EFFECT OF NONLINEAR THERMAL RADIATION ON CONVECTIVE HEAT AND MASS TRANSFER FLOW OF A ROTATING FLUID PAST POROUS VERTICAL STRETCHING SHEET WITH VARIABLE VISCOSITY WITH PARTIAL SLIP

K.Kalpana¹ and Dr.M.Sreevani²

¹Research scholar in Mathematics, Rayalaseema University, Kurnool, A.P

²Assistant Professor, Mathematics, S.K.University, Anantapur, A.P

Email: kalpananruthya@gmail.com, k.kalpana@rguktrkv.ac.in

Abstract

We analyse the combined influence of dissipation, variable viscosity effect, thermo-diffusion and rotation on the hydromagnetic free-convective flow, heat and mass transfer flow past a stretching surface in the presence of heat generation/absorption source. The governing equations have been solved by employing fifth-order Runge-Kutta-Fehlberg method along with shooting technique. The effects of various parameters on the velocity, temperature and concentration as well as on the local skin-friction coefficient, local Nusselt number and local Sherwood number are presented graphically and discussed.

Keywords : Heat and Mass transfer, Thermal Radiation, Rotating fluid, Stretching sheet, Partial slip

1. INTRODUCTION

Steady heat flow on a moving continuous flat surface was first considered by Sakiadis [41] who developed a numerical solution using a similarity transformation. Crane [14] motivated by the processes of polymer extrusion, in which the extrudate emerges from a narrow slit, examined semi-infinite fluid flow driven by a linearly stretching surface. Many researchers and academicians have advanced their studies relating to problems involving MHD in stretching sheet considering various parameters (Sharidan [44], Gupta and Gupta [17], Ishak et al. [20-22], Aziz et al. [8], Abel et al. [2], Mukhopadhyay and Mondal [32], Krishnendu [24]), Swati Mukhopadhyay et al. [32], Prasad et al. [37], Wang [51]. Rashad et al [40], Vajravelu and Hadjinicolaou [49], Hossain et al [19], Alam et al [4] Chamkha [12]). The influence of a uniform transverse magnetic field on the motion of an electrically conducting fluid past a stretching sheet was investigated by Pavlov [35]. The effect of chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [3] in the presence of transverse magnetic field. Some recent interesting contributions pertaining to heat transfer aspects of viscous dissipation and Joule heating are cited in Aziz et al.[8], Abel et al. [1], Pal et al. [34], Bharathi et al [11] have discussed Non Darcy MHD mixed convective Heat and Mass Transfer flow of a viscous fluid in a vertical channel with Heat generating sources. Balasubramanyam et al [10] have discussed Non Darcy viscous electrically conductive Heat and Mass transfer flow through a porous medium in a vertical channel in the presence of heat generating sources.

Most of the previous studies are concerning the constant thermo-physical properties of the fluid. It is well known that the fluid property most sensitive to temperature rise is viscosity. For many liquids, among them water, petroleum oils, glycerin, glycols, silicone fluids and some molten salts, the viscosity shows a rather pronounced variation with temperature. For example, when the temperature increases from 10 °C ($\mu=0.00131\text{g/cm}$) to 50 °C ($\mu=0.00548\text{g/cm}$), the viscosity of the water decreases by 240%. To predict the heat transfer rate accurately, it is necessary to take the variation of viscosity with temperature into consideration. The variation of viscosity in thermal boundary layer is large. There exist several applications of this problem, for example, in the processes of hot rolling, wire drawing, glass fiber production, paper production, gluing of labels on hot bodies, drawing of plastic films and the study of spilling pollutant crude oils over the surface of seawater. Many researchers have studied the flows with temperature dependent viscosity in different geometries and under various flow conditions (Mohamed Abd El Aziz [27]). The effect of temperature-dependent viscosity on heat and mass transfer laminar boundary layer flow has been discussed by many authors (Mukhopadhyay *et al.* [30], Mukhopadhyay and Layek [31], Ali [6], Makinde [26], Prasad *et al.* [37], Alam *et al.* [5]) in various situations. They showed that when this effect was included, the flow characteristics might change substantially compared with the constant viscosity assumption. Anjali Devi and Ganga [7] have considered the viscous dissipation effects on MHD flows past stretching porous surfaces in porous media. Mohamed Abd El Aziz [27] analyzed unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation. Xi-Yan Tian *et al.* [52] investigated the two-dimensional boundary layer flow and heat transfer in variable viscosity MHD flow over a stretching plate.

Gebhart and Mollendorf [16] have shown that viscous dissipation heat in the natural convective flow is important when the fluid is of extreme size or is at extremely low temperature or in high gravitational field. On the other hand, Zanchini [53] pointed out that relevant effects of viscous dissipation on the temperature profiles and the Nusselt number may occur in the fully developed convection in tubes. In view of this, several authors, notably, Sreevani [48] have studied the effect of viscous dissipation on the convective flows past an infinite vertical plate and through vertical channels and ducts.

Combined rotational and buoyancy effects are very common in nature and in devices and processes equipments. Most flows have regions of rotation as well as stratification. Gas flows in turbine and compressors, motion in seas and atmosphere are some of the examples where rotation affects the heat and mass transportation. Also buoyancy and rotational forces are often comparable in geophysical process like 'Dust Devil' caused by rotating atmosphere above a locally heated surfaces. Several researchers (Mahendra Mohan [28], Mahendra Mohan and Srivastava [29], Prasada Rao *et.al.* [38], Sarojamma and Krishna [42], Krishna *et.al.* [23], Seth and Ghosh [43], El-Mistikawy *et.al.* [15], Hazim Ali Attia [18], Circar and Mukherjee [13], Balasubramanyam [9] and Vijaya Bhaskar Reddy [50], Singh and Mathew [37]) have discussed the effect of rotation on convective heat and mass transfer flow of viscous fluid in different configurations under different conditions.

Sreedevi *et al* [47] have discussed the effect of radiation absorption and variable viscosity on hydromagnetic convective heat and mass transfer flow past a stretching sheet. Madhavalatha *et al* [25] have discussed the effect of variable viscosity on convective heat and mass transfer flow of an electrically conducting fluid past a stretching sheet with Hall effects, partial slip under convective boundary conditions.

In this paper we analyse the combined influence of dissipation, variable viscosity effect, thermo-diffusion and rotation on the hydromagnetic free-convective flow, heat and mass transfer flow past a stretching surface in the presence of heat generation/absorption

source. The governing equations have been solved by employing fifth-order Runge-Kutta-Fehlberg method along with shooting technique. The effects of various parameters on the velocity, temperature and concentration as well as on the local skin-friction coefficient, local Nusselt number and local Sherwood number are presented graphically and discussed.

2. FORMULATION OF THE PROBLEM

We consider the steady free-convective flow, heat and mass transfer of an incompressible, viscous and electrically conducting fluid past a stretching sheet and the sheet is stretched with a velocity proportional to the distance from a fixed origin O (Fig. 1).

The temperature and the species concentration are maintained at prescribed constant values T_w , C_w at the sheet and T_∞ and C_∞ are the fixed values far away from the sheet.

The fluid viscosity μ is assumed to vary as a reciprocal of a linear function of temperature given by

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 + \gamma_0 (T - T_\infty)] \quad (1)$$

$$\frac{1}{\mu_\infty} = a(T - T_\infty) \quad (2)$$

Where $a = \frac{\gamma_0}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\gamma_0}$

In the above equation both a and T_r are constants, and their values depend on the thermal property of the fluid, i.e., γ_0 . In general $a > 0$ represent for liquids, whereas for gases $a < 0$. Owing to the above mentioned assumptions, the boundary layer free-convection flow with mass transfer and generalized Ohm's law with Hall current effect are governed by the following system of equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$\left. \begin{aligned} \rho_m \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) + \rho_m g_0 \beta_r (T - T_\infty) \\ &\quad + 2\Omega w + \rho_m g_0 \beta_c (C - C_\infty) - \sigma B_0^2 u \end{aligned} \right\} \quad (4)$$

$$\rho_m \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} \right) = \frac{\partial}{\partial y} \left(\mu \frac{\partial w}{\partial y} \right) - 2\Omega u - \sigma B_0^2 w \quad (5)$$

$$\rho_m C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_f \frac{\partial^2 T}{\partial y^2} - Q_H (T - T_\infty) + \mu(u^2 + w^2) - \frac{\partial(q_R)}{\partial y} \quad (6)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_0 (C - C_\infty) + \frac{D_m K_T}{T_m} \frac{\partial^2 T}{\partial y^2} \quad (7)$$

where (u, v, w) are the velocity components along the (x, y, z) directions respectively. D_m is the solution diffusivity of the medium, K_T is the thermal diffusion ratio, C_s is the concentration

Figure 1. Physical Sketch of the Problem

susceptibility, C_p is the specific heat at constant pressure, T_m is the mean fluid temperature., q_R is the radiative heat flux.

The boundary conditions for the present problem can be written as

$$u = bx + L \frac{\partial u}{\partial y}, v = w = 0, T = T_w, C = C_w \text{ at } y=0 \quad (8)$$

$$u \rightarrow 0, w \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ at } y \rightarrow \infty \quad (9)$$

where $b (> 0)$ being stretching rate of the sheet. The boundary conditions on velocity in Eq. (9) are the no-slip condition at the surface $y = 0$, while the boundary conditions on velocity at $y \rightarrow \infty$ follow from the fact that there is no flow far away from the stretching surface.

We consider the solution of equation(1) as:

$$w = -w_0 \quad (10)$$

The radiation heat term by using the Rosseland approximation is given by

$$q_r = -\frac{4\sigma^*}{3\beta_R} \frac{\partial T'^4}{\partial y}, T'^4 \cong 4TT_\infty^3 - 3T_\infty^4, \quad \frac{\partial q_R}{\partial z} = -\frac{16\sigma^*T_\infty^3}{3\beta_R} \frac{\partial^2 T}{\partial y^2} \quad (11)$$

The non-dimensional temperature $\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$ can be simplified as

$$T = T_\infty (1 + (\theta_w - 1)\theta) \quad (12)$$

Where $\theta = \frac{T_w}{T_\infty}$ is the temperature parameter.

To examine the flow regime adjacent to the sheet, the following transformations are invoked

$$u = bxf'(\eta); v = -\sqrt{bv}f(\eta); w = bxg(\eta); \quad \eta = \sqrt{\frac{b}{v}}y; \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}; \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (13)$$

where f is a dimensionless stream function, h is the similarity space variable, θ and ϕ are the dimensionless temperature and concentration respectively. Clearly, the continuity Eq. (3) is satisfied by u and v defined in Eq. (13), Substituting Eq. (13) the Eqs. (4)-(7) reduce to

$$\left(\frac{\theta - \theta_r}{\theta_r}\right)(f' - f'')f'' - \left(\frac{\theta'}{\theta - \theta_r}\right)f'' \quad (14)$$

$$\left(\frac{\theta - \theta_r}{\theta_r}\right)G(\theta + N\phi) - M^2 \left(\frac{\theta' - \theta_r}{\theta_r}\right)(f') - 2Rg = 0$$

$$\left(\frac{\theta - \theta_r}{\theta_r}\right)(fg' - fg') + g'' - \left(\frac{\theta'}{\theta - \theta_r}\right)g' - M^2 \left(\frac{\theta - \theta_r}{\theta_r}\right)(g) + 2Rf' = 0 \quad (15)$$

$$\theta'' + PrQ\theta + PrEcM^2(f'^2 + g^2) + Rd(\theta_w - 1)^3(3\theta^2\theta'^2 + \theta^3\theta'') + 3Rd((\theta_w - 1)(2\theta\theta'^2 + \theta^2\theta'') + 3Rd((\theta_w - 1)(\theta'^2 + \theta\theta') + Rd\theta'' = 0 \quad (16)$$

$$\phi'' - Sc(f\phi' - \gamma\phi) = -ScSr\theta'' \quad (17)$$

Similarly, the transformed boundary conditions are given by

$$f'(0)=1+A11f''(0), f(0)=0, g(0)=0, \theta(0)=1, \phi(0)=1 \text{ at } \eta=0 \quad (18)$$

$$f'(\eta) \rightarrow 0, g(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ at } \eta \rightarrow \infty \quad (19)$$

where a prime denotes the differentiation with respect to η only and the dimensionless parameters appearing in the Eqs. (14)-(17) are respectively defined as

$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = - \left[\frac{1}{\gamma_0 (T_w - T_\infty)} \right]$ the viscosity parameter, $M = \frac{\sigma B_0^2}{\rho_\infty b}$ the magnetic parameter,
 $P_r = \frac{\rho C_p v}{k}$ the Prandtl number, $\gamma = \frac{k_0}{b} (C_w - C_\infty)$ the non-dimensional chemical reaction parameter, $G = \frac{g_0 \beta_T (T_w - T_\infty)}{b^2 x}$ the local Grashof number, $N = \frac{\beta_C (C_w - C_\infty)}{\beta_T (T_w - T_\infty)}$ the Buoyancy ratio,
 $Rd = \frac{kk_0}{4T_\infty^3 \sigma^*}$ is the thermal radiation parameter, $Sr = \frac{D_m K_T}{C_s C_p}$ the Soret parameter, $Sc = \frac{\mu}{\rho_\infty D_m}$
 the Schmidt number, $Q = \frac{Q_H}{\rho_\infty C_p b}$ is defined as the heat generation / absorption parameter and
 $Ec = \frac{\mu^2}{(T_w - T_\infty) C_p b}$ is the Eckert number.

3. METHOD OF SOLUTION

The coupled ordinary differential equations (14)-(17) are of third-order in f , and second-order in g , θ and ϕ which have been reduced to a system of nine simultaneous equations of first-order for nine unknowns. In order to solve this system of equations numerically we require nine initial conditions but two initial conditions on f and one initial condition each on g , θ and ϕ are known. However the values of f' , g , θ and ϕ are known at $\eta \rightarrow \infty$. These four end conditions are utilized to produce four unknown initial conditions at $\eta=0$ by using shooting technique. After knowing all the nine initial conditions, we solve this system of simultaneous equations using fifth-order Runge-Kutta-Fehlberg integration scheme with automatic grid generation scheme which ensures convergence at a faster rate. During the computation, the shooting error was controlled by keeping it to be less than 10^{-6} .

4. SKIN FRICTION, NUSSELT NUMBER AND SHERWOOD NUMBER

The local skin-friction coefficient C_f , the local Nusselt number Nu and the local Sherwood number Sh defined by

$$C_f = \frac{\tau_w}{\mu b x \sqrt{\frac{b}{v}}} = f''(0) \quad (21)$$

$$\text{where } \tau_w = \mu \left(\frac{\partial T}{\partial y} \right)_{y=0} = \mu b x \sqrt{\frac{b}{v}} f''(0), Nu = \frac{\dot{q}_w}{k \sqrt{\frac{b}{v}} (T_w - T_\infty)} \quad (22)$$

$$\text{where } q_w = -k_f \left(\frac{\partial T}{\partial y} \right)_{y=0} = -k_f \sqrt{\frac{b}{v}} (T_w - T_\infty) \theta'(0) \text{ and}$$

$$Sh = \frac{m_w}{D_m \sqrt{\frac{b}{v}} (C_w - C_\infty)} = -\phi'(0) \quad (23)$$

$$\text{where } m_w = -D_m \left(\frac{\partial C}{\partial y} \right)_{y=0} = -D_m \sqrt{\frac{b}{v}} (C_w - C_\infty) \phi'(0)$$

COMPARISON:

The results of this paper are compared with the results of previous published paper of Rahman et al. [39] and Shit et al. [45] as shown in Table 1 and Table 2 and the outcomes are in good concurrence.

Table 1 Comparison of Nu and Sh at $\eta=0$ with Rahman et al. [39] with $Pr=0.71$, $m= Sr = Nr=\lambda=\gamma=Ec=0$, $\theta_w=0$ for different values of θ_r

Different Values of θ_r	Rahman et al. [39] Results		Present Results	
	Nu(0)	Sh(0)	Nu(0)	Sh(0)
-2	-0.37865	4.805362	-0.37866	4.80539
-4	-0.30536	4.54266	-0.30538	4.54269
2	0.248266	4.485639	0.24829	4.48566
4	0.238773	4.33562	0.23878	4.33880

Table 2. Comparison of Nu and Sh at $\eta=0$ with Shit et al. [45] with $Sr=0, Ec=0, \theta_w=0$

M	Rd	γ	Q	θ_r	Shit et al.[45] Results		Present Results	
					Nu(0)	Sh(0)	Nu(0)	Sh(0)
0.5	1	0.5	0.5	-2	-0.6912	0.6265	-0.69085	0.62661
1.5	1	0.5	0.5	-2	-0.6977	0.6543	-0.69652	0.65464
0.5	3	0.5	0.5	-2	-12.3751	0.9278	-12.3745	0.9280
0.5	1	1.5	0.5	-2	-0.6956	1.0959	-0.6946	1.0961
0.5	1	-0.5	0.5	-2	-0.6966	0.4898	-0.69600	0.4899
0.5	1	0.5	1.5	-2	-0.6968	0.4245	-0.6969	0.4147
0.5	1	0.5	1.5	-2	-0.5974	0.4071	-0.5970	0.4072
0.5	1	0.5	1.5	-4	-0.6969	0.6253	-0.6971	0.6255

5. RESULTS AND DISCUSSION

The system of coupled non-linear Eqs. (15)-(18) together with the boundary conditions (19) and (20) have been solved numerically. In our present study the numerical values of the physical parameters have been chosen so that R, Q, Rd, γ , Sr, Ec, A, A11, θ_r are varied over a range which is listed in the following figures. (cf. Watanabe and Pop (79), Afify (5), Salem (66) and Shit and Haldar (73))

Figs.2a-2d show the variation of the flow variables with rotation parameter (R). It is found an increase in $R \leq 1.0$, enhances the axial velocity and for higher $R \geq 1.5$, it experiences a depreciation in the flow region while the temperature reduces and the cross flow velocity, concentration enhances with rising R in the flow region. This may be attributed to the fact that an increase in R decreases the thickness of the momentum and thermal boundary layers and increases the solutal boundary layer.

The effect of thermal radiation parameter Rd, on $f'(\eta)$ and $g(\eta)$ is shown in figs. 3a & 3b. It is found that higher the radiative heat flux smaller the axial and larger the cross flow velocity. It is due to the fact that an increase in Rd, the thickness of the boundary layer decreases. The temperature reduces and the concentration enhances with increase in the radiation parameter.

Figs.4a -4 d represent $f'(\eta)$ and $g(\eta)$ with chemical reaction parameter γ . It can be seen from the profiles that the axial velocity, temperature enhance and the cross flow velocity, concentration depreciate in the degenerating chemical reaction parameter ($\gamma > 0$) and a reversed effect is noticed in $f'(\eta)$ and $g(\eta)$ in generating chemical reaction ($\gamma < 0$).

Figs.5a-5d represent $f'(\eta)$ and $g(\eta)$ with heat generation parameter ($Q > 0$) and absorption parameter ($Q < 0$). From the profiles we find that the axial velocity, cross flow velocity reduces, temperature and concentration enhances with increase in the heat generation parameter ($Q > 0$) and the axial velocity reduces, temperature and the cross flow velocity, concentration enhance with increase in the heat absorption parameter ($Q < 0$).

Figs. 6a-6d represents $f'(\eta)$ and $g(\eta)$ with Soret parameter Sr . It can be observed from the profiles that higher the thermo-diffusion effects larger the axial velocity, temperature and smaller the cross flow velocity, concentration in the boundary layer. Figs.7a&7b represent $f'(\eta)$ and $g(\eta)$ with Eckert number Ec . It can be seen from the profiles that the axial velocity and temperature enhance and the cross flow velocity, concentration reduces with higher values of Ec .

Figs. 8a-8d depicts the axial velocity $f'(\eta)$ and cross flow velocity $g(\eta)$ with viscosity parameter θ_r . It can be seen from the profiles that the axial velocity enhances with $\theta_r \leq -6$ and reduces with higher $\theta_r = -10$ while the cross flow velocity increase with the increase of viscosity parameter θ_r . This observation leads to the increase of thermal boundary layer thickness. Figs. 8c&8d represent temperature and concentration with viscosity parameter θ_r . It is found that an increase in $\theta_r (< 0)$, enhances the temperature and concentration in the boundary layer.

Figs.9a-9d represent the variation of the variation of $f'(\eta)$ and $g(\eta)$ with slip parameter (A_{11}). It can be seen from the profiles that higher the slip parameter A larger the axial velocity, temperature and smaller the cross flow velocity, components.

An increase in temperature parameter ($\theta_w = A$) enhances the axial velocity and temperature and reduces the cross flow velocity and concentration in the boundary layer (figs.10a-10d).

The variation of Skin friction with different parameters is exhibited in table.3. It is found that Higher the Coriolice force /thermal radiation larger the skin friction components at the wall. With respect to the chemical reaction parameter γ , we find that the skin friction components enhance in both the degenerating and generating chemical reaction cases at the wall. An increase in the viscosity parameter θ_r leads to a reduction in τ_x and enhancement in τ_y . Higher the slip parameter (A_{11}) smaller the skin friction component τ_x and larger τ_y at the wall. The skin friction component τ_x reduce and τ_y enhances on the wall with increase in the strength of the heat generating/absorbing source. With reference to temperature parameter (A) we find that an increase in A results in an enhancement in the skin friction component τ_x and τ_y .

The rate of heat transfer (Nusselt number) at the wall $\eta=0$ is shown in Tables.3 for different parametric variations. Higher the radiative heat flux/Chemical reaction parameter smaller the rate of heat transfers. Higher the thermo-diffusion effects larger Nu and smaller with increasing values of θ_r at the wall. Nu reduces on the wall with increasing slip parameter (A_{11}) and temperature parameter (A). The rate of heat transfer at the wall reduce with increase in the strength of the heat generating source and in the generating heat source, it enhances on the wall.

The rate of mass transfer (Sherwood Number) at the wall is shown in tables .3 for different variations of the parameters. It is found that the rate of mass transfer enhances with increase in R or R_d or Sr . Higher the viscosity parameter larger the rate of mass transfer at the wall. The rate of mass transfer enhances at the wall in degenerating chemical reaction case and reduces in the generating chemical reaction case. With respect to heat source parameter (Q), we find that the rate of mass transfer increases with increase in the strength of

the heat generating source and reduces with that of heat absorbing source. An increase in the temperature parameter(A) /slip parameter(A_{11}),enhances the Sherwood number at the wall.

6. CONCLUSIONS

The problem of combined influence of rotation, variable viscosity effect, nonlinear thermal radiation on MHD free-convective flow and heat and mass transfer over a stretching sheet in the presence of heat generation/absorption has been analyzed. The main findings of this analysis are:

An increase in the rotation parameter(R) reduces the axial velocity, concentration and enhances the cross flow velocity, temperature. The skin friction components, Nusselt number and the Sherwood number enhances at the wall with R . The axial and cross flow velocities increase in the boundary layer with the increase in the viscosity parameter θ_r . With increase in the viscosity parameter the temperature and concentration of the fluid flow also increase in the boundary layer. The skin friction component τ_x , Nu reduces while τ_z , Sh enhances at the wall reduces with increase in θ_r . An increase in temperature parameter(A) enhances the axial velocity, temperature and reduces the cross flow velocity, concentration.. τ_x , τ_y , Sh enhance and Nu reduces at the wall with increase in A .

Table : 3: Skin friction(τ_x),Nusselt number(Nu) and Sherwood number(Sh) at $\eta=0$

Parameter		$\tau_x(0)$	$\tau_z(0)$	$Nu(0)$	$Sh(0)$
R	0.5	4.77205	-0.136489	0.243879	0.62626
	1.0	4.78021	-0.273653	0.236251	0.62864
	1.5	4.79943	-0.403555	0.236252	0.62898
	2.0	4.82172	-0.585929	0.236254	0.62914
Q	0.5	4.76729	-0.136459	0.247012	0.625117
	1.5	4.81659	-0.136723	0.212928	0.637138
	-0.5	4.72312	-0.136152	0.273999	0.614822
	-1.5	4.65751	-0.135699	0.316764	0.598989
γ	0.5	4.77205	-0.136489	0.243879	0.626262
	1.5	4.81659	-0.136723	0.212928	0.740796

Parameter		$\tau_x(0)$	$\tau_z(0)$	$Nu(0)$	$Sh(0)$
Sr	-0.5	4.85246	-0.136918	0.188706	0.575196
	-1.5	4.88971	-0.137109	0.163509	0.531541
	0.5	4.77205	-0.136489	0.243879	0.626262
	1.0	4.81659	-0.136723	0.212928	0.645263
	1.5	4.85246	-0.136918	0.188706	0.672697
	2.0	4.88971	-0.137109	0.163509	0.711622
Rd	0.5	4.77205	-0.136489	0.243879	0.626261
	1.5	4.81639	-0.136722	0.213063	0.637091
	3.5	4.85241	-0.136918	0.188745	0.645798
	5.0	4.88774	-0.137131	0.164855	0.654312
Ec	0.2	4.77205	-0.136489	0.243879	0.626226
	0.4	4.82113	-0.136744	0.208913	0.638447
	0.6	4.87367	-0.137009	0.169922	0.651933
	0.8	4.93899	-0.1373	0.119684	0.668889
θ_r	-2	4.77205	-0.136489	0.243879	0.626261
	-4	4.48922	-0.143089	0.213051	0.631492
	-6	4.40326	-0.145879	0.189001	0.637846
	-8	4.37695	-0.147707	0.163995	0.645503
A	0.01	4.77205	-0.136489	0.243879	0.626261
	1.5	4.81579	-0.136722	0.213517	0.636946
	2.0	4.85052	-0.136914	0.190193	0.645341
	2.5	4.88625	-0.137104	0.166227	0.653944
A11	0.2	4.77205	-0.136489	0.243879	0.626261
	0.4	3.36241	-0.143847	0.213729	0.650715
	0.6	2.64611	-0.147426	0.190375	0.665777
	0.8	2.18415	-0.149755	0.166081	0.678724
	7.00	4.88971	-0.137109	0.163509	0.654784

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