Some Types of τ^* -Generalized Closed Mappings

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ABSTRACT:

In this work, we introduce a new class of τ^* -G-closed maps called (τ^{**} -G-closed map and τ^* -Gc-closed map) in topological space and we study the relation between these mappings, as well as several properties of these maps are proved.

المستخلص:
في هذا العمل، قدمنا صفاً جديداً من التطبيقات المغلقة τ^* -G-المغلقة المعممة تدعى (τ^{**} -G-المغلقة و τ^* -Gc-المغلقة)، في الفضاء التوبولوجي وسندرس العلاقة فيما بينها، وأيضاً بعض خواص تلك التطبيقات ستبرهن.

INTRODUCTION:

In 1970, Levine [6] introduced a new class of closed sets is called generalized closed sets in topological space. In [7] (Malghan, 1982) defined and studied generalized closed maps. In [5] (Jamhour, 2002) studied and discussion new types of G-closed mappings. Recently, in [9] (Pushpalatha, Eswaran and Rajarubi 2009-2010) introduced and studied τ^* -generalized closed sets and τ^* -generalized open sets. They also introduced the notion τ^* -generalized continuous maps, see [3].

In [10], (Pushpalatha and Eswaran, 2010) studied some strong forms of τ^* -generalized continuous maps.

This paper is devoted to introduce and investigate a new class of τ^* -G-closed mappings which are (τ^{**} -G-closed and τ^* -Gc-closed) maps and give the relation between them. Also several properties of these types of mappings are proved.

PRELIMINARIES:

Some definition and basic Concepts have been given in this section.

Definition (2-1): [6]

A subset A of a topological space (X, τ) is called **generalized closed** (briefly **g-closed**) in X if $CL(A) \subseteq G$ whenever $A \subseteq G$ and G is open in X. A subset A is called **generalized open** (briefly **g-open**) in X if its complement A^c is g-closed.

Definition (2-2): [2], [4]

Let X be a topological space and A is a subset of X , the **generalized closure operator** $CL^*(A)$ is defined by the intersection of all g -closed sets containing A .

Definition (2-3): [9]

For the subset A of a topological X , the topology τ^* is defined by $\tau^* = \{G : cL^*(G^c) = G^c\}$.

Definition (2-4): [4], [9]

A subset A of a topological space (X, τ^*) is called **τ^* -generalized closed set** (briefly **τ^* -g-closed**) if $CL^*(A) \subseteq G$ whenever $A \subseteq G$ and G is τ^* -open in X . The complement of τ^* -generalized closed set is called the **τ^* -generalized open set** (briefly **τ^* -g-open**).

Remark (2-5):

In [9], it has been proved that (i) every closed set is τ^* -g-closed and (ii) every g -closed set is τ^* -g-closed.

We explain that by the following diagram:

But the converse of the Remark (2-5) need not to be true as seen from the following examples.

Example (2-1):

Consider the topological space $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}\}$, then the set $A = \{a, b, d\}$ is g -closed in X but is not closed set.

Example (2-2):

Consider the topological space $X = \{a, b, c\}$ with topology $\tau = \{X, \emptyset, \{a\}\}$. Then the set $\{a\}$ is τ^* -g-closed but not closed.

Remark (2-6): [9]

For any two subsets A and B of a topological space X, then:

1. $cL^*(A \cup B) = cL^*(A) \cup cL^*(B)$
2. The union of two τ^* -g-closed sets in X is a τ^* -g-closed set in X.

Definition (2-7): [1]

A topological space X is called $T_{1/2}$ -space if every g-closed set in X is closed.

Definition (2-8): [3], [4]

A topological space (X, τ^*) is called τ^* -Tg space if every τ^* -g-closed set in X is g-closed in X.

Definition (2-9): [5], [7], [11] :

Let X and Y be two topological space, than a mapping $f: X \rightarrow Y$ is said to be:

- (1) **Closed** if the image of every closed set in X is a closed set in Y.
- (2) **G-closed** if the image of every closed set in X is g-closed set in Y.
- (3) **τ^* -generalized closed map** (briefly **τ^* -G-closed**) if for every g-closed subset H of X, then $f(H)$ is τ^* -g-closed subset of Y.

Remark (2-10): [7]

Every closed mapping is G-closed. But the converse is not true in general, as in shown by the following example.

Example (2-3):

Let $X = \{a, b, c\}$ and $T_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $T_2 = \{\phi, X, \{a\}, \{a, b\}\}$ be two topological space in X. Define $f: (X, T_1) \rightarrow (X, T_2)$ by $f(x) = x$.

It is easily seen that f is G-closed mapping but is not closed map.

MAIN RESULTS:

In this section we introduce new types of τ^* -G-closed mappings which are [τ^{**} -G-closed and τ^* -GC-closed] mappings and studying the relations between them. Also, we give some propositions about the composition of these mapping.

Definition (3-1):

A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be **strongly τ^* -generalized closed map** (briefly **τ^{**} -G-closed**) if for every τ^* -g-closed subset H of X, then $f(H)$ is g-closed subset of Y.

Proposition (3-2):

Every τ^{**} -G-closed mapping is G-closed.

Proof:

Let $f: X \rightarrow Y$ be τ^{**} -G-closed mapping and let H is a closed set in X .

Thus, by Remark (2-5) H is τ^* -g-closed set in X . Since f is τ^{**} -G-closed mapping, then $f(H)$ is g-closed set in Y . Hence, $f: X \rightarrow Y$ is G-closed mapping.

Proposition (3-3):

Every τ^{**} -G-closed mapping is τ^* -G-closed.

Proof:

Let $f: X \rightarrow Y$ be τ^{**} -G-closed mapping and let H is a g-closed set in X .

Thus, H is τ^* -g-closed set in X . [by Remark (2-5)]. Since f is τ^{**} -G-closed mapping, then $f(H)$ is g-closed set in Y and by using Remark (2-5) we get $f(H)$ is τ^* -g-closed set in Y .

Hence, $f: X \rightarrow Y$ is τ^* -G-closed mapping.

Here, in the following examples shows the converse of Proposition (3-2) and Proposition (3-3) is not necessarily true.

Example (3-1):

Consider the topological space $X = \{a, b, c\}$, let $\tau = \{X, \phi\}$ and $\sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$.

Let $f: (X, \tau) \rightarrow (X, \sigma)$ be an identity map. Then f is

G-closed map but f is not τ^{**} -G-closed, since for the τ^* -g-closed set

$H = \{b\}$ in $\{X, \tau\}$ but $f(H) = f(\{b\}) = \{b\}$ is not g-closed set in (X, σ) .

Example (3-2):

Consider the topological space $X = \{a, b, c\}$, let $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{X, \phi, \{c\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ be an identity map. Then f is τ^* -G-closed map, but is not τ^{**} -G-closed map, since for the τ^* -g-closed set $H = \{c\}$ in $\{X, \tau\}$ but $f(H) = f(\{c\}) = \{c\}$ is not g-closed set in (X, σ) .

Now, the following propositions give the condition to make the converse of Proposition (3-2) and Proposition (3-3) is true.

Proposition (3-4):

If $f: X \rightarrow Y$ is G-closed mapping and let X be a $T_{1/2}$ -space and τ^* - T_g topological space, then f is τ^{**} -G-closed mapping.

Proof:

Let H be a τ^* -g-closed set in X . Since X is τ^* - T_g topological space and $T_{1/2}$ space. Then by

using Definitions (2-8), (2-7) we get H is closed set in X .

Thus, $f(H)$ is a g-closed set in Y . Hence, f is a τ^{**} -G-closed mapping.

Proposition (3-5):

If $f: X \rightarrow Y$ is τ^* -G-closed mapping and let X, Y be two τ^* - T_g topological spaces, then f is τ^{**} -G-closed mapping.

Proof:

Let H be a τ^* -g-closed set in X . Since X is τ^* - T_g space

Then, H is a g-closed set in X . Thus, $f(H)$ is a τ^* -g-closed set in Y . Since Y is τ^* - T_g topology space, then by Definition (2-8) we get, $f(H)$ is a g-closed set in Y .

Hence, f is a τ^{**} -G-closed mapping.

Now, we give another type of τ^* -G-closed mapping is called τ^* -Gc-closed mapping.

Definition (3-6):

A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be τ^* -Gc-closed if every τ^* -g-closed subset H of X , then $f(H)$ is a τ^* -g-closed set in Y .

Proposition (3-7):

Every τ^* -Gc-closed mapping is τ^* -G-closed.

Proof:

Let $f: X \rightarrow Y$ be a τ^* -Gc-closed mapping and let H is a g-closed set in X , then by using Remark (2-5) we get H is τ^* -g-closed set in X , and since f is τ^* -Gc-closed mapping. Then $f(H)$ is a τ^* -g-closed set in Y .

Hence, f is τ^* -G-closed mapping.

The converse of the above proposition need not to be true as seen from the following example.

Example (3-3):

Consider the topological space $X = \{a, b, c\}$ and let $\tau = \{X, \phi, \{a\}\}$ and

$\sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Let $f: (X, \tau) \rightarrow (X, \sigma)$ defined by $f(b) = f(c) = c$ and $f(a) = a$.

Then, f is τ^* -G-closed map, but not τ^* -Gc-closed map, since for the τ^* -g-closed set $H = \{a\}$ in (X, τ) but $f(H) = f(\{a\}) = \{a\}$ is not τ^* -g-closed set in (X, σ) .

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Proposition (3-8):

Every τ^{**} -G-closed mapping is τ^* -Gc-closed.

Proof: Let $f: X \rightarrow Y$ be a τ^{**} -G-closed mapping and let H be a τ^* -g-closed set in X , then $f(H)$ is a g-closed set in Y and by using Remark (2-5) we get $f(H)$ is a τ^* -g-closed set in Y . therefore, f is τ^* -Gc-closed mapping. But the converse is not true in general. It is easy seen that in Example (3-2).

Here in the following propositions addition the necessarily condition in order to the converse of Proposition (3-7) and Proposition (3-8) is true.

Proposition (3-9): If $f: X \rightarrow Y$ is τ^* -G-closed mapping and let X be a τ^* -Tg topological space, then f is τ^{**} -Gc-closed mapping.

Proof: Let H be an τ^* -g-closed set in X and since X is τ^* -Tg topology space, then by Definition (2-8) we get H is a g-closed set in X , by Remark (2-5) $f(H)$ is a τ^* -g-closed set in Y . Hence, f is τ^{**} -Gc-closed mapping.

Proposition (3-10): If $f: X \rightarrow Y$ is τ^{**} -Gc-closed mapping and let Y be a τ^* -Tg topological space, then f is τ^* -G-closed.

Proof: Let H be an τ^* -g-closed set in X . Then, $f(H)$ is a τ^* -g-closed set in Y and since Y is a τ^* -Tg topology space, then by Definition (2-8) we get $f(H)$ is a g-closed set in Y . Hence, f is τ^{**} -G-closed mapping.

Here in the following diagram illustrates the relation between the τ^* -G-closed mapping types (without using condition), where the converse is not necessarily true.

Diagram(2)

Summarized the relation between τ^* -G-closed mapping types.

Now, we give some propositions about the composition of τ^* -G-closed mapping types:

Proposition (3-11):

If $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ is a τ^* -G-closed mapping and a function.

(1) $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping, then $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is τ^* -G-closed mapping.

(2) $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping, then $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is τ^* -G-closed mapping.

Proof:

(1) Let H be a g -closed set in X , and since f is an τ^* -G-closed mapping, then $f(H)$ is an τ^* -g-closed set in Y , also since g is an τ^* -Gc-closed mapping, we get $g(f(H))$ is an τ^* -g-closed set in Z . But $g(f(H)) = g \circ f(H)$

Hence, $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is τ^* -G-closed mapping.

(2) Since $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping and by using Proposition (3-12), we get $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping.

Then, by Proposition [(3-11) – step 1] we have $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is τ^* -G-closed mapping.

Remark (3-12):

The mapping $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ in Proposition (3-11) is not necessarily τ^* -Gc-closed mapping and τ^{**} -G-closed mapping.

Here in the following proposition and corollary addition the condition in order to Remark (3-12) is true:

Proposition (3-13):

Let $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ is a τ^* -G-closed mapping and let X, Z are τ^* -Tg topological spaces.

(1) If $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \alpha)$ is a τ^{**} -G-closed mapping.

(2) If $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \alpha)$ is a τ^{**} -G-closed mapping.

Proof:

(1) Let H be a τ^* -g-closed set in X , and since X is a τ^* -Tg topological space, and by using Definition (2-8), we get H is a g -closed set in X .

Thus, $f(H)$ is a τ^* -g-closed set in Y .

Also, since g is a τ^* -Gc-closed mapping, we have $g(f(H))$ is τ^* -g-closed set in Z . But Z is a τ^* -Tg topological space. Then $g(f(H))$ is a g-closed set in Z . Since $g(f(H)) = g \circ f_{(H)}$
 Hence, $g \circ f: (X, \tau) \rightarrow (Z, \alpha)$ is a τ^{**} -G-closed mapping.

(2) Since $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping and by using Proposition (3-8) we get $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping.
 Then, by Proposition [(3-13) step 1], we have $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping.

From Proposition (3-13) and Proposition (3-8), we get the following Corollary, the proof it is easy. Thus we omitted it is:

Corollary (3-14):

Let $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ is a τ^* -G-closed mapping and let X is a τ^* -Tg topological space.

(1) If $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping, then

$g \circ f: (X, \tau) \rightarrow (Z, \alpha)$ is a τ^* -Gc-closed mapping.

(2) If $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -Gc-closed mapping, then

$g \circ f: (X, \tau) \rightarrow (Z, \alpha)$ is a τ^* -Gc-closed mapping.

Remark (3-15):

If $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ and $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ are τ^* -G-closed mappings, then $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is not necessarily (τ^{**} -G-closed, τ^* -Gc-closed and τ^* -G-closed) mapping.

The following proposition and result give the condition to make Remark (3-15) true:

Proposition (3-16):

Let $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ and $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ are τ^* -G-closed mappings and let X, Y and Z are a τ^* -Tg topological spaces, then

$g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping.

Proof:

Let H be a τ^* -g-closed set in X and since X is a τ^* -Tg topological space.

Then, H is a g-closed set in X .

Thus, $f(H)$ is a τ^* -g-closed set in Y . But Y is τ^* -Tg topological space and by using Definition (2-8), we get $f(H)$ is a g-closed set in Y . Also, since g is a τ^* -G-closed mapping, then $g(f(H))$ is a τ^* -g-closed set in Z , but Z is a τ^* -Tg topological space. Then, $g(f(H))$ is a g-closed set in Z .

But $g(f(H)) = g \circ f_{(H)}$

Hence, $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping.

Similarly, we prove the following corollary and proposition :

Corollary (3-17):

Let $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ and $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ are τ^* -G-closed mappings and let X, Y and Z are a τ^* -Tg topological spaces, then $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping and τ^* -G-closed mapping respectively.

Proposition (3-18):

Let $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ be a τ^{**} -G-closed mapping:

- (1) If $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^* -G-closed mappings, then $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping.
- (2) If $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mappings, then $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping.

Proposition (3-19):

If $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ and $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ are τ^{**} -G-closed mappings, then $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping.

Proof:

Let H be a τ^* -g-closed set in X and since f is a τ^{**} -G-closed mapping. Then, $f(H)$ is a g-closed set in Y and by using Remark (2-5) we get $f(H)$ is a τ^* -g-closed set in Y . Thus $g(f(H))$ is a g-closed set in Z . But

$$g(f(H)) = g \circ f(H)$$

Hence, $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping.

Corollary (3-20): If $f: (X, \tau^*) \rightarrow (Y, \sigma^*)$ and $g: (Y, \sigma^*) \rightarrow (Z, \alpha^*)$ are τ^{**} -G-closed mappings, then $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping and τ^* -G-closed mapping.

Proof:

Since f and g are τ^{**} -G-closed mappings, then by Proposition (3-19) we get $g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^{**} -G-closed mapping.

Also, by using Proposition (3-12) and (3-11) we have

$g \circ f: (X, \tau^*) \rightarrow (Z, \alpha^*)$ is a τ^* -Gc-closed mapping and τ^* -G-closed mapping respectively.

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