Scheduling Surgical Operations and the Post-Anesthesia Care Unit Using Work Tours and Binary Programming

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When surgical patients leave an operating room, they go to the post-anesthesia care unit (PACU). The PACU is shared with two or more operating rooms. It has a limited capacity for holding patients. A problem for the operating room scheduler is to schedule operations such that the PACU is not requested to take on more patients than it can handle. The solution is to develop work tours for operating time/PACU time pairs spread throughout the day and employ a binary integer programming formulation that not only obeys the PACU capacity constraint but also handles other scheduling constraints.

Keywords: Operating Room Scheduling, PACU, Work Tours, SAS , Binary Programming

1. Introduction

When surgical patients leave an operating room, they go to the post-anesthesia care unit (PACU) to recover. The PACU is shared by multiple operating rooms (OR) and receives patients with varying levels of acuity from multiple specialties. The volume of patients coming from the OR to the PACU can vary greatly depending upon the complexity and duration of the surgical procedures. In addition, the amount of time required in the PACU can also vary greatly depending upon the level of sedation and response of the individual patients. A problem for the operating room scheduler is to schedule operations such that the PACU is not requested to take more patients than it can handle while accommodating requests from the surgeons and the patients.

Each hospital is different and there is no generic scheduling system to assist the scheduler. A Midwest hospital sought a better solution than the spreadsheet approach they were using and approached the hospital's data analysis staff to be included with a team of surgeons, OR nurses, PACU nurses, and schedulers to address the scheduling problem.

A common approach to OR scheduling is to maximize the utilization of both the operating rooms and the surgeons without regard to the effect that this maximization might have on the downstream PACU or the patients being admitted to an inpatient unit in the hospital. A regular request from surgeons is to schedule their complex cases early in the morning and the less complex (usually shorter) cases later in the day. The result is that patients being held in the PACU because beds are not available on the receiving units and demand for PACU resources varies greatly across the day. Data analysis reveals that the primary cause is two-fold. The more complex cases are likely to require a post-surgical bed and patients are to be "held" in the PACU until a bed is available after other patients are discharged. Early surgical patients might be ready for admission before morning rounds are completed and resulting discharges can occur. The less complex cases are often shorter in duration, and with several surgeons performing cases at a high volume, the PACU cannot release patients quickly enough since the surgeries are often shorter than the recovery time.

The analytics team determined that these problems could be addressed simultaneously through modeling and worked with the team to develop a solution. This paper discusses the solution developed by that team to address these issues and provide an optimal system solution.

2. Literature Review

An excellent literature review of operating room scheduling through 2010 is provided by Cardoen *et al*. [1]. A subsequent publication, Cardoen *et al*. [2], presents a classification scheme for research papers in the area of operating room scheduling similar to the classification schemes for machine scheduling and project scheduling. Using the proposed classification system, this work can be classified as *el|other*; *cap*; *int (PACU)| det|single*; *util*.

- *el* means this article addresses elective surgeries rather than emergency surgeries.
- *other* denotes there is a different object other than cost minimization; this objective is to use the operating rooms and PACU to best advantage.
- *cap* signifies to consider capacity of the operating rooms.
- *int(PACU)* indicates the model is to link downstream to the PACU.
- *det* means the problem being addressed is deterministic rather than stochastic.
- *single* indicates that the model has a single objective.
- *util* communicates that the objective is to maximize utility as defined by the model.

Romanyuk and Silva [3] present an optimal OR scheduling approach that uses Excel's Premier Solver as the solver. They do not discuss linking to the PACU. Marcon *et al*. [4] use simulation to decide the number of beds to place in the PACU. Hsu *et al*. [5] discuss scheduling patients in an ambulatory surgical center with the objective to minimize the number of PACU nurses.

Other works that include scheduling the PACU in addition to the ORs since the 2010 literature review are Berg and Denton [6], Beaulieau *et al*. [7], Lee [8], and Rinehart *et al*. [9].

3. Model Development

Analysis of historic data and review of the current scheduling policies revealed that the hospital had an opportunity to optimally schedule most of its cases for the day of surgery. Aside from urgent or emergent cases which were typically handled in dedicated OR’s, surgeons would identify a surgery date for patients needing surgery and notify them 48 hours before the procedure of their scheduled time slot/start time. In addition, the surgical scheduling practices incorporated the use of historic data for scheduling the duration of surgical procedures that were identified by surgeon and by procedure. These two factors together revealed to the analytic team that there was an opportunity to optimize the schedule before it was finalized.

One of the challenges in healthcare is the variability that is found in the system and that is found in the OR. The team quickly recognized that developing stochastic models to optimize the solution would not only be infeasible from a resource perspective (as it would have to be completed daily) but would quickly result in an np-hard model as the scope had to be able to accommodate up to 20 operating rooms and hundreds of procedures. The approach was to aggregate historic data to create estimated values for OR procedures and PACU time that could be utilized in a linear optimization model.

The objective of the model was two-fold and addressed the two areas that were found to be a problem – delayed admissions to inpatient beds and fluctuating demand for PACU resources. Specific requirements were to delay surgical procedures with patients planned for an admission to a surgical unit until later in the day (specifically so that they would not arrive in the PACU before 11:00 AM) and to smooth the demand for PACU resources. The model had to accommodate some procedures that had a pre-assigned time slot because they involved multiple surgical specialties and could not be changed, and to maximize the utilization of the operating rooms. Because the scope was large, the initial focus and work surrounded a single hospital specialty with a plan to spread to other areas after the benefits were realized with a single group.

To develop the model, we utilized work tours to determine how surgeries would be scheduled and optimized the schedule to meet the objectives. Initial scale and proof of concept was scoped with a single surgical division with the intent of scaling the model to assess all operating rooms in the hospital.

3.1 Work Tours

We define a work tour as a vector of zeroes and ones against a discrete time domain where a zero represents a time when work is not to be performed and a one represents a time when work is to be performed. Table 1 shows an example work tour with five-minute increments, and where work is only to be performed from 0740 through 0755.

Table 1 *An Abbreviated Example Work Tour*

Time	At Work?
0730	0
0735	0
0740	1
0745	1
0750	1
0755	0
0800	0
0805	0
0810	0
…	

Since a patient leaves an operating room and goes directly to the PACU we constructed OR/PACU combination tours as in Table 2

Table 2 *Abridged Work Tours for an OR/PACU Combination*

Operating Room	OR1							…
Operation ID		O01		O01		O01		
Tour		Tour 1		Tour 2		Tour 3		
	Start Time	OR	PACU	OR	PACU	OR	PACU	
Time Slot 1	0730	1	0	0	0	0	0	
Time Slot 2	0735	1	0	1	0	0	0	
Time Slot 3	0740	1	0	1	0	1	0	
Time Slot 4	0745	1	0	1	0	1	0	
Time Slot 5	0750	0	1	1	0	1	0	
Time Slot 6	0755	0	1	0	1	1	0	
Time Slot 7	0800	0	1	0	1	0	1	
Time Slot 8	0805	0	0	0	1	0	1	
Time Slot 9	0810	0	0	0	0	0	1	
…								

In Table 2, operation O01 begins at 0730 for Tour 1, leaves the OR at 0750 going directly to the PACU, and then leaving the PACU at 0805. Tour 2 is like Tour 1 except it is shifted forward by one five-minute increment. Tour 3 is shifted five minutes forward from Tour 2. Tours are until 1730 which is ten hours past 0730 or 120 five-minute increments.

We constructed three OR/PACU combinations

- Normal OR/PACU tours are at five-minute intervals beginning at 0730, the beginning of the day.
- Early morning admission OR/PACU tours where surgeries cannot begin before 1030. These tours are similar to normal tours except all time slots before 1030 are filled with zeroes.
- Preferred starting time OR/PACU tours which are similar to early morning admissions in that all time slots before the preferred start time are filled with zeroes. Preferred times are given an inflated weight in the objective function to encourage the solver to select a preferred starting time operation over other operations.

In Table 3, operation O02 has a preferred start time of 0930 and operation O03 is an early morning admit and therefore cannot begin surgery before 1030.

Table 3 *Example Manifest of Three Operations*

Operation ID	Operating Room Time (Minutes)	PACU Time (Minutes)	Early Morning Admit?	Assigned Operating Room	Preferred Start Time
O01	20	15	0	OR1	
O02	10	10	0	OR2	0930
O03	15	10	1	OR1	

4. Mathematical Model

4.1 Constraints

There are three constraints

1. A surgery can only be performed once. (See (1) in section 4.4 below.)

2. Only one surgery can occupy an operating room at a time. (See (2) in section 4.4 below.)
3. Only three patients can occupy the PACU at a time. (See (3) in section 4.4 below.)

4.2 Objective Function Weight

In order to encourage the solver to select tours that begin early in the day and to discourage gaps in the use of the operating rooms, an exponential weight is applied in the objective function. The weight is

$$w_{sl} = \frac{1}{\exp\left(\frac{l}{15}\right)}$$

Where s is the operating room and l is the five-minute time increment of the day. The divisor of 15 was selected since it creates a not-too-steep exponential shape. See Figure 1

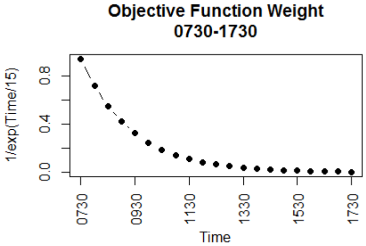

Figure 1 *High Weight Values Early in the Day Encourage the Solver to Load Operations as early as possible*

An exponential weight, rather than a quadratic weight or linear weight, was selected since it places disproportionately more weight on operations occurring early in the day when scheduling is most critical. This results in a shortest-operating-time-next schedule, while obeying the constraints, which is appealing to hospital management since it minimizes the staff resources for late-in-the-day procedures.

4.3 Sensitivity of the Solution to the Divisor in the Objective Function Weight

Divisors of 5, 10, 15, 25, and 150 were tried in the objective function weight. The run with 5 as the divisor did not schedule all operations. This may be due to the asymptotic values being too close to zero early in the day. All other divisors led to successful solutions.

4.4 The Model

$$max\ Z = \sum_{s,o,t,l} w_{sl} p_o\, r_{sotl} x_{sot}$$

subject to

$$\sum_{s,t} x_{sot} \leq 1\ \forall\ o \qquad (1)$$

$$\sum_{o,t} r_{sotl} x_{sot} \leq 1 \; \forall \; s, l \qquad (2)$$

$$\sum_{s,o,t} c_{sotl} x_{sot} \leq 3 \; \forall \; l \qquad (3)$$

where
s identifies the surgical operating room
o is the operation to be performed
t is the tour
l is the time slot
p is the preference weight, 1.0 for normal operations, 1.1 for operations with a preferred starting time
r is the 0 or 1 value from the operating room tours
c is the 0 or 1 value from the PACU tours and
x_{sot} *is* 1 *if operation o is to be performed in operating room s during tour t;* 0 *otherwise.*

5. Computer Solution

The hospital that cooperated in developing this model uses SAS. Thus, the daily tours were developed in base SAS and the model was expressed in SAS's OPTMODEL modeling language. SAS's mixed integer linear programming solver was used to find solutions.

The SAS computer code along with the input for the real-sized problem that appears below in Table 4 can be obtained from the shared Google document https://drive.google.com/folderview?id=0B1F5496LkPI9dE1Zd01mNElvN28&usp=sharing

6. A Real-Sized Problem

Table 4 shows a real-sized problem with 16 operations to be scheduled for two operating rooms. Early morning admissions cannot begin surgery before 1030. Preferred start times can be specified.

Table 4 *A Real-Sized Problem*

Operation ID	Operating Room Time	PACU Time	Early Morning Admit?	Assigned Operating Room	Preferred Start Time
O01	40	45	0	OR1	0910
O02	20	45	0	OR1	
O03	40	45	1	OR1	
O04	45	60	0	OR1	
O05	15	45	0	OR2	1410
O06	30	45	0	OR2	
O07	30	60	0	OR1	
O08	40	75	0	OR2	
O09	15	60	1	OR2	
O10	90	45	0	OR2	1500
O11	115	45	0	OR1	1050
O12	50	60	0	OR2	
O13	70	75	1	OR1	
O14	60	30	0	OR2	
O15	130	60	1	OR2	1115
O16	15	15	0	OR2	

7. Solution to the Real-Sized Problem

Figure 2 shows the solution presented in the problem of Table 4.

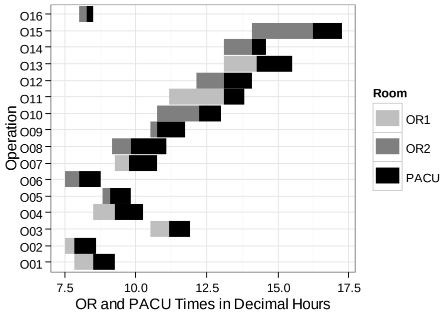

Figure 2 *Solution to Problem of Table 4*

8. Scalability

The complexity of the problem is roughly the number of operating rooms times the number of tours times the number of time periods in a day. Since there is one tour for each time period, the complexity is O ((number of tours)^{2}).

However, the number of tours in, for instance, a ten-hour day, does not change. It is the number of operating rooms that drives the amount of memory needed to run the algorithm.

A log-log plot of the number of binary variables generated as a function of the number of operating rooms is shown in Figure 3 suggesting that growth is exponential.

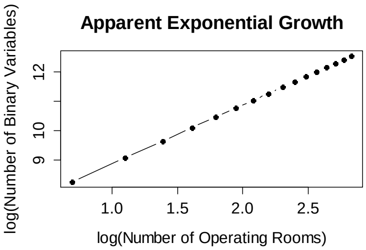

Figure 3 *Apparent Exponential Growth*

Seventeen was the largest number of operating rooms we examined sharing a single PACU. This problem solved successfully. If a very large hospital has, say, 45 operating rooms and a 30-bed PACU, the problem could be split into three 15 operating room problems each with 10 beds in a PACU. This will yield a feasible albeit suboptimal solution. Alternatively, hospital management could purchase software as a service (SaaS) on larger servers as part of a cloud computing solution.

9. Conclusion

Contemporary solvers and modern modeling languages no longer place mathematical programming solutions out of the reach of hospital administrators. This solution was tested at a Midwest hospital and found to produce results that could benefit surgical schedules and PACU usage. The system requirements to run the model daily proved to be a stumbling block for robustly integrating this solution into the daily operations of the hospital. System priorities were shifted and the analytic resources had to be redirected before the program was able be used daily. However, the learning gleaned from the development and testing of the model provided insights for the schedulers that have been considered as they schedule thousands of cases throughout the year. The proof of concept was well received, and demonstrates a step forward in integrating mathematical decision making into healthcare.

10. Acknowledgement

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11. References

1 B. Cardoen, E. Demeulemeester, and J. Belien (2010) Operating room planning and scheduling: A literature review. European Journal of Operational Research, 201(3), 921-932.

2 B. Cardoen, E. Demeulemeester, and J. Belien (2010) Operating room planning and scheduling problems: A classification scheme. International Journal of Health Management and Information, 1(1): 71-83.

3 A. Romanyuk, and A. Silva (2012) Optimization of an operating room surgical schedule, report, Washington University in St. Louis, Department of Electrical & Systems Engineering.

4 E. Marcon, S. Kharraja, N. Smolski, B. Luquet, JP. and Viale (2003) Dtermining the number of beds in the postanesthesia care unit: a computer simulation flow approach. Anesthesia & Analgesia, 96(5): 1415-23.

5 V. N.Hsu,R. de Matta, and C.-Y. Lee (2003) Scheduling patients in an ambulatory surgical center. Naval Research Logistics, 50:218-238. doi: 10.1002/nav.10060.

6 B. Berg, and B. T. Denton (2012). Appointment planning and scheduling in outpatient procedure centers. In Handbook of Healthcare System Scheduling (pp. 131-154). Springer US.

7 I. Beaulieu, M. Gendreau, and P. Soriano (2012). Operating rooms scheduling under uncertainty. In Advanced Decision Making Methods Applied to Health Care (pp. 13-32). Springer Milan.

8 S. Lee (2012). Appointment scheduling strategies in primary care clinics and surgical operating units. Purdue University dissertation.

9 D. Rinehart, C. Kleiner, M. Ballou, B. Loucks-Schultz, and A. P. S. Council (2013). An inpatient PACU scheduling approach for adequate staffing. Journal of Peri Anesthesia Nursing, *28*(3), e8-e9.

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