Bounded generalized Gaussian Mixture Model for Image Modeling

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Abstract

Many researchers studied the probability models for quantized and dequantized block Discrete Cosine transform(DCT) Coefficients in JPEG compression. However, All the studied probability models are unbounded. In many applications, the observed data are digitalized and have bounded support. A new bounded generalized Gaussian mixture model (BGGMM), which includes the Gaussian mixture model (GMM), Laplace mixture model(LMM), and GGMM(Generalized Gaussian Mixture Model) as special cases, is presented in this paper as a model for discrete cosine Coefficients. This new distribution has a flexibility to fit different shapes of observed data such as non-Gaussian and bounded support data. In order to estimate the model parameters, we propose an alternate approach to minimize the higher bound on the data negative log-likelihood function. Seven different probability distribution functions (Gaussian, Gamma, Rayleigh, Laplace, generalized Gamma, generalized Gaussian and bounded generalized Gaussian mixture) are compared for the accurate modeling of the DCT coefficients. We quantify the performance of the BGGMM with simulations and real data. It is shown via Kolmogorov-Smirnov (KS) tests that the PMFs based on the Bounded Generalized Gaussian Mixture Model (BGGMM) PDF are the most suitable distributions for DCT coefficients.

Keywords: BGGMM : Bounded Generalized Gaussian Mixture model ; GGM : Generalized Gaussian Model; GF M: Generalized Gamma Model; GGMM: Generalized Gaussian Mixture Model, LMM: Laplace Mixture Model; GMM: Gaussian Mixture Model; K-S : Kolmogorov-Smirnov test, DCT: Discrete Cosine Transform

1. Introduction

Due to the advances in various aspects of digital electronics like image acquisition, data storage and display, many new applications of the digital imaging have emerged over the past decade. However, many of these applications are not widespread because of required large storage space. As a result, the image compression has grown tremendously over the last decade and various image compression algorithms have been proposed. Joint Photographic Experts Group (JPEG) is a worldwide standard for the compression of still images.

Knowledge of the statistical distribution of the DCT coefficients is necessary in order to design the efficient quantizer with high SNR. Fig. 1.b shows a typical plot of the histograms of the DCT coefficients. The image used here is the “man” picture shown in Fig. 1.a. The upper left coefficient is called the DC coefficient while the rest are AC coefficients. The scaling of the histogram is kept the same for all ac coefficients in this plot. It has empirically been proven that the DC coefficient, the coefficient at mode (0,0) has a PDF that is different from the other modes of the DCT [9]. The AC coefficients, i.e., the coefficients at modes other than (0, 0) have been shown to have PDFs that are very similar to each other [3] [5] [9]. However, different papers have employed different PDFs to model AC block DCT coefficients. Early work on the distributions of 2D block DCT coefficients in images was carried out by Pratt (1978) [8], who conjectured that the DC coefficients should be Rayleigh distributed, since they are the sum of positive values (with the assumption of no level-shift), and that the AC coefficients, in general, must be Gaussian distributed as per the central limit theorem, with the assumption that each image pixel may be considered as being statistically independent from another. Tescher (1979) [12] and Murakami, Hatori and Yamamoto (1982) [6] indicated that the AC coefficients are best modeled as Laplacian, and the DC coefficients are best modeled as Gaussian. Reininger and Gibson (1983) summarized the findings in their seminal paper [9] and concluded that the DC coefficient is best modeled by a Gaussian PDF, while AC coefficients are best modeled by a Laplacian PDF. Reininger and Gibson used the Kolmogorov-Smirnov (KS) test [13] on standard 256x256 8-bit PCM encoded grayscale images over multiple modes, to determine the goodness of fit of the different PMFs. Muller [5] showed that the Generalized Gaussian distribution model (GG) fits the PDF of AC block DCT coefficients better than the Laplacian distribution using the χ^2 -Square test. In 2005, Chang, Shin, Kim and Mitra proposed a model for block DCT coefficients based on the Generalized Gamma distribution (GrM). All the previously studied probability models are unbounded. In many applications, the observed data are digitalized and have bounded support. A new bounded generalized Gaussian mixture model (BGGMM), which includes the Gaussian mixture model (GMM), Laplace mixture

model(LMM), and GGMM(Generalized Gaussian Mixture Model) as special cases, is presented in this paper as a model for discrete cosine Coefficients. This new distribution has a flexibility to fit different shapes of observed data such as non-Gaussian and bounded support data. Seven different probability distribution functions (Gaussian, Gamma, Rayleigh, Laplace, generalized Gamma, generalized Gaussian and bounded generalized Gaussian mixture) are compared for the accurate modeling of the DCT coefficients.

The present paper has been divided as follows. In Section 2, Bounded Generalized Gaussian Mixture model is given. In Section 3, the parametric estimator of the presented statistical model is also presented. In section 4, we provide the experimental results of seven different distributions for four images and nine DCT coefficients. The last section concludes this paper.

2. Bounded Generalized Gaussian Distribution Mixture (BGGMM)

A Mixture of Bounded Generalized Gaussian Density [BGGMM] is a parametric statistical model which assumes that the data originates from weighted sum of several Bounded generalized Gaussian sources [11]. More specifically,

$$g(x_i|\theta) = \sum_{j=1}^K m_j g_j(x_i|\mu_j, \sigma_j, \lambda_j) \quad (1)$$

Where i) $\xi_1 = (\mu_1, \mu_2, \dots, \mu_K)$, $\xi_2 = (\sigma_1, \sigma_2, \dots, \sigma_K)$, $\xi_3 = (\lambda_1, \lambda_2, \dots, \lambda_K)$, $\xi_4 = (m_1, m_2, \dots, m_K)$,

$$\theta = \bigcup_{i=1}^4 \xi_i \quad (2)$$

ii) K is the number of mixture density components,

iii) m_j is the jth mixture weight and satisfies $m_j \geq 0$, $\sum_{j=1}^K m_j = 1$

iv) $g_j(x_i|\mu_j, \sigma_j, \lambda_j)$ is an individual density of the bounded generalized Gaussian density which introduced by Thanh Minh Nguyen, Q.M. Jonathan Wu and Hui Zhang [11] defined by

$$g_j(x_i|\mu_j, \sigma_j, \lambda_j) = \frac{T_j(x_i|\mu_j, \sigma_j, \lambda_j)I(x_i|\Omega_j)}{\int_{\Omega_j} T_j(x_i|\mu_j, \sigma_j, \lambda_j)I(x_i|\Omega_j)dx_i} \quad (3)$$

The distribution $T_j(x_i|\mu_j, \sigma_j, \lambda_j)$ in (3) is as follows:

$$T_j(x_i|\mu_j, \sigma_j, \lambda_j) = A(\lambda_j) \exp\left(-B(\lambda_j) \left|\frac{x_i - \mu_j}{\sigma_j}\right|^{\lambda_j}\right) \quad (4)$$

Where

$$A(\lambda_j) = \frac{\lambda_j \sqrt{\Gamma(3/\lambda_j)}}{2\sigma_j \Gamma(1/\lambda_j) \sqrt{\Gamma(1/\lambda_j)}} \quad \text{and} \quad B(\lambda_j) = \left(\frac{\Gamma(3/\lambda_j)}{\Gamma(1/\lambda_j)} \right)^{\lambda_j/2} \quad (5)$$

The parameters μ_j and σ_j are the mean and standard deviation. The parameter λ_j controls the tails of the distribution and determines whether the latter is peaked or flat.

In (3), $\int_{\partial_j} T_j(x_i | \mu_j, \sigma_j, \lambda_j) I(x_i | \Omega_j) dx_i$ is the normalization constant and is identified as the share of $T_j(x_i | \mu_j, \sigma_j, \lambda_j)$ that belongs to the supported region ∂_j and $I(x_i | \Omega_j)$ is an indicator function defined for each component Ω_j that belongs to the supported region ∂_j . Where $I(x_i | \Omega_j)$ as:

$$I(x_i | \Omega_j) = \begin{cases} 1 & \text{if } x_i \in \partial_j \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

The importance of the bounded generalized Gaussian mixture model is that it has a flexibility to fit different shapes of observed data such as non-Gaussian and bounded support data and can be specialized to several important distributions Thanh Minh Nguyen, Q.M. Jonathan Wu and Hui Zhang [11] as shown in table 1

GMM	BGGMM: $\lambda_j = 2, I(x_i \Omega_j) = 1$
LMM	BGGMM: $\lambda_j = 1, I(x_i \Omega_j) = 1$
GGD	$K = 1, I(x_i \Omega_j) = 1$
GGMM	BGGMM: $I(x_i \Omega_j) = 1$
BGMM	BGGMM: $\lambda_j = 2, I(x_i \Omega_j) = I(x_i \Omega_k); \forall j, k = (1, 2, \dots, K)$
BLMM	BGGMM: $\lambda_j = 1, I(x_i \Omega_j) = I(x_i \Omega_k); \forall j, k = (1, 2, \dots, K)$

Table 1: some special distributions from the bounded generalized Gaussian mixture model(BGGMM)

Two important problems in the case of bounded generalized Gaussian mixture model are the estimation of the vector of parameters θ and the determination of the number of classes K . When the number of classes is known, statistical inferential methods about the parameters can be used

mostly via the Maximum-Likelihood estimation (MLE). In the Next section, we show how to estimate the model parameter and model order.

3. Boundedgeneralized Gaussian mixturemodel Parameters Estimation

Using the maximum likelihood method, the estimators of the unknown parameters vector $\theta = (\mu_j, \sigma_j, \lambda_j, m_j)$, $j = 1, 2, 3, \dots, K$ can be obtained by taking the natural log of the likelihood function of a random sample of observation with the pdf in equation (1) then setting the derivatives of the log-likelihood function to zeros. The log-likelihood function of equation (1), given by [11] is

$$E(\theta) = - \sum_{i=1}^N \sum_{j=1}^K z_{ij} \left\{ \log m_j + \log (T_j(x_i | \mu_j, \sigma_j, \lambda_j)) + \log (I(x_i | \Omega_j)) - \log \left(\int_{\partial_j} T_j(x_i | \mu_j, \sigma_j, \lambda_j) dx_i \right) \right\} \quad (7)$$

where N the sample size and z_{ij} is defined as [11]

$$z_{ij} = \frac{m_j \psi(x_i | \mu_j, \sigma_j, \lambda_j)}{\sum_{l=1}^K m_l \psi(x_i | \mu_l, \sigma_l, \lambda_l)} \text{ and } \psi(x_i | \mu_j, \sigma_j, \lambda_j) = \frac{T_j(x_i | \mu_j, \sigma_j, \lambda_j) I(x_i | \Omega_j)}{\int_{\partial_j} T_j(x_i | \mu_j, \sigma_j, \lambda_j) dx_i} \quad (8)$$

According to [11] the equations for estimating the parameters $(\mu_j, \sigma_j, \lambda_j, m_j)$ are [11]

$$\frac{\partial E(\theta)}{\partial \mu_j} = -\beta(\lambda_j) \frac{\lambda_j}{\sigma_j^{\lambda_j}} \sum_{i=1}^N z_{ij} |x_i - \mu_j|^{(\lambda_j-2)} \left\{ x_i + \mu_j - \frac{R_j}{|x_i - \mu_j|^{(\lambda_j-2)}} \right\} = 0 \quad (9)$$

Where

$$R_j = \frac{\sum_{m=1}^M \text{sign}(\mu_j^t - S_{mj}) |(S_{mj} - \mu_j^t)|^{(\lambda_j(t)-1)} I(S_{mj} | \Omega_j)}{\sum_{m=1}^M I(S_{mj} | \Omega_j)} \quad (10)$$

$$\frac{\partial E(\theta)}{\partial \sigma_j} = -\sigma_j^{-1} \sum_{i=1}^N z_{ij} \left\{ -1 + \beta(\lambda_j) |x_i - \mu_j|^{\lambda_j} \lambda_j \sigma_j^{-\lambda_j} - G_j \right\} = 0 \quad (11)$$

Where

$$G_j = \frac{\sum_{m=1}^M \left\{ -1 + \lambda_j^{(t)} \beta(\lambda_j^{(t)}) \left| (S_{mj} - \mu_j^{(t)}) \right|^{\lambda_j^{(t)}} (\sigma_j^{(t)})^{-\lambda_j^{(t)}} \right\}}{\sum_{m=1}^M I(S_{mj} | \Omega_j)} \quad (12)$$

To estimate the parameter λ we use newton raphson method [1,2,4] according to the following equation

$$\lambda_j^{(t+1)} = \lambda_j^{(t)} - \frac{\partial E(\theta)}{\partial \lambda_j} \left(\frac{\partial^2 E(\theta)}{\partial \lambda_j^2} + \gamma \right) \quad (13)$$

Where γ is a scaling factor. The derivatives of the error function with respect to λ_j is given by

$$\frac{\partial E(\theta)}{\partial \lambda_j} = - \sum_{i=1}^N z_{ij} \left\{ f(x_i | \mu_j, \sigma_j, \lambda_j) - \frac{\int_{\partial_j} T_j(x | \mu_j, \sigma_j, \lambda_j) f(x | \mu_j, \sigma_j, \lambda_j) dx_i}{\int_{\partial_j} T_j(x_i | \mu_j, \sigma_j, \lambda_j) dx} \right\} \quad (14)$$

Where $f(x | \mu_j, \sigma_j, \lambda_j) =$

$$\left[\frac{\frac{1}{\lambda_j} + 3\psi\left(\frac{1}{\lambda_j}\right) - 3\psi\left(\frac{3}{\lambda_j}\right)}{2\lambda_j^2} \right] - \beta(\lambda_j) \left| \frac{x - \mu_j}{\sigma_j} \right|^{\lambda_j} \log \left(\left| \frac{x - \mu_j}{\sigma_j} \right| \right) - \beta(\lambda_j) \left(\left(\frac{1}{2} \right) \log \left(\frac{\Gamma\left(\frac{3}{\lambda_j}\right)}{\Gamma\left(\frac{1}{\lambda_j}\right)} \right) + \psi 1 \lambda_j - 3 \psi 3 \lambda_j 2 \lambda_j x - \mu_j \sigma_j \lambda_j \right) \quad (15)$$

$$\frac{\partial E(\theta)}{\partial^2 \lambda_j} = - \sum_{i=1}^N z_{ij} \left\{ g(x | \mu_j, \sigma_j, \lambda_j) - \frac{\left(\int_{\partial_j} T_j(x | \mu_j, \sigma_j, \lambda_j) f(x | \mu_j, \sigma_j, \lambda_j) dx_i \right)^2}{\left(\int_{\partial_j} T_j(x_i | \mu_j, \sigma_j, \lambda_j) dx \right)^2} - \frac{\left(\int_{\partial_j} T_j(x | \mu_j, \sigma_j, \lambda_j) (f^2(x | \mu_j, \sigma_j, \lambda_j) + g(x | \mu_j, \sigma_j, \lambda_j)) dx \right)}{\int_{\partial_j} T_j(x_i | \mu_j, \sigma_j, \lambda_j) dx} \right\} \quad (16)$$

$$\text{where } g(x|\mu_j, \sigma_j, \lambda_j) = \frac{\partial f(x|\mu_j, \sigma_j, \lambda_j)}{\partial \lambda_j} =$$

$$\left[-\frac{1}{\lambda_j^2} - \frac{3\psi\left(\frac{1}{\lambda_j}\right)}{\lambda_j^4} - \frac{3\psi\left(\frac{1}{\lambda_j}\right)}{\lambda_j^3} + \frac{9\psi\left(\frac{3}{\lambda_j}\right)}{2\lambda_j^4} + \frac{3\psi\left(\frac{3}{\lambda_j}\right)}{\lambda_j^3} \right] -$$

$$\beta(\lambda_j) \left| \frac{x-\mu_j}{\sigma_j} \right|^{\lambda_j} \log \left(\left| \frac{x-\mu_j}{\sigma_j} \right| \right)^2 -$$

$$\beta(\lambda_j) \left(\left(\frac{1}{2} \right) \log \left(\frac{\Gamma\left(\frac{3}{\lambda_j}\right)}{\Gamma\left(\frac{1}{\lambda_j}\right)} \right) + \frac{-3\psi\left(\frac{3}{\lambda_j}\right) + \psi\left(\frac{1}{\lambda_j}\right)}{2\lambda_j} + \frac{9\psi\left(\frac{3}{\lambda_j}\right) - \psi\left(\frac{1}{\lambda_j}\right)}{2\lambda_j^3} \right) \left| \frac{x-\mu_j}{\sigma_j} \right|^{\lambda_j} - 2\beta(\lambda_j) \left(\left(\frac{1}{2} \right) \log \left(\frac{\Gamma\left(\frac{3}{\lambda_j}\right)}{\Gamma\left(\frac{1}{\lambda_j}\right)} \right) + \right.$$

$$\left. -3\psi\frac{3}{\lambda_j} + \psi\frac{1}{\lambda_j} \right) \left| \frac{x-\mu_j}{\sigma_j} \right|^{\lambda_j} \log \left(\left| \frac{x-\mu_j}{\sigma_j} \right| \right) \quad (17)$$

The term $\int_{\partial_j} T_j(x|\mu_j, \sigma_j, \lambda_j) dx$ in equations (8), (14) and (16) can be approximated as

$$\int_{\partial_j} T_j(x|\mu_j, \sigma_j, \lambda_j) dx \approx \frac{1}{M} \sum_{m=1}^M I(S_{mj}|\Omega_j) \quad (18)$$

Similarly, the term $\int_{\partial_j} T_j(x|\mu_j, \sigma_j, \lambda_j) f(x|\mu_j, \sigma_j, \lambda_j) dx$ in (14) and (16) can be approximated as :

$$\int_{\partial_j} T_j(x|\mu_j, \sigma_j, \lambda_j) f(x|\mu_j, \sigma_j, \lambda_j) dx \approx \frac{1}{M} \sum_{m=1}^M f(S_{mj}|\mu_j^{(t)}, \sigma_j^{(t)}, \lambda_j^{(t)}) I(S_{mj}|\Omega_j) \quad (19)$$

Similarly, the term $\left(\int_{\partial_j} T_j(x|\mu_j, \sigma_j, \lambda_j) f^2(x|\mu_j, \sigma_j, \lambda_j) + g(x|\mu_j, \sigma_j, \lambda_j) \right) dx$ in (16) can be approximated as :

$$\left(\int_{\partial_j} T_j(x|\mu_j, \sigma_j, \lambda_j) (f^2(x|\mu_j, \sigma_j, \lambda_j) + g(x|\mu_j, \sigma_j, \lambda_j)) dx \right)$$

$$\approx \frac{1}{M} \sum_{m=1}^M \left(f^2(x|\mu_j^{(t)}, \sigma_j^{(t)}, \lambda_j^{(t)}) + g(x|\mu_j^{(t)}, \sigma_j^{(t)}, \lambda_j^{(t)}) \right) I(S_{mj}|\Omega_j) \quad (20)$$

Solving equations (9) and (11) we get [11]

$$\mu_j^{(t+1)} = \frac{\sum_{i=1}^N Z_{ij} \left(|x_i - \mu_j|^{(\lambda_j-2)} x_i + R_j \right)}{\sum_{i=1}^N Z_{ij} \left(|x_i - \mu_j|^{(\lambda_j-2)} x_i \right)} \quad (21)$$

$$\sigma_j^{(t+1)} = \frac{\lambda_j B(\lambda_j) \sum_{i=1}^N Z_{ij} |x_i - \mu_j|^{\lambda_j}}{\sum_{i=1}^N Z_{ij} (1 + G_j)} \quad (22)$$

To estimate the parameter m_j which satisfy the constraint $m_j \geq 0$, $\sum_{j=1}^K m_j = 1$ we use the following equation

$$m_j^{(t+1)} = \frac{1}{N} \sum_{i=1}^N Z_{ij}^{(t)} \quad (23)$$

4. Experimental and results

In this experiment the performance of Bounded Generalized Gaussian mixture Model (BGGMM) is compared to six statistical distribution functions ((Gaussian, Gamma, Rayleigh, Laplace, generalized Gamma, generalized Gaussian) for the accurate modeling of the DCT coefficients on a sample of thousand images.

K-S goodness of fit test statistic is used to measure the fitting accuracy of each distribution function.

The image histogram for DCT Coefficients for man image exist in figure 1.a is shown in figure 1.b. Fitting the seven different distributions for DC, AC(0,1), AC(0,2) and AC(0,3) coefficients for image man are shown in figures 2. Figure 3 shows the mean KS test statistic values for nine discrete cosine coefficient blocks and seven statistical distribution functions of the studied images.

It can be seen from figure 2 and 3 that the Bounded Generalized Gaussian Mixture model KS statistic distance is smaller than all other KS statistic distance for the DCT coefficients. Therefore, it can be concluded that the distribution of the DCT coefficients (Both DC and AC) is Bounded Generalized Gaussian Mixture in nature. Thus, Bounded Generalized Gaussian Mixture PDF can best approximate all of DCT coefficients for the studied images.

figure: 1.a: man imag

Figure: 1.b: Histogram of the DCT Coefficients for man image

Figure 2: fitting cumulative distributions for DC Coefficient for man image with gama, gaussain, Ray, GG,GF and BGGM and laplace

Figure 3: KS Statistics Results for seven probability distribution, for nine DCT coefficients, for thousand Images

5. Conclusions

A new bounded generalized Gaussian mixture model (BGGMM), which includes the Gaussian mixture model (GMM), Laplace mixture model (LMM), and GGMM (Generalized Gaussian Mixture Model) as special cases, is presented in this paper as a model for discrete cosine coefficients. This new distribution has a flexibility to fit different shapes of observed data such as non-Gaussian and bounded support data. Seven different probability distribution functions (Gaussian, Gamma, Rayleigh, Laplace, generalized Gamma (GG), generalized Gaussian (GG) and bounded generalized Gaussian mixture (BGGMM)) are compared for the accurate modeling of the DCT coefficients. We quantify the performance of the BGGMM with simulations and real data. It is shown via Kolmogorov-Smirnov (KS) tests that the PMFs based on the Bounded Generalized Gaussian Mixture Model (BGGMM) PDF are the most suitable distributions for DCT coefficients.

References

- [1] M. Allili, Wavelet modeling using finite mixtures of generalized Gaussian distributions: application to texture discrimination and retrieval, *IEEE Trans. Image Process.* 21(4)(2012)1452–1464.
- [2] J. Ashburner, K.J. Friston, Unified segmentation, *NeuroImage* 26(3)(2005) 839–851.
- [3] J.-H. Chang, J. W. Shin, N. S. Kim, S. K. Mitra, Image Probability Distribution Based on Generalized Gamma Function, *IEEE Signal Processing Letters*, 2005.
- [4] J. Lindblom, J. Samuelsson, Bounded support Gaussian mixture modeling of speech spectra, *IEEE Trans. Speech Audio Process.* 11(1)(2003)88–99.
- [5] F. Muller, Distribution Shape of Two-Dimensional DCT Coefficients of Natural Images, *Electronics Letters*, 29, Oct. 1993, 1935-1936.
- [6] H. Murakami, Y. Hatori, and H. Yamamoto, Comparison between DPCM and Hadamard transform coding in the composite coding of the NTSC color TV signal, *IEEE Transactions on Communications*, vol COM-30, pp. 469-479, Mar. 1982.
- [7] S. Nadarajah, A generalized normal distribution, *Journal of Applied Statistics*. 32(7), pp. 685-694, 2005.
- [8] W. K. Pratt, *Digital Image Processing*, New York: Wiley-Interscience, 1978, chapter 10.
- [9] R. Reininger and J. Gibson : Distribution of the two-dimensional DCT coefficients for images, *IEEE Transactions on Communications* 31 (6) 1983.
- [10] E. W. Stacy, A Generalization of the Gamma Distribution, *The Annals of Mathematical Statistics*. Vol. 33, No. 3, pp. 1187-1192, September, 1962.
- [11] Thanh Minh Nguyen , Q. M. Jonathan Wu, Hui Zhang, Bounded generalized Gaussian mixture model, *Journal of Pattern Recognition*. Vol 47, pp. 3132–3142, (2014)

[12] A. G. Tescher, Transform image coding, in Advances in Electronics and Electron Physics. Suppl. 12. New York: Academic, 1979, pp. 113-115.

[13] E. W. Weisstein, Kolmogorov-Smirnov Test. From Math World--A Wolfram Web Resource. <http://mathworld.wolfram.com/Kolmogorov-SmimovTest.html>, retrieved March 22, 2010.