A New Intuitionistic Decagonal fuzzy number and its application

A.praveenprakash¹, N.Jose Parvin Praveena², A. Rajkumar³

¹Professor, Hindustan Institute of Technology and science, Padur, Chennai - 603103,

²Research Scholar, Hindustan Institute of Technology and science, Padur, Chennai - 603103.

Assistant Professor, KCG College of Technology, Karapakkam, Chennai-97

³Assistant Professor, Hindustan Institute of Technology and science, Padur, Chennai -603103.

Abstract

In this article we define a new intuitionistic Decagonal fuzzy number and by using this Intuitionistic Decagonal fuzzy number we find the fuzzy critical path. Here we have taken the edge weights of the network in terms of Intuitionistic Decagonal fuzzy number. For Defuzzification of this Decagonal fuzzy number we derived a magnitude measure.

Introduction

In recent days fuzzy numbers have been developed much. In this paper, we introduced Intuitionistic Decagonal fuzzy number. If uncertainty arises in 10 different points, in such cases this decagonal fuzzy numbers are applied. If the number of points is more, it will reduce the vagueness. Fuzzy critical path is one of the useful methods in project planning and project development. Network exploration is a method which decides the several sequences of activities regarding a project and the project achievement time. The main determination of CPM is to recognize critical activities on the critical path.

Chanas and Zielinski, anticipated that the task time of each activity can be represented as a crisp value, interval or a fuzzy number and deliberated the difficulty of criticality.

This paper comprising of the following sections. Section one designates the basic definitions of Decagonal fuzzy numbers, intuitionistic Decagonal fuzzy number and its membership function.

Section two describes the new formula to defuzzify this Decagonal fuzzy number and working rule to find critical path. Section three defines the working rule to find the fuzzy critical path and example. Section four concludes this paper.

The membership function and non-membership function of the Intuitionistic Decagonal fuzzy numbers are given below:

$$\mu_{\tilde{B}_{ID}}(x) = \begin{cases} 0 & x < \alpha_1 \\ \frac{1}{4} \frac{x - \alpha_1}{\alpha_2 - \alpha_1} & \alpha_1 \leq x \leq \alpha_2 \\ \frac{1}{4} + \frac{1}{4} \frac{x - \alpha_2}{\alpha_3 - \alpha_2} & \alpha_2 \leq x \leq \alpha_3 \\ \frac{1}{2} + \frac{1}{4} \frac{x - \alpha_3}{\alpha_4 - \alpha_3} & \alpha_3 \leq x \leq \alpha_4 \\ \frac{3}{4} + \frac{1}{4} \frac{x - \alpha_4}{\alpha_5 - \alpha_4} & \alpha_4 \leq x \leq \alpha_5 \\ 1 & \alpha_5 \leq x \leq \alpha_6 \\ 1 - \frac{1}{4} \frac{x - \alpha_6}{\alpha_7 - \alpha_6} & \alpha_6 \leq x \leq \alpha_7 \\ \frac{3}{4} - \frac{1}{4} \frac{x - \alpha_7}{\alpha_8 - \alpha_7} & \alpha_7 \leq x \leq \alpha_8 \\ \frac{1}{2} - \frac{1}{4} \frac{x - \alpha_8}{\alpha_9 - \alpha_8} & \alpha_8 \leq x \leq \alpha_9 \\ \frac{1}{4} \frac{\alpha_{10} - x}{\alpha_{10} - \alpha_9} & \alpha_9 \leq x \leq \alpha_{10} \\ 0 & x \geq 10 \end{cases}$$

$$\gamma_{\tilde{B}_{ID}}(x) = \begin{cases} 1 & x < \beta_1 \\ 1 - \frac{1}{4} \frac{\beta_2 - x}{\beta_2 - \beta_1} & \beta_1 \leq x \leq \beta_2 \\ \frac{3}{4} - \frac{1}{4} \frac{\beta_3 - x}{\beta_3 - \beta_2} & \beta_2 \leq x \leq \beta_3 \\ \frac{1}{2} - \frac{1}{4} \frac{\beta_4 - x}{\beta_4 - \beta_3} & \beta_3 \leq x \leq \beta_4 \\ \frac{1}{4} \frac{\beta_5 - x}{\beta_5 - \beta_4} & \beta_4 \leq x \leq \beta_5 \\ 0 & \beta_5 \leq x \leq \beta_6 \\ \frac{1}{4} \frac{\beta_7 - x}{\beta_7 - \beta_6} & \beta_6 \leq x \leq \beta_7 \\ \frac{1}{4} + \frac{1}{4} \frac{\beta_8 - x}{\beta_8 - \beta_7} & \beta_7 \leq x \leq \beta_8 \\ \frac{1}{2} + \frac{1}{4} \frac{\beta_9 - x}{\beta_9 - \beta_8} & \beta_8 \leq x \leq \beta_9 \\ \frac{3}{4} + \frac{1}{4} \frac{x - \beta_9}{\beta_{10} - \beta_9} & \beta_9 \leq x \leq \beta_{10} \\ 1 & x \geq 10 \end{cases}$$

The membership function and non-membership function of the Intuitionistic Decagonal fuzzy numbers are given above.

2. Definitions: Defuzzification of Decagonal fuzzy number:

Let $\tilde{E} = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10})$ be a Decagonal fuzzy number. It is changed to Nonagonal fuzzy number. This Nonagonal fuzzy number is defuzzified by magnitude measure of the Nonagonal fuzzy number $\tilde{E} = (\theta_1, \theta_2, \theta_3, \theta_4, \lambda, \theta_7, \theta_8, \theta_9, \theta_{10})$ where $\lambda = \frac{\theta_5 + \theta_6}{2}$

Membership function of Nonagonal fuzzy number.

$$\mu_N(x) = \begin{cases} 0 & x < \theta_1 \\ \frac{1}{4} \frac{x - \theta_1}{\theta_2 - \theta_1} & \theta_1 \leq x \leq \theta_2 \\ \frac{1}{4} + \frac{1}{4} \frac{x - \alpha_2}{\alpha_3 - \alpha_2} & \theta_2 \leq x \leq \theta_3 \\ \frac{1}{2} + \frac{1}{4} \frac{x - \alpha_3}{\alpha_4 - \alpha_3} & \theta_3 \leq x \leq \theta_4 \\ \frac{3}{4} + \frac{1}{4} \frac{x - \alpha_4}{\alpha_5 - \alpha_4} & \theta_4 \leq x \leq \lambda \\ 1 - \frac{1}{4} \frac{x - \lambda}{\theta_7 - \lambda} & \lambda \leq x \leq \theta_7 \\ \frac{3}{4} - \frac{1}{4} \frac{x - \alpha_7}{\alpha_8 - \alpha_7} & \theta_7 \leq x \leq \theta_8 \\ \frac{1}{2} - \frac{1}{4} \frac{x - \alpha_8}{\alpha_9 - \alpha_8} & \theta_8 \leq x \leq \theta_9 \\ \frac{1}{4} \frac{\alpha_{10} - x}{\alpha_{10} - \alpha_9} & \theta_9 \leq x \leq \theta_{10} \\ 0 & x \geq 10 \end{cases}$$

$$\gamma_N(x) = \begin{cases} 1 & x < \theta_1 \\ 1 - \frac{1}{4} \frac{x - \theta_1}{\theta_2 - \theta_1} & \theta_1 \leq x \leq \theta_2 \\ \frac{3}{4} - \frac{1}{4} \frac{x - \theta_2}{\theta_3 - \theta_2} & \theta_2 \leq x \leq \theta_3 \\ \frac{1}{2} - \frac{1}{4} \frac{x - \theta_3}{\theta_4 - \theta_3} & \theta_3 \leq x \leq \theta_4 \\ \frac{1}{4} \frac{\lambda - x}{\lambda - \theta_4} & \theta_4 \leq x \leq \lambda \\ \frac{1}{4} \frac{x - \lambda}{\theta_7 - \lambda} & \lambda \leq x \leq \theta_7 \\ \frac{1}{4} + \frac{1}{4} \frac{x - \theta_7}{\theta_8 - \theta_7} & \theta_7 \leq x \leq \theta_8 \\ \frac{1}{2} + \frac{1}{4} \frac{\beta_9 - x}{\beta_9 - \beta_8} & \theta_8 \leq x \leq \theta_9 \\ \frac{3}{4} + \frac{1}{4} \frac{x - \beta_9}{\beta_{10} - \beta_9} & \theta_9 \leq x \leq \theta_{10} \\ 1 & x \geq 10 \end{cases}$$

Magnitude measure of the Nonagonal fuzzy number

The α -cut of the fuzzy set $\tilde{E}$ of the universe of discourse X is defined as

$$\tilde{E} = \{x \in X / \mu(x) \geq \alpha\} \text{ Where } \alpha \in [0,1]$$

α – cut of Nonagonal Fuzzy number is given by

$$\tilde{E}_\alpha = \begin{cases} [u_l(\alpha), u_r(\alpha)] & \alpha \in [0, 0.25) \\ [v_l(\alpha), v_r(\alpha)] & \alpha \in [0.25, 0.5) \\ [w_l(\alpha), w_r(\alpha)] & \alpha \in [0.5, 0.75) \\ [s_l(\alpha), s_r(\alpha)] & \alpha \in [0.75, 1] \end{cases}$$

Where

$$\begin{aligned} u_l(\alpha), u_r(\alpha) &= [4\alpha(\theta_2 - \theta_1) + \theta_1, -4\alpha(\theta_{10} - \theta_9) + \theta_{10}] & \alpha \in [0, 0.25) \\ v_l(\alpha), v_r(\alpha) &= [4\alpha(\theta_3 - \theta_2) + \theta_2, -4\alpha(\theta_9 - \theta_8) + \theta_9] & \alpha \in [0.25, 0.5) \\ w_l(\alpha), w_r(\alpha) &= [4\alpha(\theta_4 - \theta_3) + \theta_3, -4\alpha(\theta_8 - \theta_7) + \theta_8] & \alpha \in [0.5, 0.75) \\ s_l(\alpha), s_r(\alpha) &= [4\alpha(\lambda - \theta_4) + \theta_4, -4\alpha(\theta_7 - \lambda) + \theta_7] & \alpha \in [0.75, 1] \end{aligned}$$

Magnitude measure of $\tilde{E}$ is given by

$$\begin{aligned} M(\tilde{E}) &= \frac{1}{2} \int_0^1 (u_l(\alpha) + u_r(\alpha) + v_l(\alpha) + v_r(\alpha) + w_l(\alpha) + w_r(\alpha) + s_l(\alpha) + s_r(\alpha)) \alpha d\alpha \\ &= \frac{1}{2} \int_0^1 \left([4\alpha(\theta_2 - \theta_1) + \theta_1, -4\alpha(\theta_{10} - \theta_9) + \theta_{10}] + [4\alpha(\theta_3 - \theta_2) + \theta_2, -4\alpha(\theta_9 - \theta_8) + \theta_9] + \right. \\ &\quad \left. [4\alpha(\theta_4 - \theta_3) + \theta_3, -4\alpha(\theta_8 - \theta_7) + \theta_8] + [4\alpha(\lambda - \theta_4) + \theta_4, -4\alpha(\theta_7 - \lambda) + \theta_7] \right) \alpha d\alpha \\ &= \frac{1}{12} (-5\theta_1 + 3\theta_2 + 3\theta_3 + 3\theta_4 + 16\lambda + 3\theta_7 + 3\theta_8 + 3\theta_9 + 11\theta_{10}) \end{aligned}$$

Definition :

α – cut ranking technique for IntuitionisticDecagonal fuzzy number

$$\text{Let } \tilde{E}_1 = ((\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}), (\theta_1', \theta_2', \theta_3', \theta_4', \theta_5', \theta_6', \theta_7', \theta_8', \theta_9', \theta_{10}'))$$

$\tilde{E}_2 = ((\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8, \lambda_9, \lambda_{10}), (\lambda_1', \lambda_2', \lambda_3', \lambda_4', \lambda_5', \lambda_6', \lambda_7', \lambda_8', \lambda_9', \lambda_{10}'))$ be two
 Decagonal intuitionistic fuzzy numbers. Then

$$L_{\max} = \max(\tilde{E}_1, \tilde{E}_2) = \left(\begin{array}{l} (\max(\theta_1, \lambda_1), \max(\theta_2, \lambda_2), \max(\theta_3, \lambda_3), \max(\theta_4, \lambda_4), \max(\theta_5, \lambda_5), \max(\theta_6, \lambda_6), \\ \max(\theta_7, \lambda_7), \max(\theta_8, \lambda_8), \max(\theta_9, \lambda_9), \max(\theta_{10}, \lambda_{10})), (\max(\theta_1', \lambda_1'), (\max(\theta_2', \lambda_2'), \\ (\max(\theta_3', \lambda_3'), (\max(\theta_4', \lambda_4'), (\max(\theta_5', \lambda_5'), (\max(\theta_6', \lambda_6'), (\max(\theta_7', \lambda_7'), (\max(\theta_8', \lambda_8'), \\ (\max(\theta_9', \lambda_9'), (\max(\theta_{10}', \lambda_{10}')) \end{array} \right)$$

Let $\tilde{E}_1 = ((\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}), (\theta_1', \theta_2', \theta_3', \theta_4', \theta_5', \theta_6', \theta_7', \theta_8', \theta_9', \theta_{10}'))$ be a Decagonal intuitionistic fuzzy number.

The α – cut of Decagonal fuzzy number is

$$D_\alpha = \left\{ \begin{array}{l} [4\alpha(\theta_2 - \theta_1) + \theta_1, -4\alpha(\theta_{10} - \theta_9) + \theta_{10}] \quad \alpha \in [0, 0.25) \\ [4\alpha(\theta_3 - \theta_2) + 2\theta_2 - \theta_3, -4\alpha(\theta_9 - \theta_8) + 2\theta_9 - \theta_8] \quad \alpha \in (0.25, 0.5) \\ [4\alpha(\theta_4 - \theta_3) + 3\theta_3 - 2\theta_4, -4\alpha(\theta_8 - \theta_7) + 3\theta_8 - 2\theta_7] \quad \alpha \in (0.5, 0.75) \\ [4\alpha(\theta_5 - \theta_4) + 4\theta_4 - 3\theta_5, -4\alpha(\theta_7 - \theta_6) + 4\theta_7 - 3\theta_6] \quad \alpha \in (0.75, 1] \end{array} \right\}$$

α – cut **Ranking technique for Intuitionistic Decagonal fuzzy number:**

For membership function

$$\begin{aligned} R(\tilde{E}_1) &= \int_0^1 (4\alpha(\theta_2 - \theta_1) + \theta_1) \alpha d\alpha + \\ &\int_0^1 -(4\alpha(\theta_{10} - \theta_9) + \theta_{10}) \alpha d\alpha + \\ &\int_0^1 (4\alpha(\theta_3 - \theta_2) + 2\theta_2 - \theta_3) \alpha d\alpha + \\ &\int_0^1 -(4\alpha(\theta_9 - \theta_8) + 2\theta_9 - \theta_8) \alpha d\alpha + \\ &\int_0^1 (4\alpha(\theta_4 - \theta_3) + 3\theta_3 - 2\theta_4) \alpha d\alpha + \\ &\int_0^1 -(4\alpha(\theta_8 - \theta_7) + 3\theta_8 - 2\theta_7) \alpha d\alpha + \\ &\int_0^1 (4\alpha(\theta_5 - \theta_4) + 4\theta_4 - 3\theta_5) \alpha d\alpha + \\ &\int_0^1 -(4\alpha(\theta_7 - \theta_6) + 4\theta_7 - 3\theta_6) \alpha d\alpha \\ &= \frac{-5\theta_1 + 6\theta_2 - 3\theta_3 + 2\theta_4 + 22\theta_7 + 6\theta_8 + 6\theta_9 - 5\theta_{10}}{6} \end{aligned}$$

For non-membership function

$$\begin{aligned}
R(\tilde{E}_1) &= \int_0^1 -(4\alpha(\theta_2' - \theta_1') + 5\theta_2' - 4\theta_1')\alpha d\alpha + \int_0^1 (4\alpha(\theta_{10}' - \theta_9') + 4\theta_9' - 3\theta_{10}')\alpha d\alpha + \int_0^1 (4\alpha(\theta_3' - \theta_2') + 3\theta_2' - 2\theta_3')\alpha d\alpha + \\
&\int_0^1 -(4\alpha(\theta_9' - \theta_8') - 2\theta_8' + 3\theta_9')\alpha d\alpha + \int_0^1 (4\alpha(\theta_4' - \theta_3') + 2\theta_3' - \theta_4')\alpha d\alpha + \int_0^1 -(4\alpha(\theta_8' - \theta_7') - \theta_7' + 2\theta_8')\alpha d\alpha + \\
&\int_0^1 (-4\alpha(\theta_5' - \theta_4') + \theta_5')\alpha d\alpha + \int_0^1 -(4\alpha(\theta_7' - \theta_6') + \theta_7')\alpha d\alpha \\
&= -\left(\frac{-4\theta_1' + 8\theta_2' + 13\theta_4' - 5\theta_5' + 8\theta_6' + 5\theta_9' - \theta_{10}'}{6}\right)
\end{aligned}$$

Therefore, α – cut ranking for Decagonal Intuitionistic Fuzzy number is given by

$$\begin{aligned}
R(\tilde{E}_1) &= \\
&\left(\left(\frac{-5\theta_1 + 6\theta_2 - 3\theta_3 + 2\theta_4 + 22\theta_7 + 6\theta_8 + 6\theta_9 - 5\theta_{10}}{6}\right), \left(\frac{-4\theta_1' + 8\theta_2' + 13\theta_4' - 5\theta_5' + 8\theta_6' + 5\theta_9' - \theta_{10}'}{6}\right)\right)
\end{aligned}$$

A Euclidean Ranking technique for Intuitionistic Decagonal Fuzzy number is given by

Let $P_i = ((\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10}), (\theta_1', \theta_2', \theta_3', \theta_4', \theta_5', \theta_6', \theta_7', \theta_8', \theta_9', \theta_{10}'))$ be the i^{th} fuzzy path length and

$$\begin{aligned}
P_{\max} &= ((\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}), (\alpha_1', \alpha_2', \alpha_3', \alpha_4', \alpha_5', \alpha_6', \alpha_7', \alpha_8', \alpha_9', \alpha_{10}')) \\
&= (P_{\max}, p_{\max})
\end{aligned}$$

be the fuzzy longest length, then the Euclidean ranking of P_i is denoted by

$$ER(P_i) = \left(\frac{\sqrt{(\alpha_1 - \theta_1)^2 + (\alpha_2 - \theta_2)^2 + (\alpha_3 - \theta_3)^2 + (\alpha_4 - \theta_4)^2 + (\alpha_5 - \theta_5)^2 + (\alpha_6 - \theta_6)^2 + (\alpha_7 - \theta_7)^2 + (\alpha_8 - \theta_8)^2 + (\alpha_9 - \theta_9)^2 + (\alpha_{10} - \theta_{10})^2}}{\sqrt{(\alpha_1' - \theta_1')^2 + (\alpha_2' - \theta_2')^2 + (\alpha_3' - \theta_3')^2 + (\alpha_4' - \theta_4')^2 + (\alpha_5' - \theta_5')^2 + (\alpha_6' - \theta_6')^2 + (\alpha_7' - \theta_7')^2 + (\alpha_8' - \theta_8')^2 + (\alpha_9' - \theta_9')^2 + (\alpha_{10}' - \theta_{10}')^2}} \right) =$$

(Euclidean ranking of membership function, Euclidean ranking of non-membership function)

$\tilde{E}_1$ and $\tilde{E}_2$ are two Decagonal Intuitionistic fuzzy numbers. If $\tilde{E}_1 \geq \tilde{E}_2$ then $ER(E_1) \leq ER(E_2)$

Algorithm for Fuzzy Critical path problem using Decagonal Intuitionistic Fuzzy numbers:

Step: 1 Create a network which is not cyclic. The Vertex set is denoted by V and the edge set is denoted by E.

Let

$d_{mn} = ((a_{mn}, b_{mn}, c_{mn}, d_{mn}, e_{mn}, f_{mn}, g_{mn}, h_{mn}, i_{mn}, j_{mn}), (a'_{mn}, b'_{mn}, c'_{mn}, d'_{mn}, e'_{mn}, f'_{mn}, g'_{mn}, h'_{mn}, i'_{mn}, j'_{mn}))$
be the length of edge (m,n). Each length denotes cost, time, etc.,

Step: 2 Calculate all possible paths P_i , $i=1 \dots n$ and the corresponding path lengths L_i , $i=1 \dots n$

$$L_i = ((a_i, b_i, c_i, d_i, e_i, f_i, g_i, h_i, i_i, j_i), (a'_i, b'_i, c'_i, d'_i, e'_i, f'_i, g'_i, h'_i, i'_i, j'_i))$$

Step: 3 find the value of $L_{\max}$.

$$L_{\max} = ((a, b, c, d, e, f, g, h, i, j), (a', b', c', d', e', f', g', h', i', j')) = (L_{\max}, l_{\max})$$

Step: 4 Finally find out the ranking techniques for each possible path lengths L_i , $i=1$ to n .

- i) Choose the path which is having highest α – cut ranking for membership function and lowest α – cut ranking for non-membership function. This path will be the Fuzzy critical path.
- ii) Take the route which is having lowest Euclidean ranking for both membership and non-membership function. This will be the Fuzzy critical path.

Numerical Example:

Step: 1

Construct a fuzzy acyclic network $G(V, E)$ of Type V graph fuzziness where the edge weights are taken as Decagonal Intuitionistic fuzzy numbers.

Step: 2

Calculate all the possible paths and possible lengths.

Table: 1 Possible paths and path lengths

Possible paths	Path Length
1-3-6-8	((80,102,127,154,168,189,207,226,245,269),(77,107,141,164,176,198,217,232,244,265))
1-4-8	((52,73,96,106,111,123,128,132,155,167),(56,76,95,106,111,124,137,154,169,179))
1-2-5-7-8	((112,142,168,208,238,269,288,313,331,350),(115,143,172,208,231,274,278,305,323,341))
1-4-5-7-8	((107,142,184,213,240,270,293,308,336,356),(110,145,181,205,224,261,275,294,321,346))
1-3-4-8	((75,94,119,136,147,164,179,200,229,255),(74,103,132,146,157,171,197,219,235,254))
1-3-4-5-7-8	((130,163,207,243,276,311,344,376,410,444),(128,172,218,245,270,308,335,359,387,421))

Step:3

Calculate $P_{\max}$ and P_{MAX}

$$P_{MAX} = (130,163,207,243,276,311,344,376,410,444)$$

$$P_{\max} = (128,172,218,245,270,308,335,359,387,421)$$

Paths	R(P _i)	ER(P _i)
1-3-6-8	(1029,-723)	(382,339)
1-4-8	(634,-478)	(596,543)
1-2-5-7-8	(1442,-950)	(170,154)
1-4-5-7-8	(1454,-935)	(161,162)
1-3-4-8	(890,-655)	(438,382)
1-3-4-5-7-8	(1710,-1113)	(0,0)

$$\text{Where } R(P_i) = \begin{pmatrix} \frac{1}{6}(-5\theta_1 + 6\theta_2 - 3\theta_3 + 2\theta_4 + 22\theta_7 + 6\theta_8 + 6\theta_9 - 5\theta_{10}), \\ \frac{1}{6}(-4\theta_1' + 8\theta_2' + 13\theta_4' - 5\theta_5' + 8\theta_6' + 5\theta_9' - \theta_{10}') \end{pmatrix}$$

$$ER(P_i) = \begin{pmatrix} \sqrt{\frac{(\alpha_1 - \theta_1)^2 + (\alpha_2 - \theta_2)^2 + (\alpha_3 - \theta_3)^2 + (\alpha_4 - \theta_4)^2 + (\alpha_5 - \theta_5)^2 + (\alpha_6 - \theta_6)^2 + (\alpha_7 - \theta_7)^2 + (\alpha_8 - \theta_8)^2 + (\alpha_9 - \theta_9)^2 + (\alpha_{10} - \theta_{10})^2}{(\alpha_1' - \theta_1')^2 + (\alpha_2' - \theta_2')^2 + (\alpha_3' - \theta_3')^2 + (\alpha_4' - \theta_4')^2 + (\alpha_5' - \theta_5')^2 + (\alpha_6' - \theta_6')^2 + (\alpha_7' - \theta_7')^2 + (\alpha_8' - \theta_8')^2 + (\alpha_9' - \theta_9')^2 + (\alpha_{10}' - \theta_{10}')^2}}, \\ \sqrt{\frac{(\alpha_1' - \theta_1')^2 + (\alpha_2' - \theta_2')^2 + (\alpha_3' - \theta_3')^2 + (\alpha_4' - \theta_4')^2 + (\alpha_5' - \theta_5')^2 + (\alpha_6' - \theta_6')^2 + (\alpha_7' - \theta_7')^2 + (\alpha_8' - \theta_8')^2 + (\alpha_9' - \theta_9')^2 + (\alpha_{10}' - \theta_{10}')^2}{(\alpha_1 - \theta_1)^2 + (\alpha_2 - \theta_2)^2 + (\alpha_3 - \theta_3)^2 + (\alpha_4 - \theta_4)^2 + (\alpha_5 - \theta_5)^2 + (\alpha_6 - \theta_6)^2 + (\alpha_7 - \theta_7)^2 + (\alpha_8 - \theta_8)^2 + (\alpha_9 - \theta_9)^2 + (\alpha_{10} - \theta_{10})^2}} \end{pmatrix}$$

Therefore 1-3-4-5-7-8 is the fuzzy critical path.

Procedure for the fuzzy critical path problem using Decagonal fuzzy numbers

i) Consider an acyclic network $N(V, E)$ where V is the vertex set and E is the edge set.

$e_{ij} = (e_{ij(1)}, e_{ij(2)}, e_{ij(3)}, e_{ij(4)}, e_{ij(5)}, e_{ij(6)}, e_{ij(7)}, e_{ij(8)}, e_{ij(9)}, e_{ij(10)})$ represent the decagonal fuzzy number, where ij denotes edge.

ii) Find out the dynamic programming recursion of the critical path problem in the fuzzy sense is given by

$$f^*(i) = \max(e_{ij}^* + f^*(j)) \quad i, j \in E$$

And $f^*(n) = 0$ where $f^*(i)$ is the length of the longest path in the fuzzy sense from vertex i to vertex n

The defuzzified value of e_{ij} is taken as e_{ij}^* , where

$$e_{ij}^* = \frac{1}{12} \left(-5e_{ij(1)} + 3e_{ij(2)} + 3e_{ij(3)} + 3e_{ij(4)} + \frac{16}{2}(e_{ij(5)} + e_{ij(6)}) + 3e_{ij(7)} + 3e_{ij(8)} + 3e_{ij(9)} + 11e_{ij(10)} \right)$$

Numerical Example:

Step: 1

Consider the previous Numerical Example. Consider the same edge weights. But take the edge weights in terms Decagonal fuzzy numbers. We have considered only the membership function.

Step: 2 Find out the defuzzified edge weights for each edge e_{ij} which is denoted by e_{ij}^*

$$e_{12}^* = 209.42, e_{13}^* = 205.83, e_{14}^* = 215.25, e_{25}^* = 220.83, e_{45}^* = 231.08, e_{34}^* = 198.58, e_{36}^* = 259.75, \\ e_{68}^* = 250.92, e_{48}^* = 244.67, e_{57}^* = 273.42, e_{78}^* = 271$$

$$f^*(8) = 0, f^*(7) = e_{78}^* + f^*(8) = 271 + 0 = 271$$

$$f^*(6) = e_{68}^* + f^*(8) = 250.92 + 0 = 250.92$$

$$f^*(5) = e_{57}^* + f^*(7) = 273.42 + 271 = 544.42$$

$$f^*(4) = \max(e_{48}^* + f^*(8), e_{45}^* + f^*(5)) = \max(244.67 + 0, 231.08 + 544.42) \\ \max(244.67, 775.5) = 775.5$$

$$f^*(3) = \max(e_{34}^* + f^*(4), e_{36}^* + f^*(6)) = \max(198.58 + 775.5, 259.75 + 250.92) \\ \max(974.08, 510.67) = 974.08$$

$$f^*(2) = e_{25}^* + f^*(5) = 220.83 + 544.42 = 765.25$$

$$f^*(1) = \max(e_{13}^* + f^*(3), e_{14}^* + f^*(4), e_{12}^* + f^*(2)) = \max(205.83 + 974.08, 215.25 + 775.5, 209.42 + 765.25) \\ \max(1179.91, 990.75, 974.67) = 1179.91$$

$$f^*(1) = e_{13}^* + f^*(3) = e_{13}^* + e_{34}^* + f^*(4) = e_{13}^* + e_{34}^* + e_{45}^* + f^*(5) = e_{13}^* + e_{34}^* + e_{45}^* + e_{57}^* + f^*(7) \\ = e_{13}^* + e_{34}^* + e_{45}^* + e_{57}^* + e_{78}^* + f^*(8) = e_{13}^* + e_{34}^* + e_{45}^* + e_{57}^* + e_{78}^*$$

Therefore 1-3-4-5-7-8 is the fuzzy critical path.

Conclusion:

The above method is another way to find Fuzzy Critical Path. If the duration between activities are in terms of linguistic variables, it is better to use fuzzy number. In this paper the Decagonal fuzzy number and Intuitionistic Decagonal fuzzy number have been used to change the linguistic variables. Here two methods are developed to find fuzzy critical path. This method is very useful to handle multifaceted projects. If uncertainty arises in 10 different points this Decagonal fuzzy number will be very useful.

References:

[1] K. Atanassov, Intuitionistic Fuzzy sets, Fuzzy sets and systems, 20(1) 1986, 87-96.

- [2] C.T. Chen and S.F. Huang, Applying fuzzy method for measuring criticality in project network.
- [3] S.Elizabeth and L.Sujatha, Critical Path problem under Fuzzy Environment, International Journal of Computer Applications, 75(1) (2013), 7-11.
- [4] S.Elizabeth and L.Sujatha, Project scheduling method using Triangular Intuitionistic fuzzy numbers and Triangular fuzzy numbers, 9(4) (2015), 185 - 198.
- [5] Felix, Christopher, Victor Devadoss, A Nonagonal fuzzy number and its arithmetic operations, International journal of mathematics and its applications, 3(2) 2015, 185-195
- [6] Felix, Victor Devadoss, A New decagonal fuzzy number under uncertain linguistic environment, International journal of mathematics and its applications, 3(1) 2015, 89-97.
- [7] D. Dubois and H. Prade, Operations on Fuzzy Numbers, International Journal of Systems Science, 9(6) (1978), 613-626.
- [8] [19] L.A.Zadeh, Fuzzy Logic and Approximate Reasoning, Syntheses, 30(1975b), 407-428.
- [9] H.J.Zimmermann, Fuzzy Set Theory and its Application, Fourth Edition, Springer, (2011).