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BINARY SOFT PRE-SEPARATION AXIOMS IN BINARY SOFT TOPOLOGICAL SPACES

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ABSTRACT

In this article, we introduce binary soft pre-separation axioms in binary soft Topological space along with several properties of binary soft pre τ_{Δ_i} , $i = 0; 1; 2$, binary soft pre-regular, binary soft pre τ_{Δ_3} , binary soft pre-normal and binary soft τ_{Δ_4} axioms using binary soft points. We also mention some binary soft invariance properties namely binary soft topological property and binary soft hereditary property. We hope that these results will be useful for the future study on binary soft topology to carry out general background for the practical applications and to solve the thorny problems containing doubts in different grounds.

KEYWORDS

Binary soft topology, Binary soft pre-open and binary soft pre-closed sets Binary soft pre-separation axioms.

1. INTRODUCTION

Based on a study, the concept of soft sets was first introduced in 1999 as a general mathematical technique for dealing with uncertain substances [1]. A researcher magnificently applied the soft theory in numerous ways, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on [2]. Point soft set topology deals with a non-empty set X together with a collection τ of sub set X under some set of parameters satisfying certain conditions. Such a collection τ is called a soft topological structure on X . General soft topology studied the characteristics of sub set of X by using the members of τ . Therefore, the study of point soft topology can be thought of the study of information. But in the real-world situation there may be two or more universal sets. Our attempt is to introduce a single structure which carries the sub sets of X and Y for studying the information about ordered pair of sub sets of X and Y . Such a structure is called a binary soft structure from X to Y .

In 2016, a group of researchers introduced the notion of binary soft set Theory on two master sets and studied some basic characteristics [3]. In prolongation, another researcher planned the idea of binary soft topology and linked fundamental properties which are defined over two master sets with appropriate parameters [4]. In continuation, in the present paper we have defined and explored several properties of binary soft pre τ_{Δ_i} , $i = 0; 1; 2$ binary soft pre regular, binary soft pre τ_{Δ_3} , binary soft pre normal and binary soft pre τ_{Δ_4} axioms using binary soft points. Also, we have talked over some binary soft invariance properties i.e. binary soft topological property and binary soft hereditary property in binary soft topological spaces.

The arrangement of this paper is as follows: Section 2 briefly reviews some basic concepts about soft sets, binary soft sets and their related properties; Section 3 some hereditary properties are discussed in a beautiful way.

Section 4 is devoted to Binary Soft Pre-Separation Axioms. Section 5 is devoted to Binary Soft Pre-Regular, Binary Soft pre-Normal and Binary pre-Soft τ_{Δ_i} ($i=4, 3$) Spaces

2. PRELIMINARIES

Definition 1: Let X be an initial universe and let E be a set of parameters [5]. Let $P(X)$ denote the power set of X and let A be a nonempty subset of E . A pair (F, A) is called a soft set over X , where F is a mapping given by: $A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $\epsilon \in A$, $F(\epsilon)$ may be considered as the set of ϵ -approximate elements of the soft set (F, A) . Clearly, a soft set is not a set.

Let U_1, U_2 be two initial universe sets and E be a set of parameters.

Let $P(U_1), P(U_2)$ denote the power set of U_1, U_2 respectively. Also, let $A, B, C \subseteq E$.

Definition 2: A pair (F, A) is said to be a binary soft set over U_1, U_2 where F is defined as below:

$$F: A \rightarrow P(U_1) \times P(U_2), F(e) = (X, Y) \text{ for each } e \in A \text{ such that } X \subseteq U_1, Y \subseteq U_2$$

Definition 2.1: A binary soft set (G, A) over U_1, U_2 is called a binary absolute soft set, denoted by $\tilde{A}$ if $F(e) = U_1, U_2$ for each $e \in A$.

Definition 3: The intersection of two binary soft sets of (F, A) and (G, B) over the common U_1, U_2 is the binary soft set (H, C) , where $C = A \cap B$ and for all $e \in C$

$$H(e) = \begin{cases} (X_1, Y_1) & \text{if } e \in A - B \\ (X_2, Y_2) & \text{if } e \in B - A \\ (X_1 \cup X_2, Y_1 \cup Y_2) & \text{if } e \in A \cap B \end{cases}$$

Such that $F(e) = (X_1, Y_1)$ for each $e \in A$ and $G(e) = (X_2, Y_2)$ for each $e \in B$. We denote it $(F, A) \tilde{\cap} (G, B) = (H, C)$

Definition 5: The intersection of two binary soft sets (F, A) and (G, B) over a common U_1, U_2 is the binary soft set (H, C) , where $C = A \cap B$, and

$H(e) = (X_1 \cap X_2, Y_1 \cap Y_2)$ for each $e \in C$ such that $F(e) = (X_1, Y_1)$ for each $e \in A$ and $G(e) = (X_2, Y_2)$ for each $e \in B$. We denote it as $(F, A) \tilde{\cap} (G, B) = (H, C)$

Definition 6: Let (F, A) and (G, B) be two binary soft sets over a common U_1, U_2 . (F, A) is called a binary soft subset of (G, B) if

(i) $A \subseteq B$,

(ii) $X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$ Such that $F(e) = (X_1, Y_1)$, $G(e) = (X_2, Y_2)$ for each $e \in A$. We denote it as $(F, A) \tilde{\subseteq} (G, B)$.

Definition 7: A binary soft set (F, A) over U_1, U_2 is called a binary null soft set, denoted by $F(e) = (\emptyset, \emptyset)$ for each $e \in A$.

Definition 8: The difference of two binary soft sets (F, A) and (G, B) over the

Common U_1, U_2 is the binary soft set (H, A) , where $H(e) = (X_1 - X_2, Y_1 - Y_2)$ for each $e \in A$ such that $F(e) = (X_1, Y_1)$ and $G(e) = (X_2, Y_2)$.

Definition 9: Let τ_Δ be the collection of binary soft sets over U_1, U_2 then τ_Δ is said to be a binary soft topology on U_1, U_2 if

(i) $\tilde{\emptyset}, \tilde{X} \in \tau_\Delta$

(ii) The union of any member of binary soft sets in τ_Δ belongs to τ_Δ .

(iii) The intersection of any two binary soft sets in τ_Δ belongs to τ_Δ .

Then $(U_1, U_2, \tau_\Delta, E)$ is called a binary soft topological space over U_1, U_2 .

Definition 10: Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on $\tilde{X}$ over $U_1 \times U_2$ and $\tilde{Y}$ be non-empty binary soft subset of $\tilde{X}$.

Then $\tau_{\Delta_Y} = \{^Y(F, E)/(F, E) \in \Delta\}$ is said to be the binary soft relative topology on $\tilde{Y}$ and $(\tilde{Y}, \tau_{\Delta_Y}, E)$ is called a binary soft subspace of $(U_1, U_2, \tau_\Delta, E)$. We can easily verify that τ_{Δ_Y} is a binary soft topology on $\tilde{Y}$.

Example 1: Any binary soft subspace of a binary soft indiscrete topological space is binary soft indiscrete topological space.

3. SOME HEREDITARY PROPERTIES

Definition 11: Let (F, A) be any binary soft sub set of a binary soft topological space $(U_1, U_2, \tau_\Delta, E)$ then (F, A) is called

1) (F, A) binary soft pre-open set of $(U_1, U_2, \tau_\Delta, E)$ if $(F, A) \subseteq \text{int}(\text{cl}((F, A)))$ and

2) (F, A) binary soft pre-closed set of $(U_1, U_2, \tau_\Delta, E)$ if $(F, A) \supseteq \text{cl}(\text{int}(F, A))$

The set of all binary pre-open soft sets is denoted by $\text{bsp}_o(U)$

Proposition 1. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on $\tilde{X}$ over $U_1 \times U_2$ and $\tilde{Y}$ be a non-empty binary soft subset of $\tilde{X}$. Then $(U_1, U_2, \tau_{\Delta_Y}, \alpha)$ is subspace of $(U_1, U_2, \tau_\Delta, E)$ for each $\alpha \in \tilde{E}$.

Proof. Let $(U_1, U_2, \tau_{\Delta_Y}, \alpha)$ is a binary soft topological space for each $\alpha \in E$. Now by definition for any $\alpha \in E$

$$\tau_{\Delta_Y} = \{^Y F(\alpha)/(F, E) \text{ is binary soft pre-open set}\}$$

$$= \{\tilde{Y} \cap F(\alpha)/(F, E) \text{ is binary soft pre-open set}\}$$

$$= \{\tilde{Y} \cap F(\alpha)/(F, E) \text{ is binary soft pre-open set}\}$$

$$= \{\tilde{Y} \cap F(\alpha)/F(\alpha) \in \tau_{\Delta_\alpha}\}$$

Thus $(U_1, U_2, \tau_{\Delta_Y}, \alpha)$ is a subspace of $(U_1, U_2, \tau_\Delta, \alpha)$.

Proposition 2. Let $(U_1, U_2, \tau_{\Delta_Y}, E)$ be a binary soft subspace of a binary soft topological space $(U_1, U_2, \tau_\Delta, E)$ and (G, E) be a binary soft pre-open in $\tilde{Y}$. If $\tilde{Y} \in \tau_\Delta$

Then $(G, E) \in \tau_\Delta$.

Proof. Let (G, E) be a binary soft pre-open set in $\tilde{Y}$, then there exists a binary soft pre-open set (H, E) in $\tilde{X}$ over $U_1 \times U_2$ such that $(G, E) = \tilde{Y} \cap (H, E)$. Now, if $\tilde{Y} \in \tau_\Delta$

Then $\tilde{Y} \cap (H, E) \in \tau_\Delta$ by the third axiom of the definition of binary soft topological space and hence $(G, E) \in \tau_\Delta$.

Proposition 3. Let $(U_1, U_2, \tau_{\Delta_Y}, E)$ be a binary soft subspace of a binary soft

Topological space $(U_1, U_2, \tau_\Delta, E)$ and (G, E) be a binary soft pre-open set of $\tilde{X}$ over $U_1 \times U_2$, then

(i) (G, E) is binary soft pre-open in $\tilde{Y}$ if and only if $(G, E) = \tilde{Y} \cap (H, E)$ for some $(H, E) \in \tau_\Delta$.

(ii) (G, E) is binary soft pre-closed in $\tilde{Y}$ if and only if $(G, E) = \tilde{Y} \cap (H, E)$ for some binary soft pre-closed set in $(H, E) \in \tilde{X}$ over $U_1 \times U_2$.

Proof. (i) Follows from the definition of binary soft subspace.

(ii) If (G, E) is binary soft pre-closed in $\tilde{Y}$ then we have $(G, E) = \tilde{Y} \cap (H, E)$ then we

have $(G, E) = \widetilde{Y} - (H, E)$, for some binary soft pre-open $(H, E) \in \tau_{\Delta_Y}$, now $(H, E) = \widetilde{Y} \widetilde{\cap} (H, E)$

For some binary soft pre-open $(K, E) \in \tau_{\Delta}$ for any $\beta \in E$, $G(\beta) = \widetilde{Y}(\beta) - H(\beta)$

$$= \widetilde{Y} - H(\beta)$$

$$= \widetilde{Y} - [\widetilde{Y}(\beta) \cap K(\beta)]$$

$$= \widetilde{Y} - [\widetilde{Y} \cap K(\beta)]$$

$$= \widetilde{Y} - K(\beta)$$

$$= \widetilde{Y} \widetilde{\cap} (\widetilde{X} - K(\beta))$$

$$= \widetilde{Y} \widetilde{\cap} [K(\beta)]$$

$$= \widetilde{Y}(\beta) \widetilde{\cap} [K(\beta)]^c$$

Thus $(G, E) = \widetilde{Y}(\beta) \widetilde{\cap} [K(\beta)]^c$ Where $(K, E)^c$ is binary soft pre-closed set in $\widetilde{X}$ over $U_1 \times U_2$ as $(K, E) \in \tau_{\Delta}$.

Conversely, assume that $(G, E) = \widetilde{Y} \cap (H, E)$ for some binary soft pre-closed set (H, E) in $\widetilde{X}$ over $U_1 \times U_2$ which means that $(H, E) \in \tau_{\Delta}$. Now if $(H, E) = \widetilde{X} - (K, E)$ where $(K, E) \in \tau_{\Delta}$ then for any $\beta \in E$,

$$G(\beta) = \widetilde{Y}(\beta) \widetilde{\cap} H(\beta)$$

$$= \widetilde{Y} \widetilde{\cap} H(\beta)$$

$$= \widetilde{Y} \widetilde{\cap} [\widetilde{X} - K(\beta)]$$

$$= \widetilde{Y} - [\widetilde{Y} \widetilde{\cap} K(\beta)]$$

$$= \widetilde{Y}(\beta) - [\widetilde{Y}(\beta) \widetilde{\cap} K(\beta)]$$

Thus $= \widetilde{Y} - [\widetilde{Y} \widetilde{\cap} (K, E)]$. Since $(K, E) \in \tau_{\Delta}$, so $[\widetilde{Y} \widetilde{\cap} (K, E)] \in \tau_{\Delta_Y}$ and hence (G, E) is binary soft closed pre-open set in $\widetilde{Y}$. This finishes the proof.

Let $(U_1, U_2, \tau_{\Delta}, E)$ be a binary soft topological space. Let $(U_1, U_2, \tau_{\Delta_Y}, E)$

be a binary soft subspace of $(U_1, U_2, \tau_{\Delta}, E)$. Let $(F, E) \subseteq \widetilde{Y}$ be a binary soft subset of $\widetilde{Y}$. Then we can find the binary soft pre-closure of (F, E) in the space $(U_1, U_2, \tau_{\Delta_Y}, E)$. The binary soft pre-closure of (F, E) in $(U_1, U_2, \tau_{\Delta_Y}, E)$

is denoted by $\overline{(F, E)}^y$.

Topological space $(U_1, U_2, \tau_{\Delta}, E)$. Let $(F, E) \subseteq \widetilde{Y}$ be a binary soft subset of $\widetilde{Y}$. Then we have the following results as follows.

$$(i) \overline{(F, E)}^y = \widetilde{Y} \widetilde{\cap} \overline{(F, E)}.$$

$$(ii) (F, E)^{y*} = \widetilde{Y} \widetilde{\cap} (F, E)^*$$

$$(iii) \underline{\underline{(F, E)}}_y \subseteq \widetilde{Y} \widetilde{\cap} \underline{\underline{(F, E)}}$$

Proof. (i) To prove, let $\overline{(F, E)}^y = \widetilde{Y} \widetilde{\cap} \overline{(F, E)}$. we have $\overline{(F, E)}^y$ = the binary soft intersection of all the binary soft Pre-closed sets containing $(F, E) = \widetilde{\cap}\{(G, E)_y : (G, E)_y \text{ is } \tau_{\Delta_Y}\text{-binary soft pre-closed set and } (G, E)_y \widetilde{\supset} (F, E)\} = \widetilde{\cap}\{\widetilde{Y} \widetilde{\cap} (G, E) : (G, E) \text{ is -binary soft pre-closed set and } \widetilde{Y} \widetilde{\cap} (G, E) \widetilde{\supset} (F, E)\} = \widetilde{\cap}\{\widetilde{Y} \widetilde{\cap} (G, E) : (G, E) \text{ is } \tau_{\Delta_Y}\text{-binary soft pre-closed set and } (G, E) \widetilde{\supset} (F, E)\}$

$$= \widetilde{Y} \widetilde{\cap} \{(G, E) : (G, E) \text{ is } \tau_{\Delta}\text{-binary soft pre-closed set and } (G, E) \widetilde{\supset} (F, E)\}$$

$$= \widetilde{Y} \widetilde{\cap} \overline{(F, E)}. \text{ Thus } \overline{(F, E)}^y = \widetilde{Y} \widetilde{\cap} \overline{(F, E)}.$$

(ii) To prove that $(F, E)^y = \widetilde{Y} \widetilde{\cap} (F, E)^*$, we know that, $(F, E)^{ey}$ = The binary soft union of all the τ_{Δ_Y} -binary soft pre-open sets contained in (F, E) .

$$= \widetilde{\cup} \{(H, E) : (H, E) \text{ is } \tau_{\Delta_Y}\text{-binary soft pre-open and } (H, E) \widetilde{\supset} (F, E)\}$$

$$= \widetilde{\cup} \{(H, E) = \widetilde{Y} \widetilde{\cap} (K, E) : (K, E) \text{ is } \tau_{\Delta}\text{-binary soft pre-open set and } \widetilde{Y} \widetilde{\cap} (K, E) \widetilde{\supset} (F, E)\}.$$

Also we know that $(F, E)^e = \widetilde{Y} \widetilde{\cap} [\widetilde{\cup}(L, E)_\gamma] : (L, E) \text{ is } \tau_{\Delta}\text{-binary soft pre-open set and } (L, E)_\gamma \widetilde{\subseteq} (F, E)$

Now let $(M, E) \subseteq \widetilde{Y} \widetilde{\cap} (F, E)^*$ which implies $(M, E) \subseteq \widetilde{Y}$ and $(M, E) \subseteq (F, E)^*$

$(M, E) \subseteq \widetilde{Y}$ and $(M, E) \subseteq \widetilde{\cup}(L, E)_\gamma : (L, E)_\gamma \text{ is } \tau_{\Delta}\text{-binary soft pre-open set and}$

$$(L, E)_\gamma \widetilde{\subseteq} (F, E)$$

$$\Rightarrow (M, E) \subseteq \widetilde{Y} \text{ and } (M, E) \subseteq (L, E)_\gamma, \text{ where } (L, E)_{\gamma_1} \text{ is } \tau_{\Delta}\text{-pre-open and } (L, E)_{\gamma_1} \widetilde{\subseteq} (F, E)$$

$$\Rightarrow (M, E) \subseteq \widetilde{Y} \widetilde{\cap} (L, E)_{\gamma_1} \text{ Where } (L, E)_{\gamma_1} \text{ is } \tau_{\Delta}\text{-pre-open and } (L, E)_{\gamma_1} \widetilde{\subseteq} (F, E) \text{ that is } \widetilde{Y} \widetilde{\cap} (L, E)_{\gamma_1} \widetilde{\subseteq} \widetilde{Y} \widetilde{\cap} (F, E)$$

$$\Rightarrow (M, E) \subseteq \widetilde{\cup} \{\widetilde{Y} \widetilde{\cap} (K, E) : (K, E) \text{ is } \tau_{\Delta}\text{-pre-open } \widetilde{Y} \widetilde{\cap} (K, E) \widetilde{\subseteq} (F, E)\}$$

$$\Rightarrow (M, E) \subseteq (F, E)^{y*}. \quad \text{Thus} \quad (M, E) \subseteq \widetilde{Y} \widetilde{\cap} (F, E)^* \text{ which implies } (M, E) \subseteq (F, E)^*.$$

Therefore $\widetilde{Y} \widetilde{\cap} (F, E)^* \subseteq (F, E)^*$.

Proposition 4. Let $(U_1, U_2, \tau_{\Delta_Y}, E)$ be a binary soft subspace of binary soft

(iii) To prove, $\underline{(F, E)}_y \tilde{\subseteq} \tilde{Y} \tilde{\cap} \underline{(F, E)}$

Now consider $\underline{(F, E)}_y = \overline{\underline{(F, E)}}^y \tilde{\cap} \overline{\underline{(F, E)}}^y$

$$\Rightarrow [\tilde{Y} \tilde{\cap} \underline{(F, E)}] \tilde{\cap} \tilde{Y} \tilde{\cap} [\underline{(F, E)}]$$

Since using the result (i) $[\tilde{Y} \tilde{\cap} \underline{(F, E)}] \tilde{\cap} \tilde{Y} \tilde{\cap} [\underline{(F, E)}] \tilde{\subseteq} \tilde{Y} \tilde{\cap} \underline{(F, E)}$ (since $\tilde{Y} \tilde{\subseteq} \tilde{X}$).

$$= \tilde{Y} \tilde{\cap} \underline{(F, E)}$$

Thus $\underline{(F, E)}_y \tilde{\subseteq} \tilde{Y} \tilde{\cap} \underline{(F, E)}$

This finishes the proof.

4. BINARY SOFT PRE-SEPARATION AXIOMS

In this section binary soft pre-separation axioms in Binary Soft Topological Spaces are discussed.

Definition 12: Let $(U_1, U_2, \tau_\Delta, A)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\subseteq} \tilde{X}_A$ such that $F_e \not\tilde{\subseteq} G_e$. Then the binary soft topological space is said to be a binary soft pre- τ_0 space denoted as pre- T_{Δ_0} . If there exists at least one binary soft pre-open set (F_1, A) or (F_2, A) such that $F_e \tilde{\subseteq} (F_1, A)$, $G_e \tilde{\subseteq} (F_1, A)$ or $F_e \tilde{\subseteq} (F_2, A)$, $G_e \tilde{\subseteq} (F_2, A)$.

Definition 13: Let $(U_1, U_2, \tau_\Delta, A)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\subseteq} \tilde{X}_A$ such that $F_e \not\tilde{\subseteq} G_e$. Then the binary soft topological space is said to be a binary soft pre- τ_1 space denoted as pre- T_{Δ_1} . If there exists at least one binary soft pre-open set (F_1, A) or (F_2, A) such that $F_e \tilde{\subseteq} (F_1, A)$, $G_e \tilde{\subseteq} (F_1, A)$ or $F_e \tilde{\subseteq} (F_2, A)$, $G_e \tilde{\subseteq} (F_2, A)$.

Definition 14: Let $(U_1, U_2, \tau_\Delta, A)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\subseteq} \tilde{X}_A$ such that $F_e \not\tilde{\subseteq} G_e$. Then the binary soft topological space is said to be a binary soft pre- τ_2 space denoted as pre- T_{Δ_2} . If there exists at least one binary soft pre-open set (F_1, A) or (F_2, A) such that $F_e \tilde{\subseteq} (F_1, A)$, $H_e \tilde{\subseteq} (F_2, A)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}_A$.

Proposition 5. (i) Every pre- T_{Δ_1} -space is pre- T_{Δ_0} space.

(ii) Every T_{Δ_2} -space is a T_{Δ_1} -space.

Proof. (i) is obvious. (ii) If $(U_1, U_2, \tau_\Delta, A)$ is a T_{Δ_2} -space then by definition, for $F_e, G_e \tilde{\subseteq} \tilde{X}_A$, $F_e \not\tilde{\subseteq} G_e$ there exists at least one binary soft pre-open set (F_1, A) and (F_2, A) such that $F_e \tilde{\subseteq} (F_1, A)$, $H_e \tilde{\subseteq} (F_2, A)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}_A$. Since $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}_A$; $F_e \tilde{\subseteq} (F_2, A)$ and $G_e \tilde{\subseteq} (F_1, A)$. Thus it follows that $(U_1, U_2, \tau_\Delta, A)$ is pre- T_{Δ_1} space.

Note that every pre- T_{Δ_1} space is pre- T_{Δ_0} space. Every pre- T_{Δ_2} space is pre- T_{Δ_1} space.

Proposition 6. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $\tilde{Y}$ be a non-empty subset of $\tilde{X}$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre-

T_{Δ_0} Space then $(U_1, U_2, \tau_{\Delta_Y}, E)$ is a binary soft pre- T_{Δ_0} space.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ Now let $F_e, G_e \tilde{\subseteq} \tilde{Y}$ such that $F_e \not\tilde{\subseteq} G_e$. If there exist a binary soft pre-open set (F_1, E) in $\tilde{X}$ such that $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_1, E)$. Now if $F_e \tilde{\subseteq} \tilde{Y}$ implies that $F_e \tilde{\subseteq} \tilde{Y}$. So $F_e \tilde{\subseteq} \tilde{Y}$ and $F_e \tilde{\subseteq} (F_1, E)$. Hence $F_e \tilde{\subseteq} \tilde{Y} \tilde{\cap} (F_1, E) = [{}^Y(F_1, E)]$, where, (F_1, E) is binary soft pre-open

set. That is $(F_1, E) \in \tau_\Delta$. let $G_e \tilde{\subseteq} (F_1, E)$, this means that $G_e \tilde{\subseteq} F(\beta)$ for some $\beta \tilde{\subseteq} E$. $G_e \tilde{\subseteq} \tilde{Y} \tilde{\cap} (F_1, E)_\alpha = Y_\beta(F_1, E)_\alpha$. Therefore, $G_e \tilde{\subseteq} \tilde{Y} \tilde{\cap} (F_1, E) = [{}^Y(F_1, E)]$. Similarly, it can prove that if $G_e \tilde{\subseteq} (F_2, E)$ and $F_e \tilde{\subseteq} (F_2, E)$ then $G_e \tilde{\subseteq} [{}^Y(F_2, E)]$ and $F_e \tilde{\subseteq} [{}^Y(F_2, E)]$. Thus $(U_1, U_2, \tau_{\Delta_Y}, E)$ is a binary soft pre- T_{Δ_0} space.

Example2: Let $U_1 = \{c_1, c_2, c_3\}$, $U_2 = \{m_1, m_2\}$, $E = \{e_1, e_2\}$ and

$$\begin{aligned} \tau_\Delta &= \{\tilde{X}, \tilde{\phi}, \{(e_1(\{c_2\}\{m_2\}))\}, (e_2(\{c_1\}\{m_1\}))\}, \\ &\{(e_1(\{c_1\}\{m_1\}))\}, (e_2(\{c_2\}\{m_2\}))\}, \{(e_1(\{c_1\}\{m_1\}))\}, \{(e_1(\{\tilde{X}\}\{\tilde{X}\}))\}, (e_2(\{c_1\}\{m_1\}))\}, \\ &\text{where } (F_1, E) = \{(e_1(\{c_2\}\{m_2\}))\}, (e_2(\{c_1\}\{m_1\}))\}, \\ (F_2, E) &= \{(e_1(\{c_1\}\{m_1\}))\}, (e_2(\{c_2\}\{m_2\}))\}, \\ (F_3, E) &= \{(e_1(\{c_1\}\{m_1\}))\} \\ (F_4, E) &= \{(e_1(\{\tilde{X}\}\{\tilde{X}\}))\}, (e_2(\{c_1\}\{m_1\}))\} \end{aligned}$$

Clearly $(U_1, U_2, \tau_\Delta, E)$ is binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$.

Note that $\tau_{\Delta_1} = \{\tilde{X}, \tilde{\phi}, \{(e_1(\{c_1\}\{m_1\}))\}, (e_1(\{c_2\}\{m_2\}))\}$

$T_{\Delta_2} = \{\tilde{X}, \tilde{\phi}, \{(e_2(\{c_1\}\{m_1\}))\}, (e_2(\{c_2\}\{m_2\}))\}$ are binary soft topological spaces on $\tilde{X}$ over $(U_1 \times U_2)$. There are two pairs of distinct binary soft points namely

$F_{e_1} = \{(e_1(\{c_2\}\{m_2\}))\}$, $G_{e_1} = \{(e_1(\{c_1\}\{m_1\}))\}$ and $F_{e_2} = \{(e_2(\{c_1\}\{m_1\}))\}$,

$G_{e_2} = \{(e_2(\{c_2\}\{m_2\}))\}$. Then for binary soft pair $F_{e_1} \neq G_{e_1}$ of points there are binary soft open sets (F_1, E) and (F_2, E) such that $F_{e_1} \tilde{\subseteq} (F_1, E)$, $G_{e_1} \tilde{\subseteq} (F_1, E)$ and $G_{e_1} \tilde{\subseteq} (F_2, E)$. Similarly for the pair $F_{e_2} \neq G_{e_2}$, there are binary soft pre-open sets (F_1, E) and (F_2, E) such that $F_{e_2} \tilde{\subseteq} (F_2, E)$, $G_{e_2} \tilde{\subseteq} (F_2, E)$ and $G_{e_2} \tilde{\subseteq} (F_1, E)$, $F_{e_2} \tilde{\subseteq} (F_1, E)$. This shows that $(U_1, U_2, \tau_\Delta, E)$ is binary soft space pre- T_{Δ_1} -space and hence a binary soft pre- T_{Δ_0} space. Note that $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre- T_{Δ_2} space.

Proposition 7. Let $(U_1, U_2, \tau_\Delta, E)$ is binary soft topological space on $\tilde{X}$ over $(U_1 \times U_2)$. Then each binary soft point is binary soft pre-closed if and only if $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre- T_{Δ_1} space.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ is binary soft topological space on $\tilde{X}$ over $(U_1 \times U_2)$. Now to prove let $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre- T_{Δ_1} space, suppose binary soft points $F_{e_1} \tilde{\subseteq} (F, E)$, $G_{e_1} \tilde{\subseteq} (G, E)$ are binary soft pre-closed and $F_{e_1} \neq G_{e_1}$. Then $(F, E)^c$ and $(G, E)^c$ are binary soft pre-open in $(U_1, U_2, \tau_\Delta, E)$. Then by definition $(F, E)^c = (F^c, E)$ where $F^c(e_1) = \tilde{X} - F(e_1)$ and $(G, E)^c = (G^c, E)$, where $G^c(e_1) = \tilde{X} - G(e_1)$. Since $F(e_1) \tilde{\cap} G(e_1) = \tilde{\phi}$. This implies

$F(e_1) = \tilde{X} - G(e_1) = G^c(e_1) \quad \forall e$. This implies $F(e_1) = (F, E) \tilde{\subseteq} (G, E)^c$. Similarly $G(e_1) = (G, E) \tilde{\subseteq} (F, E)^c$. Thus we have $(e_1) \tilde{\subseteq} (G, E)^c$, $G(e_1) \tilde{\subseteq} (G, E)^c$ and $F(e_1) \tilde{\subseteq} (F, E)^c$, $G(e_1) \tilde{\subseteq} (F, E)^c$. this prove that $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre- T_1 space.

Conversely, let $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre- T_{Δ_1} space, to prove that $F(e_1) = (F, E) \tilde{\subseteq} \tilde{X}$ is binary soft pre-closed, we show that $(F, E)^c$ is binary soft pre-open in $(U_1, U_2, \tau_\Delta, E)$. Let $G_{e_1} = (G, E) \tilde{\subseteq} (F, E)^c$ is binary soft pre-closed. Then $F_{e_1} \neq G_{e_1}$, since $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre- T_{Δ_1} space, there exists binary soft pre-open set

(L, E) such that $G(e_1) \tilde{\subseteq} (L, E) \tilde{\subseteq} (F, E)^c$ and hence $\tilde{U}_{G_{e_1}} \{(L, E), G_{e_1} \tilde{\subseteq} (F, E)^c\}$. This proves that $(F, E)^c$ is binary soft pre-open in $(U_1, U_2, \tau_\Delta, E)$ that is $F_{e_1} = (F, E)$ is binary soft pre-closed in $(U_1, U_2, \tau_\Delta, E)$. Which completes the proof.

Proposition 8. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\subseteq} \tilde{X}$ such that $F_e \neq G_e$. If there exist binary soft pre-open sets (F_1, E) , (F_2, E) such that $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_2, E)^c$, then $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space and $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space for each $e \tilde{\subseteq} E$.

Proof. Clearly $G_{e_1} \tilde{\subseteq} (F_1, E)^c = (F_1^c, E)$ implies $G_{e_1} \tilde{\subseteq} (F_2, E)$ similarly $F_e \tilde{\subseteq} (F_2, E)^c = (F_2^c, E)$ implies $F_e \tilde{\subseteq} (F_2, E)$. Thus we have $F_e \tilde{\subseteq} (F_1, E)$, $G_e \tilde{\subseteq} (F_1, E)$ or $G_e \tilde{\subseteq} (F_2, E)$, $F_e \tilde{\subseteq} (F_2, E)$. This proves $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space. Now for any $\tilde{e} \in E$, $(U_1, U_2, \tau_\Delta, E)$ is a binary soft topological space and $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_1, E)^c$ or $G_e \tilde{\subseteq} (F_2, E)$ and $F_e \tilde{\subseteq} (F_2, E)^c$. So that $F_e \tilde{\subseteq} F_1(e)$, $G_e \tilde{\subseteq} F_1(e)$, $G_e \tilde{\subseteq} F_2(e)$, $G_e \tilde{\subseteq} F_2(e)$. Thus $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space.

Proposition 9. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\subseteq} \tilde{X}$ such that $F_e \neq G_e$. If there exist binary soft pre-open sets (F_1, E) , (F_2, E) such that $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_1, E)^c$ or $F_e \tilde{\subseteq} (F_2, E)$ and $G_e \tilde{\subseteq} (F_2, E)$, then $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space and $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space for each $e \in E$.

Proof. Clearly $G_e \tilde{\subseteq} (F_1, E)^c = (F_1^c, E)$ implies $G_e \tilde{\subseteq} (F_2, E)$ similarly $F_e \tilde{\subseteq} (F_2, E)^c = (F_2^c, E)$ implies $F_e \tilde{\subseteq} (F_2, E)$. Thus we have $F_e \tilde{\subseteq} (F_1, E)$, $G_e \tilde{\subseteq} (F_1, E)$ or $G_e \tilde{\subseteq} (F_2, E)$, $F_e \tilde{\subseteq} (F_2, E)$. This proves $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space. Now for any $\tilde{e} \in E$, $(U_1, U_2, \tau_\Delta, E)$ is a binary soft topological space and $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_1, E)^c$ or $G_e \tilde{\subseteq} (F_2, E)$ and $F_e \tilde{\subseteq} (F_2, E)^c$. So that $F_e \tilde{\subseteq} F_1(e)$, $G_e \tilde{\subseteq} F_1(e)$ or $G_e \tilde{\subseteq} F_2(e)$, $F_e \tilde{\subseteq} F_1(e)$. Thus $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space.

Proposition 10. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $F_e, G_e \tilde{\subseteq} \tilde{X}$ such that $F_e \neq G_e$. If there exist binary soft pre-open sets (F_1, E) , (F_2, E) such that $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_1, E)^c$ or $G_e \tilde{\subseteq} (F_2, E)$ and $F_e \tilde{\subseteq} (F_2, E)^c$, then $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space and $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_1} space and $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_1} space for each $e \in E$.

Proof. The proof is similar to the proof 9.

Now we shall discuss some of the binary soft hereditary properties of T_{Δ_i} ($i = 0, 1$) spaces.

Proposition 11. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $\tilde{Y} \tilde{\subseteq} \tilde{X}$. Then if $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space then $(U_1, U_2, \tau_{\Delta_Y}, E)$ is binary soft pre- T_{Δ_0} space.

Proof. $F_e, G_e \tilde{\subseteq} \tilde{Y}$ such that $F_e \neq G_e$. Then $F_e, G_e \tilde{\subseteq} \tilde{X}$. Since $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_0} space, thus there exists binary soft pre-open sets (F, E) and (G, E) in $(U_1, U_2, \tau_\Delta, E)$ such that $F_e \tilde{\subseteq} (F, E)$ and $G_e \tilde{\subseteq} (F, E)$ or $G_e \tilde{\subseteq} (G, E)$ and $F_e \tilde{\subseteq} (G, E)$. Therefore $F_e \tilde{\subseteq} \tilde{Y} \cap (F, E) = {}^Y(F, E)$. Similarly I can be shown that if $G_e \tilde{\subseteq} (G, E)$ and $F_e \tilde{\subseteq} (G, E)$, then $G_e \tilde{\subseteq} {}^Y(G, E)$ and $F_e \tilde{\subseteq} {}^Y(G, E)$ and $F_e \tilde{\subseteq} {}^Y(G, E)$. Thus $(U_1, U_2, \tau_{\Delta_Y}, E)$ is binary soft pre- T_{Δ_0} space.

Proposition 12. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$ and $\tilde{Y} \tilde{\subseteq} \tilde{X}$. Then if $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_1} space then $(U_1, U_2, \tau_{\Delta_Y}, E)$ is binary soft pre- T_{Δ_1} space.

Proof. The proof is similar to the proof 11.

Proposition 13. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on $\tilde{X}$ over $(U_1 \times U_2)$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- τ_{Δ_2} space on $\tilde{X}$ over $(U_1 \times U_2)$ then $(U_1, U_2, \tau_{\Delta_e}, E)$ is binary soft pre- T_{Δ_2} space for each $e \in E$.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on $\tilde{X}$ over $(U_1 \times U_2)$. For any $\tilde{e} \in E$, $\tau_{\Delta_e} = \{F(e): (F, E) \tilde{\subseteq} \tau_\Delta\}$ is a binary soft topology on $\tilde{X}$ over $(U_1 \times U_2)$. Let $x, y \tilde{\subseteq} \tilde{X}$ such that $x \neq y$, since $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_2} space, therefore binary soft points $F_e, G_e \tilde{\subseteq} \tilde{X}$ such that $F_e \neq G_e$ and $x \tilde{\subseteq} F(e)$, $y \tilde{\subseteq} G(e)$, there exists binary soft pre-open sets (F_1, E) , (F_2, E) such that $F_e \tilde{\subseteq} (F_1, E)$, $G_e \tilde{\subseteq} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$. Which implies that $\tilde{e} \tilde{\subseteq} F_1(e)$, $y \tilde{\subseteq} G(e) \tilde{\subseteq} F_2(e)$ and $F_1(e) \tilde{\cap} F_2(e) = \tilde{\phi}$. This proves that $(U_1, U_2, \tau_{\Delta_e}, E)$ is binary soft pre- T_{Δ_2} space.

Proposition 14. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space on $\tilde{X}$ over $(U_1 \times U_2)$ and $\tilde{Y} \tilde{\subseteq} \tilde{X}$. Then if $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre-

τ_{Δ_2} space then $(U_1, U_2, \tau_{\Delta_Y}, E)$ is binary soft pre- T_{Δ_2} space and $(U_1, U_2, \tau_{\Delta_e}, E)$ is binary soft pre- T_{Δ_2} space for each $e \in E$.

Proof. Let $F_e, G_e \tilde{\subseteq} \tilde{Y}$ such that $F_e \neq G_e$. Then $F_e, G_e \tilde{\subseteq} \tilde{X}$. Since $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_2} space, thus there exists binary soft pre-open sets (F_1, E) and (F_2, E) such that $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_1, E)^c$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$. Thus Therefore $F_e \tilde{\subseteq} \tilde{Y} \cap (F_1, E) = {}^Y(F_1, E)$ and ${}^Y(F_2, E) \tilde{\cap} {}^Y(F_1, E) = \tilde{\phi}$. Thus it proves that $(U_1, U_2, \tau_{\Delta_Y}, E)$ is binary soft pre- T_{Δ_2} space.

Proposition 15. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over $(U_1 \times U_2)$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- T_{Δ_2} space and for any two binary soft points $F_e, G_e \tilde{\subseteq} \tilde{X}$ such that $F_e \neq G_e$. Then there exist binary soft pre-closed sets (F_1, E) and (F_2, E) such that $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_1, E)^c$ or $G_e \tilde{\subseteq} (F_2, E)$ and $F_e \tilde{\subseteq} (F_2, E)^c$.

$(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over

$(U_1 \times U_2)$. Since $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- τ_{Δ_2} space and $F_e, G_e \tilde{\subseteq} \tilde{X}$ such that $F_e \neq G_e$ there exists binary soft pre-open sets (H, E) and (L, E) such that $F_e \tilde{\subseteq} (H, E)$ and $G_e \tilde{\subseteq} (L, E)$ and $(H, E) \tilde{\cap} (L, E) = \tilde{\phi}$. Clearly $(H, E) \tilde{\subseteq} (L, E)^c$ and $(L, E) \tilde{\subseteq} (H, E)^c$. Hence $F_e \tilde{\subseteq} (L, E)^c$, put $(L, E)^c = (F_1, E)$ which gives $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \notin (F_1, E)$. Also $G_e \tilde{\subseteq} (F_1, E)^c$, then put $(H, E)^c = (F_2, E)$. Therefore $F_e \tilde{\subseteq} (F_1, E)$ and $G_e \tilde{\subseteq} (F_2, E)$. Moreover, $(F_1, E) \tilde{\cap} (F_2, E) = (L, E)^c \tilde{\cap} (H, E)^c = \tilde{\phi}$. Which completes the proof.

5. BINARY SOFT PRE- T_{Δ_i} ($i=4, 3$) SPACES

In this section binary soft pre-separation axioms in Binary Soft Topological Spaces are discussed.

In this section, we define binary soft pre-regular and binary soft pre- T_{Δ_1} spaces using binary soft points. We also characterize binary soft pre-regular and binary soft pre-normal spaces. Moreover, we prove that binary soft pre-regular and binary soft pre- T_{Δ_3} properties are binary soft hereditary, whereas binary soft pre-normal and binary soft pre- T_{Δ_4} are binary soft pre-closed hereditary properties.

Now we define binary soft pre-regular space as follows:

Definition 15: Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over

$(U_1 \times U_2)$. Let (F, E) be a binary soft pre-closed set in $(U_1, U_2, \tau_\Delta, E)$ and $F_e \tilde{\subseteq} (F, E)$. If there exists binary soft pre-open sets (G, E) and (H, E) such that $F_e \tilde{\subseteq} (G, E)$, $(F, E) \tilde{\subseteq} (H, E)$ and $(F, E) \tilde{\cap} (H, E) = \tilde{\phi}$, then $(U_1, U_2, \tau_\Delta, E)$ is called a binary soft pre-regular space.

Proposition 16. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft topological space of $\tilde{X}$ over

$(U_1 \times U_2)$. Then the following statements are equivalent:

- (i) $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre-regular.
- (ii) For any binary soft pre-open set (F, E) in $(U_1, U_2, \tau_\Delta, E)$ and $G_e \tilde{\subseteq} (F, E)$, there is binary soft pre-open set (G, E) containing G_e such that $G_e \tilde{\subseteq} \overline{(G, E)} \tilde{\subseteq} (F, E)$.
- (iii) Each binary soft point in $(U_1, U_2, \tau_\Delta, E)$ has a binary soft neighborhood base consisting of binary soft pre-closed sets.

Proof. (i) $\Rightarrow$ (ii)

Let (F, E) be a binary soft pre-open set in $(U_1, U_2, \tau_\Delta, E)$ and $G_e \tilde{\subseteq} (F, E)$. Then

$(F, E)^c$ is binary soft pre-closed set such that $G_e \tilde{\subseteq} (F, E)^c$. By the binary soft regularity of $(U_1, U_2, \tau_\Delta, E)$ there are binary soft pre-open sets (F_1, E) , (F_2, E) such that $G_e \tilde{\subseteq} (F_1, E)$, $(F, E)^c \tilde{\subseteq} (F_2, E)$

and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$. Clearly $(F_2, E)^c$ is a binary soft set contained in (F, E) . Thus, $(F_1, E) \tilde{\subseteq} (F_2, E)^c \tilde{\subseteq} (F, E)$. This gives $\overline{(F_1, E)} \tilde{\subseteq} (F_2, E)^c \tilde{\subseteq} (F, E)$, put $(F_1, E) = (G, E)$. Consequently $G_e \tilde{\subseteq} (G, E)$ and $\overline{(G, E)} \tilde{\subseteq} (F, E)$.

This proves (ii)

(ii) $\Rightarrow$ (iii)

Let $G_e \tilde{\subseteq} \tilde{X}$, for binary soft pre-open set (F, E) in $(U_1, U_2, \tau_\Delta, E)$ there is a binary soft pre-open set (G, E) containing G_e such that $G_e \tilde{\subseteq} (G, E)$, $\overline{(G, E)} \tilde{\subseteq} (F, E)$. Thus for each $G_e \tilde{\subseteq} \tilde{X}$, the sets $\overline{(G, E)}$ from a binary soft neighborhood base consisting of binary soft pre-closed sets of $(U_1, U_2, \tau_\Delta, E)$ which proves (iii).

(iii) $\Rightarrow$ (i)

Let (F, E) be a binary soft pre-closed set such that $G_e \tilde{\not\subseteq} (F, E)$. Then $(F, E)^c$ is a binary soft open neighbourhood of G_e . By (iii) there is a binary soft pre-closed set (F_1, E) Which contains G_e and is a binary soft neighborhood of G_e with $(F_1, E) \tilde{\subseteq} (F_1, E)^c$. Then $G_e \tilde{\not\subseteq} (F, E)^c$, $(F, E) \tilde{\subseteq} (F_1, E)^c = (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \tilde{\phi}$. Therefore $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre-regular.

Proposition 17. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft pre-regular space on $\tilde{X}$ over

$(U_1 \times U_2)$. Then every binary soft subspace of $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre-regular.

Proof. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft subspace of a binary soft pre-regular space $(U_1, U_2, \tau_\Delta, E)$. Suppose (F, E) is a binary soft pre-closed set in $(U_1, U_2, \tau_{\Delta_Y}, E)$ and $F_e \tilde{\subseteq} \tilde{Y}$ such that $F_e \tilde{\not\subseteq} (F, E)$. Then $(F, E) = (G, E) \tilde{\cap} \tilde{Y}$;

Where (G, E) is binary soft pre-closed set in $(U_1, U_2, \tau_\Delta, E)$. Then $F_e \tilde{\not\subseteq} (F, E)$, since $(U_1, U_2, \tau_\Delta, E)$ be a binary soft subspace of a binary soft pre-regular, there exists soft disjoint binary pre-open sets (F_1, E) , (F_2, E) in $(U_1, U_2, \tau_\Delta, E)$. Then $F_e \tilde{\not\subseteq} (G, E)$. Since $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre-regular, there exist binary soft disjoint binary pre-open sets (F_1, E) , (F_2, E) in $(U_1, U_2, \tau_\Delta, E)$ such that $F_e \tilde{\subseteq} (F_1, E)$, $(G, E) \tilde{\subseteq} (F_2, E)$. Clearly $F_e \tilde{\subseteq} (F_1, E) \tilde{\cap} \tilde{Y} = {}^Y(F_2, E)$ and $(F, E) \tilde{\subseteq} (F_2, E) \tilde{\cap} \tilde{Y} = {}^Y(F_2, E)$ such that ${}^Y(F_1, E) \tilde{\cap} {}^Y(F_2, E) = \tilde{\phi}$. Therefore, it proves that $(U_1, U_2, \tau_{\Delta_Y}, E)$ is a binary soft pre-regular subspace of $(U_1, U_2, \tau_\Delta, E)$.

Proposition 18. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on $\tilde{X}$ over

$(U_1 \times U_2)$. A binary space $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre-regular if and only if for each $F_e \tilde{\subseteq} \tilde{X}$ and a binary soft pre-closed set (F, E) in $(U_1, U_2, \tau_\Delta, E)$ such that $F_e \tilde{\not\subseteq} (F, E)$ there exist binary soft pre-open sets $\overline{(F_1, E)}$, $\overline{(F_2, E)}$ in $(U_1, U_2, \tau_\Delta, E)$ such that $F_e \tilde{\subseteq} \overline{(F_1, E)}$, $\overline{(F_1, E)} \tilde{\subseteq} \overline{(F_2, E)}$ and $\overline{(F_1, E)} \tilde{\cap} \overline{(F_2, E)} = \tilde{\phi}$.

Proof. For each $F_e \tilde{\subseteq} \tilde{X}$ and a binary soft pre-closed set (G, E) such that $F_e \tilde{\not\subseteq} (F, E)$ by theorem 16 there is a binary soft pre-open set (G, E) such that $F_e \tilde{\subseteq} (G, E)$, $\overline{(G, E)} \tilde{\subseteq} (F_1, E)^c$. Again by theorem 16 there is a binary soft pre-open (F_1, E) containing F_e such that $\overline{(F_1, E)} \tilde{\subseteq} (G, E)$. Let $(F_2, E) = \overline{((G, E))^c}$, then

$\overline{(F_1, E)} \tilde{\subseteq} (G, E) \tilde{\subseteq} \overline{(G, E)} \tilde{\subseteq} (F_1, E)^c$ Implies $\overline{(F_1, E)} \tilde{\subseteq} \overline{((G, E))^c} = (F_2, E)$ or $(F, E) \tilde{\subseteq} (F_2, E)$.

Also

$$\begin{aligned} \overline{(F_1, E)} \tilde{\cap} \overline{(F_2, E)} \\ = \overline{(F_1, E)} \tilde{\cap} \overline{((G, E))^c} \tilde{\subseteq} (G, E) \tilde{\cap} \overline{((G, E))^c} \tilde{\subseteq} \overline{(G, E)} \tilde{\cap} \overline{((G, E))^c} = \tilde{\phi} = \varphi. \end{aligned}$$

Thus $(F_1, E), (F_2, E)$ are the required binary soft pre-open sets in $(U_1, U_2, \tau_\Delta, E)$. This proves the necessity. The sufficiency is immediate.

Definition 16: Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on $\tilde{X}$ over $(U_1 \times U_2)$. $(F, E), (G, E)$ are binary soft pre-closed sets over $(U_1 \times U_2)$ such that $(F, E) \tilde{\cap} (G, E) = \tilde{\phi}$. If there exist binary soft pre-open sets (F_1, E) , and (F_2, E) such that $(F, E) \tilde{\subseteq} (F_1, E)$, $(G, E) \tilde{\subseteq} (F_2, E)$ and $(F_1, E) \tilde{\cap} (F_2, E) = \varphi$, then $(U_1, U_2, \tau_\Delta, E)$ is called a binary soft pre-normal space.

Definition 17: Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on $\tilde{X}$ over $(U_1 \times U_2)$. Then $(U_1, U_2, \tau_\Delta, E)$ is said to be a binary soft pre- τ_{Δ_3} space if it is binary soft pre-regular and a binary soft pre- τ_{Δ_1} space.

Proposition 19. Let $(U_1, U_2, \tau_\Delta, E)$ be a binary soft regular space on $\tilde{X}$ over $(U_1 \times U_2)$ and $\tilde{Y} \tilde{\subseteq} \tilde{X}$. If $(U_1, U_2, \tau_\Delta, E)$ is a binary soft pre- τ_{Δ_3} space then $(U_1, U_2, \tau_{\Delta_Y}, E)$ is a binary soft pre- τ_{Δ_3} space.

Definition 18: A binary soft topological space $(U_1, U_2, \tau_\Delta, E)$ on $\tilde{X}$ over

$(U_1 \times U_2)$ is said to be a binary soft pre- τ_{Δ_4} space if it is binary soft pre-normal and binary soft pre- τ_{Δ_1} space.

τ_{Δ_3} -space.

Proposition 20. A binary soft topological space $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre-normal if and only if for soft pre-closed set (F, E) and a binary soft pre-open set (G, E) , such that $(F, E) \tilde{\subseteq} (G, E)$ these exist at least one binary soft pre-open set (H, E) containing (F, E) such that $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (G, E)$.

Proof. Let us suppose that $(U_1, U_2, \tau_\Delta, E)$ is a binary soft normal space and (F, E) is any binary soft pre-closed subset of $(U_1, U_2, \tau_\Delta, E)$ and (G, E) is a binary soft pre-open set such that $(F, E) \tilde{\subseteq} (G, E)$. Then $(G, E)^c$ is binary soft pre-closed and $(F, E) \tilde{\cap} (G, E)^c = \varphi$. So by supposition, there are binary soft pre-open sets (H, E) and (K, E) such that $(F, E) \tilde{\subseteq} (H, E)$, $(G, E)^c \tilde{\subseteq} (K, E)$ and $\tilde{\cap} (K, E) = \tilde{\phi}$. Since $(H, E) \tilde{\cap} (K, E) = \tilde{\phi}$, $(H, E) \tilde{\subseteq} (K, E)^c$. But $(K, E)^c$ is binary soft pre-closed, so that $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (K, E)^c \tilde{\subseteq} (G, E)$.

Hence $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (K, E)^c \tilde{\subseteq} (G, E)$.

Conversely, suppose that for every binary soft pre-closed set (F, E) and a binary soft pre-open set (G, E) such that $(F, E) \tilde{\subseteq} (H, E)$, there is a binary soft pre-open set (H, E) such that $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (G, E)$. Let $(F_1, E), (F_2, E)$ be any two-soft disjoint pre-closed sets, then $(F_1, E) \tilde{\subseteq} (F_2, E)^c$ where $(F_2, E)^c$ binary soft pre-open. Hence there is a binary soft pre-open set (H, E) such that $(F, E) \tilde{\subseteq} (H, E) \tilde{\subseteq} \overline{(H, E)} \tilde{\subseteq} (F_2, E)^c$. But then $(F_2, E) \tilde{\subseteq} \overline{((H, E))^c}$ and $(H, E) \tilde{\cap} \overline{((H, E))^c} \neq \varphi$.

Hence $(F_1, E) \tilde{\subseteq} (H, E)$ and $(F_2, E) \tilde{\subseteq} \overline{((H, E))^c}$ with $(H, E) \tilde{\cap} \overline{((H, E))^c} = \varphi$.

Hence $(U_1, U_2, \tau_\Delta, E)$ is binary soft pre-normal space.

6. CONCLUSION

A soft topology between two sets other than the product soft topology has been touched through proper channel. A soft set with single specific topological structure is unable to shoulder up the responsibility to build the whole theory. So, to make the theory strong, some additional structures on soft set has to be introduced. It makes it bouncier to grow the soft topological spaces with its infinite applications. In this regards we familiarized soft topological structure known as binary soft pre-separation axioms in binary soft topological structure with respect to soft pre-open sets.

Topology is the most important branch of pure mathematics which deals with mathematical structures by one way or the others. Recently, many scholars have studied the soft set theory which is carefully applied to many difficulties which contain uncertainties in our social life. A few researchers familiarized and profoundly studied the foundation of soft topological spaces. They also studied topological structures and displayed their

several properties with respect to ordinary points.

In the present work, we constantly study the behavior of binary soft pre-separation axioms in binary soft topological spaces with respect to soft points as well as ordinary points. We introduce (pre- τ_{Δ_0} , pre- τ_{Δ_1} , pre- τ_{Δ_2} , pre- τ_{Δ_3} and pre- τ_{Δ_4}) structures with respect to soft points. In future we will plant these structures in different results. We also planted these axioms to different results. These binary soft pre-separation structure would be valuable for the development of the theory of soft in binary soft topology to solve complicated problems, comprising doubts in economics, engineering, medical etc. We also attractively discussed some soft transmissible properties with respect to ordinary as well as soft points. I have fastidiously studied numerous homes on the behalf of Soft Topology. And lastly, I determined that soft Topology is totally linked or in other sense we can correctly say that Soft Topology (Separation Axioms) are connected with structure. Provided if it is related with structures then it gives the idea of non-linearity beautifully. In other ways we can rightly say Soft Topology is somewhat directly proportional to non-linearity. Although we use non-linearity in Applied Math. So, it is not wrong to say that Soft Topology is applied Math in itself. It means that Soft Topology has the taste of both of pure and applied math. In future I will discuss Separation Axioms in Soft Topology With respect to soft points. We expect that these results in this article will do help the researchers for strengthening the toolbox of soft topological structures. Soft topology provides less informations on the behalf of a few choices. The reason for this is that we use a single set in soft topology and in binary soft topology we use double sets. It means that binary soft topology exceeds soft topology in all respect. In the light of above mentioned discussion I can literary say that number of sets is directly proportional to choices. Therefore, all mathematicians are kindly informed to emphasize upon it.

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