is regular open in Y , h is almost α rps -- continuous. So we have $h^{-1}(\mathcal{M}^c)$ is α rps - open in X , but $h^{-1}(\mathcal{M}^c) = X - h^{-1}(\mathcal{M}) = (h^{-1}(\mathcal{M}))^c$. Hence, $h^{-1}(\mathcal{M})$ is α rps-closed in X . Reciprocally, let $\mathcal{M}$ be regular open in Y . Then, $\mathcal{M}^c$ is regular closed Y . By assumption, we get $h^{-1}(\mathcal{M}^c)$ is α rps - closed set in X . But, $h^{-1}(\mathcal{M}^c) = X - h^{-1}(\mathcal{M}) = (h^{-1}(\mathcal{M}))^c$. Hence, $h^{-1}(\mathcal{M})$ is α rps-open in X . Therefore, h is almost α rps—continuous.

Proposition(3-11):-

A function $h : (X, \mathbf{t}) \rightarrow (Y, q)$ is almost α rps-- continuous if h is

- 1- Completely α rps-- continuous
- 2- Completely continuous
- 3- Almost continuous
- 4- α rps-- continuous
- 5- Strongly α rps-- continuous

Proof(1) :-

Let h be completely α rps—continuous and $\mathcal{M}$ be regular open in Y . Since (all regular open set is α rps- open). So $\mathcal{M}$ is α rps- open in Y . This lead $h^{-1}(\mathcal{M})$ is regular open in X . Also, since (each regular open is α rps- open). Hence, $h^{-1}(\mathcal{M})$ is α rps-open in X . Therefore, h is almost α rps—continuous.

In the same way, we prove the rest of the points, (2), (3), (4) and (5).

The examples below show that the inverse of the above proposition does not have to be correct in general.

Example (3-12):- Let $X = \{i, j, \ell\}$, $\mathbf{t} = \{X, \{i\}, \{i, \ell\}, \phi\}$. where $\mathbf{RO}(X, \mathbf{t}) = \{X, \phi\}$, $\alpha\mathbf{RPSO}(X, \mathbf{t}) = \{X, \{i\}, \{i, j\}, \{i, \ell\}, \phi\}$, define $h : (X, \mathbf{t}) \rightarrow (X, \mathbf{t})$ by $h(i)=i$, $h(j)=j$ and $h(\ell)=\ell$, it certainly h is almost α rps--continuous, but is not completely continuous. since the set $\mathcal{M} = \{i\}$ is open in X , but $h^{-1}(\mathcal{M}) = h^{-1}(\{i\}) = \{i\}$ is not regular open in X .

Example (3-13):- Let $X = Y = \{i, j, \ell\}$, $\mathbf{t} = \{X, \{i\}, \phi\}$ and $\lambda = \{Y, \{i\}, \{j\}, \{i, j\}, \phi\}$. where $\mathbf{RO}(Y, \lambda) = \{Y, \phi, \{i\}, \{j\}\}$, $\alpha\mathbf{RPSO}(X, \mathbf{t}) = \{X, \{i\}, \{i, j\}, \{i, \ell\}, \phi\}$, $\mathbf{RO}(X, \mathbf{t}) = \{X, \phi\}$ and let $h : (X, \mathbf{t}) \rightarrow (Y, \lambda)$ define by $h(i)=h(j)=i$ and $h(\ell)=\ell$, clearly h is almost α rps--continuous, but is not almost continuous. since the set $\mathcal{M} = \{i\}$ is regular open in Y , but $h^{-1}(\mathcal{M}) = h^{-1}(\{i\}) = \{i, j\}$ is not open in X .

Example (3-14):-

Let $X = Y = \{i, j, \ell\}$, $\mathbf{t} = \{X, \{i\}, \{i, \ell\}, \phi\}$ and $\lambda = \{Y, \{i\}, \phi\}$. where $\mathbf{RO}(Y, \lambda) = \{Y, \phi\}$, $\alpha\mathbf{RPSO}(X, \mathbf{t}) = \{X, \{i\}, \{i, \ell\}, \{i, j\}, \phi\}$, $\mathbf{RO}(X, \mathbf{t}) = \{X, \phi\}$ and let $h : (X, \mathbf{t}) \rightarrow (Y, \lambda)$ define by $h(i)=j$, $h(j)=i$ and $h(\ell)=\ell$, it clear that h is almost α rps--continuous, but h is not (completely α rps--continuous and strongly α rps--continuous). because, the set $\mathcal{M} = \{i, j\}$ is α rps--open in Y , but $h^{-1}(\mathcal{M}) = h^{-1}(\{i, j\}) = \{i, j\}$ is not regular open and is not open set in X . Also, h is not α rps--continuous, since the set $\mathcal{M} = \{i\}$ is open in Y , but $h^{-1}(\mathcal{M}) = h^{-1}(\{i\}) = \{j\}$ is not α rps--open in X .

Now, we will present the conditions that make the proposition (3-12) correct

Proposition (3-15):- If $h : (X, \mathbf{t}) \rightarrow (Y, \mathbf{q})$ be almost α rps--continuous and X and Y are both discrete spaces, then h is Completely α rps--continuous

Proof (1) :- Let $\mathcal{M}$ be α rps--open in Y . Since Y is discrete space. So, we get $\mathcal{M}$ is α rps--open in Y . h is almost α rps--continuous, hence $h^{-1}(\mathcal{M})$ is α rps--open in X . Also, since X is discrete space. Hence, $h^{-1}(\mathcal{M})$ is regular open in X . Therefore, h is completely α rps--continuous

Corollary (3-16):- If $h : (X, \mathbf{t}) \rightarrow (Y, \mathbf{q})$ be almost α rps--continuous and X and Y are both discrete spaces, then h is

- 1- Completely continuous
- 2- Almost continuous
- 3- α rps--continuous
- 4- Strongly α rps--continuous

Proof :-

It is clear from proposition (3-15) and proposition (3-4).

The composition between these kinds of functions, we will present in the following.

Proposition (3-17):- The composition of two completely α rps--continuous functions is also completely α rps--continuous.

Proof :- Let $h : (X, \mathbf{t}) \rightarrow (Y, \lambda)$ and $\mathcal{P} : (Y, \lambda) \rightarrow (Z, \delta)$ are two completely α rps--continuous and a set $\mathcal{N}$ be α rps--open in Z , since $\mathcal{P}$ is completely α rps--continuous, then $\mathcal{P}^{-1}(\mathcal{N})$ is regular open in Y .

since (each regular open is α rsps – open). This lead $\mathcal{P}^{-1}(\mathcal{N})$ is α rsps – open set in $\mathcal{Y}$. h is completely α rsps -- continuous . Thus, $h^{-1}(\mathcal{P}^{-1}(\mathcal{N}))$ is α rsps -open in $\mathcal{X}$. But , $h^{-1}(\mathcal{P}^{-1}(\mathcal{N})) = (\mathcal{P} \circ h)^{-1}(\mathcal{N})$. Therefore, $\mathcal{P} \circ h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Z}, \delta)$ is completely α rsps-- continuous .

Proposition (3-18):-

If $h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Y}, \lambda)$ is completely α rsps-- continuous and $\mathcal{P} : (\mathcal{Y}, \lambda) \rightarrow (\mathcal{Z}, \delta)$ is strongly α rsps-- continuous, then $\mathcal{P} \circ h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Z}, \delta)$ is completely α rsps-- continuous.

Proof :-

let $\mathcal{N}$ be α rsps –open in $\mathcal{Z}$, since $\mathcal{P}$ is strongly α rsps -- continuous , then $\mathcal{P}^{-1}(\mathcal{N})$ is open in $\mathcal{Y}$. since (each open is α rsps – open). This lead $\mathcal{P}^{-1}(\mathcal{N})$ is α rsps – open set in $\mathcal{Y}$. h is completely α rsps --continuous . Thus, $h^{-1}(\mathcal{P}^{-1}(\mathcal{N}))$ is α rsps -open in $\mathcal{X}$. But , $h^{-1}(\mathcal{P}^{-1}(\mathcal{N})) = (\mathcal{P} \circ h)^{-1}(\mathcal{N})$. Therefore, $\mathcal{P} \circ h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Z}, \delta)$ is completely α rsps—continuous.

Likewise , the following propositions prove :

Proposition (3-19):-

Let $h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Y}, \lambda)$ be function and $\mathcal{P} : (\mathcal{Y}, \lambda) \rightarrow (\mathcal{Z}, \delta)$ be completely α rsps-- continuous Then , $h \circ \mathcal{P} : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Z}, \delta)$ is completely α rsps—continuous . If h is

- 1- Completely continuous.
- 2- Totally continuous .

Proposition (3-20):-

Let $h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Y}, \lambda)$ be function and $\mathcal{P} : (\mathcal{Y}, \lambda) \rightarrow (\mathcal{Z}, \delta)$ be completely α rsps-- continuous Then , $h \circ \mathcal{P} : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Z}, \delta)$ is

- 1- Strongly α rsps—continuous . If h almost continuous.
- 2- α rsps—continuous . If h is α rsps—continuous.

Remark (3-21):-

The composition of two almost α rsps—continuous is not to be necessary almost α rsps—continuous. the example below shows that :

Example (3-22):-

Let $\mathcal{X} = \mathcal{Y} = \mathcal{Z} = \{i, j, \ell\}$, $\mathfrak{t} = \{\mathcal{X}, \{i\}, \{j, \ell\}, \phi\}$ and $\lambda = \{\mathcal{Y}, \{i\}, \phi\}$. $\delta = \{\mathcal{Z}, \{i\}, \{j\}, \{i, j\}, \phi\}$, where $\mathcal{RO}(\mathcal{Z}, \delta) = \{\mathcal{Z}, \{i\}, \{j\}, \phi\}$, $\alpha\mathcal{RPSO}(\mathcal{X}, \mathfrak{t}) = \{\mathcal{X}, \{i\}, \{j, \ell\}, \phi\}$, $\mathcal{RO}(\mathcal{Y}, \lambda) = \{\mathcal{Y}, \phi\}$ and let $h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Y}, \lambda)$ define by $h(i) = i$, $h(j) = j$, $h(\ell) = \ell$ and $\mathcal{P} : (\mathcal{Y}, \lambda) \rightarrow (\mathcal{Z}, \delta)$ define by $\mathcal{P}(i) = \mathcal{P}(j) = i$ and $\mathcal{P}(\ell) = \ell$, very clear the functions h and $\mathcal{P}$ are almost α rsps --continuous , but $h \circ \mathcal{P} : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Z}, \delta)$ not almost α rsps—continuous. because , the set $\mathcal{M} = \{i\}$ is regular open in $\mathcal{Z}$, but $h^{-1}(\mathcal{P}^{-1}(\mathcal{M})) = h^{-1}(\mathcal{P}^{-1}(\{i\})) = h^{-1}(\{i, j\}) = \{i, j\}$ is not α rsps– open in $\mathcal{X}$.

The proposition below present the condition that makes Remark(3-22) correct :

Proposition (3-23):-

If $h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Y}, \lambda)$, $\mathcal{P} : (\mathcal{Y}, \lambda) \rightarrow (\mathcal{Z}, \delta)$ are two almost α rsps -- continuous and $\mathcal{Y}$ is discrete space . Then, $\mathcal{P} \circ h : (\mathcal{X}, \mathfrak{t}) \rightarrow (\mathcal{Z}, \delta)$ is also almost α rsps-- continuous.

Proof :-

Let a set N be regular open in Z , a function $\mathcal{P}$ is almost α rps -- continuous, then $\mathcal{P}^{-1}(N)$ is α rps -- open in Y . since Y is discrete space. This lead $\mathcal{P}^{-1}(N)$ are both open and closed (which is mean clopen), since (each clopen is regular open), so we obtain $\mathcal{P}^{-1}(N)$ regular open set in Y . h is almost α rps -- continuous. Thus, $h^{-1}(\mathcal{P}^{-1}(N))$ is α rps -- open in X . But, $h^{-1}(\mathcal{P}^{-1}(N)) = (\mathcal{P} \circ h)^{-1}(N)$. Therefore, $\mathcal{P} \circ h : (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is almost α rps -- continuous.

Proposition (3-24):- Let $h : (X, \mathfrak{t}) \rightarrow (Y, \lambda)$ be completely α rps -- continuous and $\mathcal{P} : (Y, \lambda) \rightarrow (Z, \delta)$ be any function. Then, $\mathcal{P} \circ h : (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is almost α rps -- continuous. If $\mathcal{P}$ is

- 1- almost continuous.
- 2- completely continuous.
- 3- totally continuous.
- 4- α rps -- continuous.
- 5- strongly α rps -- continuous.

Proof (1):- Let N be regular open in Z , by hypotheses $\mathcal{P}$ is almost continuous, then $\mathcal{P}^{-1}(N)$ is open in Y . Since (all open is α rps -- open), hence $\mathcal{P}^{-1}(N)$ α rps -- open in Y . h is completely α rps -- continuous. Thus, $h^{-1}(\mathcal{P}^{-1}(N))$ is α rps -- open in X . But, $h^{-1}(\mathcal{P}^{-1}(N)) = (\mathcal{P} \circ h)^{-1}(N)$. Therefore, $\mathcal{P} \circ h : (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is almost α rps -- continuous.

The points -2-, -3-, -4- and -5- are prove in a similar way.

Proposition (3-25):-

The composition $\mathcal{P} \circ h : (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is almost α rps -- continuous if $\mathcal{P} : (Y, \lambda) \rightarrow (Z, \delta)$ is completely α rps -- continuous and $h : (X, \mathfrak{t}) \rightarrow (Y, \lambda)$ is

- 1- almost continuous.
- 2- completely continuous.
- 3- totally continuous.
- 4- α rps -- continuous.
- 5- strongly α rps -- continuous.

Proof (1):- Let N be regular open in Z , since (each regular open is α rps -- open), then N is α rps -- open, from the imposition a function $\mathcal{P}$ is completely α rps -- continuous, then $\mathcal{P}^{-1}(N)$ is regular open in Y . h is almost continuous. This lead $h^{-1}(\mathcal{P}^{-1}(N))$ is open in X . hence $h^{-1}(\mathcal{P}^{-1}(N))$ is α rps -- open in X . But, $h^{-1}(\mathcal{P}^{-1}(N)) = (\mathcal{P} \circ h)^{-1}(N)$. Therefore, $\mathcal{P} \circ h : (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is almost α rps -- continuous.

As for the points (2), (3), (4) and (5) are prove by the same way.

Below, we will introduce another kind of stronger of α rps -- continuous functions

Definition (3-26):-

A function $h : (X, \mathfrak{t}) \rightarrow (Y, \lambda)$ is called **α rps -- totally continuous** if $h^{-1}(\mathcal{M})$ is clopen in X , for all α rps -- open set $\mathcal{M}$ in Y .

Example (3-27):-

Let $X = Y = \{i, j, \ell\}$, $\mathfrak{t} = \{X, \{i\}, \{j\}, \{i, j\}, \{j, \ell\}, \phi\}$, $\lambda = \{Y, \{j\}, \{i, \ell\}$ where $\alpha \mathcal{RPSO}(X, \mathfrak{t}) = \{X, \{i\}, \{j, \ell\}, \phi\}$ and $\mathcal{RO}(Y, \lambda) = \{Y, \{j\}, \{i, \ell\}$ and let $h : (X, \mathfrak{t}) \rightarrow (Y, \lambda)$ by $h(i) = j$, $h(j) = \ell$ and $h(\ell) = i$, we can see h is α rps -- totally continuous

Proposition(3-28):-Each α rps--totally continuous is completely α rps--continuous

Proof :-Let a function $h : (X, \tau) \rightarrow (Y, \eta)$ is α rps--totally continuous and $\mathcal{M}$ be α rps--open in Y . h is α rps--totally continuous. Hence, $h^{-1}(\mathcal{M})$ is clopen in X , since (all clopen is regular open). Thus, $h^{-1}(\mathcal{M})$ is in X . Therefore, a function h is α rps--totally continuous

Corollary (3-29):- Each α rps--totally continuous function is

- 1- totally continuous.
- 2- completely continuous
- 3- Almost continuous
- 4- α rps--continuous
- 5- Strongly α rps--continuous
- 6- Almost α rps--continuous

Proof :- This is evident by proposition (3-28), proposition(3-4) and proposition (3-11)step-1-

In the below some examples prove that the converse of above proposition(3-28) and corollary(3-29) are not have to be correct in general.

Example (3-30):-Let $X = \{i, j, \ell\}, \tau = \{X, \{j\}, \{\ell\}, \{j, \ell\}, \phi\}$. where $\mathcal{RO}(X, \tau) = \{X, \phi, \{j\}, \{\ell\}\}$, $\alpha\mathcal{RPSO}(X, \tau) = \{X, \{j\}, \{\ell\}, \{j, \ell\}, \phi\}$, define $h : (X, \tau) \rightarrow (X, \tau)$ by $h(i) = i, h(j) = j$ and $h(\ell) = \ell$, apparent that h is strongly α rps--continuous and (α rps--continuous, almost continuous and almost α rps--continuous), but is not α rps--totally continuous. since the set $\mathcal{M} = \{j\}$ is α rps--open in X , but $h^{-1}(\mathcal{M}) = h^{-1}(\{j\}) = \{j\}$ is not clopen in X .

Example(3-31):-Let $X = Y = \{i, j, \ell\}, \tau = \{X, \{i\}, \{j, \ell\}, \phi\}, \lambda = \{Y, \{i\}, \phi\}$, where $\alpha\mathcal{RPSO}(Y, \lambda) = \{Y, \{i\}, \{i, j\}, \{i, \ell\}, \phi\}$. define $h : (X, \tau) \rightarrow (Y, \lambda)$ by $h(i) = i, h(j) = \ell$ and $h(\ell) = j$, we can see a function h is totally continuous. But is not α rps--totally continuous. since the set $\mathcal{M} = \{i, j\}$ is α rps--open in Y , but $h^{-1}(\mathcal{M}) = h^{-1}(\{i, j\}) = \{i, \ell\}$ is not clopen in X .

Example(3-32):- Let $X = \{i, j, \ell, s\}, Y = \{i, j, \ell\}, \tau = \{X, \{i\}, \{j\}, \{\ell\}, \{i, j\}, \{i, \ell\}, \{j, \ell\}, \{i, j, \ell\}, \phi\}, \lambda = \{Y, \{i\}, \{j\}, \{i, j\}, \phi\}$, where $\alpha\mathcal{RPSO}(Y, \lambda) = \{Y, \{i\}, \{j\}, \{i, j\}, \phi\}$ and $\mathcal{RO}(X, \tau) = \{i\}, \{j\}, \{\ell\}, \{i, j\}, \{i, \ell\}, \{j, \ell\}, \phi\}$, define $h : (X, \tau) \rightarrow (Y, \lambda)$ by $h(i) = i, h(j) = j$ and $h(\ell) = h(s) = \ell$, apparent that h is completely α rps--continuous. But is not α rps--totally continuous. since the set $\mathcal{M} = \{i\}$ is α rps--open in Y , but $h^{-1}(\mathcal{M}) = h^{-1}(\{i\}) = \{i\}$ is not clopen in X .

Now, we will present the conditions that make the proposition (3-28) and corollary (3-29) correct.

Proposition(3-32):-

A function $h : (X, \tau) \rightarrow (Y, \eta)$ is α rps--totally continuous. If h is

- 1- Completely α rps – continuous and X is extremely disconnected .
- 2- Strongly α rps – continuous and X is Locally Indiscrete.
- 3- α rps – continuous, X is discrete space and Y is $\mathcal{T}_{*1/2}$ – space.
- 4- Completely continuous, X is extremely disconnected and Y is $\mathcal{T}_{*1/2}$ – space.
- 5- Almost continuous , X and Y are discrete spaces.
- 6- Totally continuous and Y is $\mathcal{T}_{*1/2}$ – space.

Proof(1):- Let $h : (X, \tau) \rightarrow (Y, \eta)$ is completely α rps -- continuous and $\mathcal{M}$ be α rps—open in Y . h is completely α rps -- continuous . Hence , $h^{-1}(\mathcal{M})$ is regular open in X . By imposition a space X is extremely disconnected and by theorem (2-6) we obtain , $h^{-1}(\mathcal{M})$ is clopen in X . Therefore , h is α rps – totally continuous .

Proof(2):- Let $h : (X, \tau) \rightarrow (Y, \eta)$ is strongly α rps -- continuous and $\mathcal{M}$ be α rps—open in Y . h is strongly α rps -- continuous . Hence , $h^{-1}(\mathcal{M})$ is open in X . By imposition , X is locally indiscrete and by definition(2-4) we conclude (each open set is closed). That mean , $h^{-1}(\mathcal{M})$ is open and closed set in X . Thus , $h^{-1}(\mathcal{M})$ is clopen in X . Therefore , h is α rps – totally continuous

Likewise , prove the other points (3) , (4) , (5) and (6) .

Proposition (3-33):- If $h : (X, \tau) \rightarrow (Y, \eta)$ and $\mathcal{P} : (Y, \eta) \rightarrow (Z, \delta)$ are two α rps -- totally continuous . Then , $\mathcal{P} \circ h : (X, \tau) \rightarrow (Z, \delta)$ is α rps-- totally continuous.

Proof :-

Let set N be α rps – open in Z , since $\mathcal{P}$ is α rps -- totally continuous , then $\mathcal{P}^{-1}(N)$ is clopen in Y . That is mean $\mathcal{P}^{-1}(N)$ is open and closed since (each open is α rps – open). This lead $\mathcal{P}^{-1}(N)$ is α rps – open set in Y . h is α rps -- totally continuous . Thus , $h^{-1}(\mathcal{P}^{-1}(N))$ is clopen in X . But , $h^{-1}(\mathcal{P}^{-1}(N)) = (\mathcal{P} \circ h)^{-1}(N)$. Therefore, $\mathcal{P} \circ h : (X, \tau) \rightarrow (Z, \delta)$ is α rps—totally continuous .

Proposition (3-34):- If $h : (X, \tau) \rightarrow (Y, \eta)$ is α rps--totally continuous and $\mathcal{P} : (Y, \eta) \rightarrow (Z, \delta)$ is completely α rps-- continuous , then $\mathcal{P} \circ h : (X, \tau) \rightarrow (Z, \delta)$ is α rps -- totally continuous.

Proof :- Let N be α rps – open in Z , since $\mathcal{P}$ is completely α rps -- continuous , then $\mathcal{P}^{-1}(N)$ is regular open in Y . Since (each regular open is α rps – open). This lead $\mathcal{P}^{-1}(N)$ is α rps – open set in Y . h is α rps – totally continuous . Thus , $h^{-1}(\mathcal{P}^{-1}(N))$ is clopen in X . But , $h^{-1}(\mathcal{P}^{-1}(N)) = (\mathcal{P} \circ h)^{-1}(N)$. Therefore, $\mathcal{P} \circ h : (X, \tau) \rightarrow (Z, \delta)$ is α rps—totally continuous .

In the identical manner we show the following results .

Corollary (3-34):- If $h : (X, \tau) \rightarrow (Y, \eta)$ is α rps--totally continuous and

- 1- $\mathcal{P} : (Y, \eta) \rightarrow (Z, \delta)$ is strongly α rps-- continuous , then $\mathcal{P} \circ h : (X, \tau) \rightarrow (Z, \delta)$ is α rps -- totally continuous.
- 2- $\mathcal{P} : (Y, \eta) \rightarrow (Z, \delta)$ is completely continuous , then $\mathcal{P} \circ h : (X, \tau) \rightarrow (Z, \delta)$ is totally continuous.

- 3- $\mathcal{P}: (\mathbb{Y}, \lambda) \rightarrow (Z, \delta)$ is almost α rp $\mathcal{S}$ —continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ almost continuous.
- 4- $\mathcal{P}: (\mathbb{Y}, \lambda) \rightarrow (Z, \delta)$ is almost continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is almost continuous (resp. almost α rp $\mathcal{S}$ —continuous) .
- 5- $\mathcal{P}: (\mathbb{Y}, \lambda) \rightarrow (Z, \delta)$ is α rp $\mathcal{S}$ --continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is totally continuous.

Corollary (3-35):- If $\mathcal{P}: (\mathbb{Y}, \lambda) \rightarrow (Z, \delta)$ is α rp $\mathcal{S}$ -- totally continuous and

- 1- $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is completely α rp $\mathcal{S}$ -- continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is completely α rp $\mathcal{S}$ -- continuous.
- 2- If $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is strongly α rp $\mathcal{S}$ -- continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is almost continuous.
- 3- If $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is completely continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is completely α rp $\mathcal{S}$ -- continuous.
- 4- If $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is almost α rp $\mathcal{S}$ —continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ almost continuous.
- 5- If $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is almost continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is strongly α rp $\mathcal{S}$ -- continuous) .
- 6- If $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is α rp $\mathcal{S}$ --continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is strongly α rp $\mathcal{S}$ -- continuous) .
- 7- If $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is totally continuous, then $\mathcal{P} \circ h: (X, \mathfrak{t}) \rightarrow (Z, \delta)$ is α rp $\mathcal{S}$ --totally continuous.

Another kind of weaker α rp $\mathcal{S}$ -- continuous functions, we will present in the following .

Definition (3-36):- A function $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is called **totally α rp $\mathcal{S}$ --continuous** if $h^{-1}(\mathcal{M})$ is α rp $\mathcal{S}$ —clopen in X (which is mean $h^{-1}(\mathcal{M})$ are both α rp $\mathcal{S}$ —open and α rp $\mathcal{S}$ —closed) set, for all open set $\mathcal{M}$ in $\mathbb{Y}$.

Example(3-37):-

Let $X = \mathbb{Y} = \{i, j, \ell\}$, $\mathfrak{t} = \{X, \{i\}, \{j, \ell\}, \emptyset\}$, $\lambda = \{\mathbb{Y}, \{i\}, \emptyset\}$, where $\alpha\mathcal{RPSO}(X, \mathfrak{t}) = \alpha\mathcal{RPSO}(X, \mathfrak{t}) = \{X, \{i\}, \{j, \ell\}, \emptyset\}$. Let $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ by $h(i) = i$, $h(j) = j$ and $h(\ell) = \ell$, apparent that h is totally α rp $\mathcal{S}$ —continuous .

Proposition(3-38):- Each totally α rp $\mathcal{S}$ --continuous function is

- 1- almost α rp $\mathcal{S}$ --continuous.
- 2- α rp $\mathcal{S}$ --continuous

Proof (1):-

Let $h: (X, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is totally α rp $\mathcal{S}$ --continuous and $\mathcal{M}$ be regular open in $\mathbb{Y}$ Thus, $\mathcal{M}$ is open in $\mathbb{Y}$. h is totally α rp $\mathcal{S}$ --continuous. Hence, $h^{-1}(\mathcal{M})$ is clopen in X , since (all open (closed) set is α rp $\mathcal{S}$ —open (α rp $\mathcal{S}$ — closed) . So, we obtain $h^{-1}(\mathcal{M})$ are both α rp $\mathcal{S}$ —open and α rp $\mathcal{S}$ — closed, hence $h^{-1}(\mathcal{M})$ is α rp $\mathcal{S}$ —open in X . Therefore, h is almost α rp $\mathcal{S}$ —continuous.

Likewise, we show step -2- .

Proposition(3-39):-

- (1) Each totally continuous is totally α rps—continuous.
- (2) Each α rps--totally continuous is totally α rps—continuous.

Proof (1):-

Let $h : (X, \tau) \rightarrow (Y, \sigma)$ be α rps—totally continuous and $\mathcal{M}$ be open in Y , h is totally continuous. Hence, $h^{-1}(\mathcal{M})$ is clopen in X , since all open (closed) set is α rps —open (α rps — closed). So, we have $h^{-1}(\mathcal{M})$ are both α rps —open and α rps — closed, hence $h^{-1}(\mathcal{M})$ is α rps —clopen in X . Therefore, h is totally α rps--continuous.

Proof(2):-

This is clear from the corollary(3-29) step-1- and proposition (3-39) step-1-.

Remark (3-40):-

The concepts of completely α rps--continuous and strongly α rps--continuous are independent to totally α rps--continuous.

The Examples below demonstrate that the opposite of the proposition(3-38), proposition(3-39) and Remark(3-40) are not generally correct.

Example (3-41):- Let $X = \{i, j, \ell, s\}$, $Y = \{i, j, \ell\}$, $\tau = \{X, \{i\}, \{j\}, \{\ell\}, \{i, j\}, \{i, \ell\}, \{j, \ell\}, \{i, j, \ell\}, \emptyset\}$, $\sigma = \{Y, \{i, j\}, \emptyset\}$, where $\alpha\mathcal{RPSO}(Y, \sigma) = \{Y, \{i, j\}, \emptyset\}$ and $\mathcal{RO}(X, \tau) = \{X, \{i\}, \{j\}, \{\ell\}, \{i, j\}, \{i, \ell\}, \{j, \ell\}, \emptyset\}$, define $h : (X, \tau) \rightarrow (Y, \sigma)$ by $h(i)=i$, $h(j)=j$ and $h(\ell)=h(s)=\ell$, apparent that h is completely α rps—continuous and (strongly α rps—continuous, α rps — continuous and almost α rps — continuous). But is not totally α rps—continuous. since the set $\mathcal{M} = \{i, j\}$ is open in Y , but $h^{-1}(\mathcal{M}) = h^{-1}(\{i, j\}) = \{i, j\}$ is α rps—open but is not α rps—closed. This lead $\mathcal{M}$ is not α rps—clopen in X .

Example (3-42):- Let $X = Y = \{i, j, \ell\}$, $\tau = \{X, \{i\}, \{j, \ell\}, \emptyset\}$, $\sigma = \{Y, \{i\}, \emptyset\}$, where $\alpha\mathcal{RPSO}(X, \tau) = \alpha\mathcal{RPSO}(X, \tau) = \{X, \{i\}, \{j, \ell\}, \emptyset\}$. Let $h : (X, \tau) \rightarrow (Y, \sigma)$ by $h(i)=i$, $h(j)=j$ and $h(\ell)=\ell$, apparent that h is totally α rps—continuous. But h is not α rps--totally continuous and is not (totally continuous, completely α rps--continuous and strongly α rps—continuous) functions. Because the set $\mathcal{M} = \{i, j\}$ is α rps—open in Y , but $h^{-1}(\mathcal{M}) = h^{-1}(\{i, j\}) = \{i, j\}$ is not clopen and is not regular open in X .

Below are the conditions that make the propositions(3-38), (3-39) and Remark(3-40) correct.

Proposition(3-43):- If $h : (X, \tau) \rightarrow (Y, \sigma)$ is totally α rps --continuous and

- 1- X is $\mathcal{T}_{*1/2}$ — space. Then h is totally continuous.
- 2- X, Y are $\mathcal{T}_{*1/2}$ — space. Then h is α rps --totally continuous.
- 3- X is discrete space and Y is $\mathcal{T}_{*1/2}$ — space. Then h is completely α rps--continuous (resp. strongly α rps—continuous).

Proof(1):-

Let $\mathcal{M}$ be open in $\mathbb{Y}$. h is totally α rps -- continuous. Hence, $h^{-1}(\mathcal{M})$ is α rps--clopen in $\mathbb{X}$. By imposition a space $\mathbb{X}$ is $\mathcal{T}_{*1/2}$ -- space. Thus, $h^{-1}(\mathcal{M})$ is clopen in $\mathbb{X}$. Therefore, h is totally continuous.

Proof(2):-

Let $\mathcal{M}$ be α rps--clopen in $\mathbb{Y}$. Since $\mathbb{Y}$ is $\mathcal{T}_{*1/2}$ -- space, then $\mathcal{M}$ is clopen in $\mathbb{Y}$. So we get $\mathcal{M}$ is open in $\mathbb{Y}$. h is totally α rps -- continuous. Hence, $h^{-1}(\mathcal{M})$ is α rps--clopen in $\mathbb{X}$. By imposition a space $\mathbb{X}$ is $\mathcal{T}_{*1/2}$ -- space. Thus, $h^{-1}(\mathcal{M})$ is clopen in $\mathbb{X}$. Therefore, h is α rps -- totally continuous.

Same method we show the step(3).

Proposition(3-44):- A function $h: (\mathbb{X}, \mathfrak{t}) \rightarrow (\mathbb{Y}, \mathfrak{q})$ is totally α rps--continuous.

- 1- α rps--continuous and $\mathbb{X}$ is discrete spaces.
- 2- Almost α rps--continuous, $\mathbb{X}$ is discrete space and $\mathbb{Y}$ locally indiscrete
- 3- Completely α rps--continuous and $\mathbb{X}$ is extremely disconnected
- 4- Strongly α rps --continuous and $\mathbb{X}$ is discrete space.

Proof(1):- Let $\mathcal{M}$ be open in $\mathbb{Y}$. h is α rps -- continuous. Hence, $h^{-1}(\mathcal{M})$ is α rps-open in $\mathbb{X}$. By imposition $\mathbb{X}$ is discrete space. Thus, $h^{-1}(\mathcal{M})$ is clopen in $\mathbb{X}$ and hence $h^{-1}(\mathcal{M})$ is α rps-clopen in $\mathbb{X}$. Therefore, h is totally α rps--continuous.

Similarly, we show the steps(2) (3), and (4).

Proposition(3-45):- If $h: (\mathbb{X}, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is totally α rps -- continuous and $\mathcal{P}: (\mathbb{Y}, \lambda) \rightarrow (\mathbb{Z}, \mathfrak{d})$ is completely α rps-- continuous. Then $h \circ \mathcal{P}: (\mathbb{X}, \mathfrak{t}) \rightarrow (\mathbb{Z}, \mathfrak{d})$ is totally α rps-- continuous

Proof (1):- Let $\mathcal{N}$ be open in $\mathbb{Z}$, since (all open set is α rps -open), hence $\mathcal{N}$ is $\mathcal{P}$ is α rps -open in $\mathbb{Y}$. completely α rps -- continuous, then $\mathcal{P}^{-1}(\mathcal{N})$ is regular open in $\mathbb{Y}$. Since (each regular open is open). This lead $\mathcal{P}^{-1}(\mathcal{N})$ is open set in $\mathbb{Y}$. h is totally α rps--continuous. Thus, $h^{-1}(\mathcal{P}^{-1}(\mathcal{N}))$ is α rps--clopen in $\mathbb{X}$. But, $h^{-1}(\mathcal{P}^{-1}(\mathcal{N})) = (\mathcal{P} \circ h)^{-1}(\mathcal{N})$. Therefore, $\mathcal{P} \circ h: (\mathbb{X}, \mathfrak{t}) \rightarrow (\mathbb{Z}, \mathfrak{d})$ is totally α rps--continuous. Likewise, we show the following results.

Corollary (3-46):-

If $h: (\mathbb{X}, \mathfrak{t}) \rightarrow (\mathbb{Y}, \lambda)$ is totally α rps -- continuous and $\mathcal{P}: (\mathbb{Y}, \lambda) \rightarrow (\mathbb{Z}, \mathfrak{d})$ is any function Then $\mathcal{P} \circ h: (\mathbb{X}, \mathfrak{t}) \rightarrow (\mathbb{Z}, \mathfrak{d})$ is totally α rps -- continuous. If $\mathcal{P}$ is α rps--totally continuous (resp. strongly α rps--continuous, totally continuous, completely continuous and almost continuous).

Corollary (3-47):-

If $h : (X, \tau) \rightarrow (Y, \lambda)$ is any function and $\mathcal{P} : (Y, \lambda) \rightarrow (Z, \delta)$ is totally α rps--continuous . Then $\mathcal{P} \circ h : (X, \tau) \rightarrow (Z, \delta)$ is

- 1- Almost α rps -- continuous. If h is completely α rps--continuous (resp. strongly α rps--continuous).
- 2- Totally α rps--continuous . If h is α rps--totally continuous .

Remark (3-48):- The diagram below clarify the relation between the stronger and weaker of α rps--continuous functions kinds. where the opposite dose not to be correct .

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