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MATHEMATICAL MODELLING THE ASSEMBLY CELL OPERATION IN THE CONTEXT OF INCREASING ITS RELIABILITY

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ABSTRACT

In the article, by limiting the mathematical description to one assembly cell connected with the workpiece dispenser, the considerations concerning the time redundancy are partially eliminated and the preferred material redundancy, in the form of the workpiece reserves in the dispensers, is achieved thanks to the reserve of the assembly cell productivity. Additionally, it has been assumed in the article that all random variables have the Erlang distribution of two-grade. Two cases were presented in the model: when the workpiece dispenser fails and when the dispenser has absolute reliability.

1. INTRODUCTION

The assembly line is a group of assembly cells equipped with assembling machines and auxiliary operation work stands which are located in accordance with the sequence of the operations performed and which are connected with mechanical transport equipment [1,2]. In order to simplify and facilitate analysing phenomena on the assembly line, the events occurring in the elementary component of the assembly line are studied. The elementary components of the line under scrutiny may be the following: module, work centre, flexible production machine, assembly cell [3]. The results obtained are generalized and approximated with regard to next elementary components of the line. This approach enables drawing conclusions which concern functioning of the whole line.

On the one hand, the task of increasing the assembly line reliability can be accomplished on the basis of increasing the reliability of separate units which constitute the whole system [4,5]. On the other hand, the task can be accomplished by applying the apparatus and time redundancy [6]. The second method is used much more frequently since it enables simultaneous increasing of the automated system flexibility [7]. Time redundancy consists in introducing time reserve during which failures of individual line components do not lead to interruption of the line functioning [8]. The time reserve is not introduced to the facility but to the rule (algorithm) which governs the utilization of the facility [9,10]. Practical experience indicates that, in order to assure the production stability, guarantee reserves should increase in parallel with the production increase, widening of nomenclature and acceleration of the production renewal rate, which consequently leads to freezing of the company circulating assets [11-14]. Introduction of time redundancy on the assembly line necessitates defining the following: time redundancy implementation method, conditions for time redundancy utilization and spatial distribution of time redundancy (for each line component or for a group of components) [15].

Frequently, in order to achieve time redundancy, intermediate production feeders (dispensers) are located on the line between separate assembly line cells [16,17]. Application of feeders (dispensers) enables increasing line reliability and capacity which results in reduced loss due to line failures [18].

Research problems focusing on description of the assembly line models, taking into account the time redundancy applied on the lines, are presented in [8]. Two methods for constructing models are applied: the first one which enables obtaining analytical expressions relies on the theory of Markow processes; the second one uses the iteration processes and enables calculating the average number of workpieces in the dispensers.

The assembly system presented in is described by means of the Markow processes taking into consideration the following states: work, renewal and standstill [19]. The notion of the resistance system, i.e. the system whose equipment and dispensers are regenerated immediately, and the dispensers are filled completely. The functions of the resistance system were described using the transition chain of the Markow system. This model was developed into another model in which failure time was included in the actual working time amount with the application of equivalent transformation of unreliable equipment into absolutely reliable equipment [4].

Construction of reliable models was carried out basing on the following assumptions [20]:

- 1 – failures of all components of the system (technological work stands, and dispensers are independent; time until the first failure is defined according to the exponential dependences with parameter: λ .
- 2 – when failure of any component of the system occurs, the repair of the component starts immediately (returning to work); time of returning to work of any component of the system was also defined according to the exponential dependences with parameter μ .

There have been many works analyzing the performance of manufacturing systems using queueing theory [21-22]. Some researcher presented a DDX algorithm to calculate the performance of n-machines serial production line with unreliable machines and finite buffers [23]. Also presented the overlapping decomposition method that enabled the numerical evaluation of the production rates of lines having complex structures. Author suggested a new approximation approach for estimating throughput in a serial line with multiple reliable machines, exponential service times and finite buffers [24].

2. MATHEMATICAL MODELLING WITH REGARD TO FUNCTIONING OF THE ASSEMBLY CELLS

Most of the research works concerning modelling of the multi-phase systems relies on the approximated method of calculating the probability of the system standstill. The multi-phase system analysed in presents the line of serial structure which includes n technological sections interconnected by dispensers [19]. Equal significance is assigned to each line section while analysing the standstill of each section caused by this section failure and by the failure of the remaining sections. Expressions of standstill probability were obtained about n - section line which describes all the structures. Equations derived were obtained for exponential distribution of random parameters concerning the system components. The model of the multi-phase system, described as above, was analysed in [25]. However, this analysis was carried out without assuming the condition that the units have equal capacity and that the units remain simultaneously in the failure state.

Complex structure of failures: complex events were analysed in when failure and repair streams were divided into the streams conditioned by the assembly cell equipment and by the technological tools [4]. Probability of failure-free operation of the system during present time was applied for describing reliability coefficients. Main stages of the approximate evaluation of reliability and special cases of obtaining such evaluations for the most frequent configurations of the system were presented. Probability of failure-free operation of the system was calculated according to the principle of complete probability limited to the number of the most probable hypotheses. The above analyses are supported by the calculations concerning the simple structure including one cell and one dispenser of limited capacity. Exponential distribution of working time until failure and normal distribution of returning to work time are applied.

In processing time, and the time to failure (TTF) and time to repair (TTR) were geometrically distributed and operation dependent [22]. In introduced an additional analysis of the operation of production lines to closed-loop systems as well as other models such as exponential and continuous production system models with different service rates of machines and exponential machine reliability [26]. Accepted that processing time is constant but dissimilar, and the distribution functions of TTF and TTR are exponentially and time dependent.

Inaccuracy of the purely exponential models was described in [25]. The assumption was made concerning independent functioning of the assembly line components and the approximation was made applying models of the mass service systems.

The serial system is modelled by means of the Markow processes [8,25]. The disadvantage of such model is that the stationary distribution of the entering Markow chains is located only at the entrance of the two-phase system. In this case, as well as while using the exponential distribution of the failure-free operation time and the failure time, the results analogous to the ones described were obtained [15].

Simplified diagram concerning the two-phase system modelling was presented in [23]. The system occurred to be the system with the non-exponential distribution of the working time until failure and the failure time. The model was applied to the highly reliable systems and, as a result of the calculations, the probability values were obtained concerning the running out of the dispenser reserves, during the failure time close to zero and with the probability of failure in time of transferring the dispenser to the previous state (after incomplete running out of the dispenser reserves) lower than a unit.

3. MODELLING ASSURES RELIABILITY AND CAPACITY OF THE ASSEMBLY CELL

Assumptions for the modelling process:

- 1 - Production workpieces are delivered in a uniform way to the entrance of the section and taken out of the line at the exit of the line.
- 2 - Time τ_P for filling H_k dispenser, when the assembly cell does not work, defined by the difference between the number of workpieces in m_k dispenser and the production reserve in this dispenser L_k , is longer than the failure time of AC. That is why the analysis will be carried out for the case $\tau_B < \tau_P$ (time of filling the dispenser is longer than the failure time of the cell situated behind the dispenser).
- 3 - The processes which occur during the operation of the assembly cell (AC) with dispenser H are the Markow processes.

For the analysis we use the system consisting of AC and the reliable H_k dispenser. Work states are presented on the Figure 1 and Table 1.

where: $t_{\xi 0}$ – failure-free operation time of AC, $t_{\eta 0}$ – time of returning to work from state S_1 to S_0 , $t_{\xi 1}$ – time of stopping the dispenser operation, starting from state S_1 when the dispenser is full, $t_{\eta 2}$ – time of returning to work from state S_2 to S_0 .

Figure 1: General diagram showing the dependences between the work states of the assembly line section

Table 1. Identification of the work states of the assembly cell and the dispenser

State	Assembly cell	Dispenser
S_0	Efficient	Receives parts
S_1	Inefficient	Still has spare space, receives parts
S_2	<i>Inefficient</i>	Efficient but does not receives parts due to lack of space

The event $S_0 \rightarrow S_1$ is described by $F_{01}(t)$ distribution function and by the function of probability distribution density $f_{01}(t)$ between the failures as well as by expected value of $M_0^{KM}[t]$ random variable:

$$F_{01}(t) = P(t_{\xi 0} < t) = \frac{\lambda_2^{0,1} [1 - e^{-\lambda_1^{0,1}(t-\tau)}] - \lambda_1^{0,1} [1 - e^{-\lambda_2^{0,1}(t-\tau)}]}{\lambda_2^{0,1} - \lambda_1^{0,1}} \quad t \geq \tau \quad (1)$$

$$f_{0,1}(t) = \begin{cases} \frac{\lambda_1^{0,1} \lambda_2^{0,1}}{\lambda_2^{0,1} - \lambda_1^{0,1}} [e^{-\lambda_1^{0,1}(t-\tau)} - e^{-\lambda_2^{0,1}(t-\tau)}] & t \geq \tau \\ 0 & t < \tau \end{cases} \quad (2)$$

$$M_0^{KM}[t_{\xi 0}] = \overline{t_{\xi 0}} = \frac{\lambda_1^{0,1} + \lambda_2^{0,1}}{\lambda_2^{0,1} \lambda_1^{0,1}} + \tau \quad (3)$$

Transition time $t_{\eta 1}$ of AC from state S_1 to state S_0 and the time $t_{\xi 1}$ of stopping the dispenser's operation when the dispenser is full are described by the distribution functions $F_{10}(t)$, $F_{12}(t)$; distribution density $f_{10}(t)$, $f_{12}(t)$ and mathematic expected value $M_B^{KM}[t]$ and $M_P^{KM}[t]$ are described by the dependences: Returning to work from state $S_1 \rightarrow S_0$

$$F_{10} = \begin{cases} P(t_{\eta 1} < t) = \frac{\mu_2^{1,0}[1 - e^{-\mu_1^{1,0}(t-\tau_B)}] - \mu_1^{1,0}[1 - e^{-\mu_2^{1,0}(t-\tau_B)}]}{\mu_2^{1,0} - \mu_1^{1,0}} & t \geq \tau_B \\ 0 & t < \tau_B \end{cases} \quad (4)$$

$$f_{1,0}(t) = \begin{cases} \frac{\mu_1^{1,0} \mu_2^{1,0}}{\mu_2^{1,0} - \mu_1^{1,0}} [e^{-\mu_1^{1,0}(t-\tau_B)} - e^{-\mu_2^{1,0}(t-\tau_B)}] & t \geq \tau_B \\ 0 & t < \tau_B \end{cases} \quad (5)$$

Transition from state $S_1 \rightarrow S_2$

$$F_{12} = \begin{cases} P(t_{\xi 1} < t) = \frac{\lambda_2^{1,2}[1 - e^{-\lambda_1^{1,2}(t-\tau_P)}] - \lambda_1^{1,2}[1 - e^{-\lambda_2^{1,2}(t-\tau_P)}]}{\lambda_2^{1,2} - \lambda_1^{1,2}} & t \geq \tau_P \\ 0 & t < \tau_P \end{cases} \quad (6)$$

$$f_{1,2}(t) = \begin{cases} \frac{\lambda_1^{1,2} \lambda_2^{1,2}}{\lambda_2^{1,2} - \lambda_1^{1,2}} [e^{-\lambda_1^{1,2}(t-\tau_P)} - e^{-\lambda_2^{1,2}(t-\tau_P)}] & t \geq \tau_P \\ 0 & t < \tau_P \end{cases} \quad (7)$$

where: τ_B – failure time, τ_P – time for filling the dispenser.

The system having the probability equal to a unit undergoes the transition from state S_2 (Figure 1) to state S_0 . Time of remaining in state S_2 is defined by the difference of times ($t_{\eta 1} - t_{\xi 1}$), where $t_{\eta 1} > t_{\xi 1}$. Thus, the time of remaining in state S_2 is $t_{\xi 2}$ random variable of distribution function:

$$F_{2,0}(t) = P(t_{\xi 20} \leq t) = P[t_{\eta 1} - t_{\xi 1} \leq t, \quad t_{\eta 1} > t_{\xi 1}] \quad (8)$$

Random value of the distribution function is defined by the right-hand part of the expression (8) designated as $[t_{\eta 1} - t_{\xi 1}]^+$ which is specified:

$$F_{2,0}(t) = P\{[t_{\eta 1} - t_{\xi 1}]^+ \leq t\} = \frac{\int_0^\infty [F_{10}(t+y) - F_{10}(y)] dF_{12}(y)}{\int_0^\infty [1 - F_{10}(y)] dF_{12}(y)} \quad (9)$$

Introducing τ_P (time for filling the dispenser when the adjacent assembly cell is idle) and τ_B (failure time of AC) in the distribution function $F_{10}(t)$ and $F_{12}(t)$, the formula (9) will have the following form:

$$F_{20}(t) = \begin{cases} \frac{A(1 - e^{-\mu_1^{2,0}t}) - B(1 - e^{-\mu_2^{2,0}t})}{A - B}, & \tau_B < \tau_P \\ \frac{C(1 - e^{-\mu_1^{2,0}(t-\tau_B+\tau_P)}) - D(1 - e^{-\mu_2^{2,0}(t-\tau_B+\tau_P)})}{C - D}, & t \geq \tau_B - \tau_P \quad \text{dla } \tau_B \geq \tau_P \\ 0 & t < \tau_B - \tau_P \quad \text{dla } \tau_B \geq \tau_P \end{cases} \quad (10)$$

Where: A, B, C, D determining from formula:

$$A = \mu_2(\mu_2 + \mu_1)(\mu_2 + \mu_2)e^{-\mu_1(\tau_P - \tau_B)} \quad (11)$$

$$B = \mu_1(\mu_1 + \mu_1)(\mu_1 + \mu_2)e^{-\mu_2(\tau_P - \tau_B)} \quad (12)$$

$$C = \frac{\mu_2 e^{-\mu_1(\tau_B - \tau_P)}}{\mu_1 + \mu_1} - \frac{\mu_2 e^{-\mu_2(\tau_B - \tau_P)}}{\mu_1 + \mu_2} \quad (13)$$

$$D = \frac{\mu_1 e^{-\mu_1(\tau_B - \tau_P)}}{\mu_2 + \mu_1} - \frac{\mu_1 e^{-\mu_2(\tau_B - \tau_P)}}{\mu_2 + \mu_2} \quad (14)$$

Expected value of random value $t_{\xi 1,2} = [t_{\eta 1} - t_{\xi 1}]^+$ equal to the average time $\overline{t_{\xi 2}}$ of remaining in state S_2 :

$$M[t_{\xi_2}] = \overline{t_{\xi_2}} = \begin{cases} \frac{1}{A-B} \left(\frac{A}{\mu_1^{1,0}} - \frac{B}{\mu_2^{1,0}} \right), & \tau_p > \tau_B \\ \frac{1}{C-D} \left(\frac{C}{\mu_1^{1,0}} e^{-\mu_1^{1,0}(\tau_B - \tau_p)} - \frac{D}{\mu_2^{1,0}} e^{-\mu_2^{1,0}(\tau_B - \tau_p)} \right) + (\tau_B - \tau_p), & \tau_B \geq \tau_p \end{cases} \quad (15)$$

Probability of transition between the work states presented on Figure 1 is defined in the following way:

$$\begin{aligned} P_{01} &= 1 \\ P_{20} &= 1 \\ P_{10} &= P(t_{\eta_1} < t_{\xi_1}) = \int_0^\infty F_{10}(t) dF_{12}(t) = \int_0^\infty \overline{F_{12}}(t) dF_{10}(t) \\ P_{12} &= P(t_{\xi_1} < t_{\eta_1}) = \int_0^\infty F_{12}(t) dF_{10}(t) = \int_0^\infty \overline{F_{10}}(t) dF_{12}(t) \end{aligned} \quad (16)$$

On the basis of (16) getting:

$$P_{12} = \begin{cases} \frac{\mu_1^{1,0} \mu_2^{1,0}}{\mu_2^{1,0} - \mu_1^{1,0}} \left(\frac{\mu_2^{1,0} e^{-\mu_1^{1,0}(\tau_p - \tau_B)}}{(\mu_1^{1,0} + \lambda_1^{1,2})(\mu_1^{1,0} + \lambda_2^{1,2})} - \frac{\mu_1^{1,0} e^{-\mu_2^{1,0}(\tau_p - \tau_B)}}{(\mu_2^{1,0} + \lambda_1^{1,2})(\mu_2^{1,0} + \lambda_2^{1,2})} \right), & \tau_B < \tau_p \\ \frac{\lambda_1^{1,2} \lambda_2^{1,2}}{\lambda_2^{1,2} - \lambda_1^{1,2}} \left(\frac{(\mu_2^{1,0} + \mu_1^{1,0} + \lambda_1^{1,2}) e^{-\mu_1^{1,0}(\tau_p - \tau_B)}}{(\mu_1^{1,0} + \lambda_1^{1,2})(\mu_2^{1,0} + \lambda_1^{1,2})} - \frac{(\mu_2^{1,0} + \mu_1^{1,0} + \lambda_2^{1,2}) e^{-\lambda_2^{1,2}(\tau_p - \tau_B)}}{(\mu_1^{1,0} + \lambda_2^{1,2})(\mu_2^{1,0} + \lambda_2^{1,2})} \right), & \tau_B \geq \tau_p \end{cases} \quad (17)$$

$$P_{10} = \begin{cases} 1 - \frac{\mu_1^{1,0} \mu_2^{1,0}}{\mu_2^{1,0} - \mu_1^{1,0}} \left(\frac{\mu_2^{1,0} e^{-\mu_1^{1,0}(\tau_p - \tau_B)}}{(\mu_1^{1,0} + \lambda_1^{1,2})(\mu_1^{1,0} + \lambda_2^{1,2})} - \frac{\mu_1^{1,0} e^{-\mu_2^{1,0}(\tau_p - \tau_B)}}{(\mu_2^{1,0} + \lambda_1^{1,2})(\mu_2^{1,0} + \lambda_2^{1,2})} \right), & \tau_B < \tau_p \\ 1 - \frac{\lambda_1^{1,2} \lambda_2^{1,2}}{\lambda_2^{1,2} - \lambda_1^{1,2}} \left(\frac{(\mu_2^{1,0} + \mu_1^{1,0} + \lambda_1^{1,2}) e^{-\mu_1^{1,0}(\tau_p - \tau_B)}}{(\mu_1^{1,0} + \lambda_1^{1,2})(\mu_2^{1,0} + \lambda_1^{1,2})} - \frac{(\mu_2^{1,0} + \mu_1^{1,0} + \lambda_2^{1,2}) e^{-\lambda_2^{1,2}(\tau_p - \tau_B)}}{(\mu_1^{1,0} + \lambda_2^{1,2})(\mu_2^{1,0} + \lambda_2^{1,2})} \right), & \tau_B \geq \tau_p \end{cases} \quad (18)$$

Average time $\overline{t_{\xi_i}}$ for $i = (0, 1, 2)$ of remaining in states S_i ($i = 0, 1, 2$) will be specified as the mathematical expected value of random values described by the distribution functions.

Value $\overline{t_{\xi_0}} = M_0^{KM}[t]$ determining (3), $\overline{t_{\xi_2}}$ - (15) and $\overline{t_{\xi_1}}$ - (23, 24).

State S_0 :

$$F_0(t) = F_{01}(t) \quad (19)$$

State S_1 :

$$\begin{aligned} F_1(t) &= P(t_{\xi_1} < t) = P(\min(t_{\eta_1}, t_{\xi_1}) < t) = 1 - P(\min(t_{\eta_1}, t_{\xi_1}) > t) = \\ &= [1 - F_{10}(t)][1 - F_{12}(t)] = 1 - \overline{F_{10}}(t) \overline{F_{12}}(t) \end{aligned} \quad (20)$$

State S_2 :

$$F_2(t) = F_{20}(t) \quad (21)$$

Solve (20) getting:

$$F_1(t) = \begin{cases} 0, & t < \tau_B, t < \tau_p \\ \frac{\mu_2^{1,0} [1 - e^{-\mu_1^{1,0}(t - \tau_B)}] - \mu_1^{1,0} [1 - e^{-\mu_2^{1,0}(t - \tau_B)}]}{\mu_2^{1,0} - \mu_1^{1,0}}, & \tau_B \leq t \leq \tau_p \\ \frac{\mu_2^{1,0} [1 - e^{-\mu_1^{1,0}(t - \tau_p)}] - \mu_1^{1,0} [1 - e^{-\mu_2^{1,0}(t - \tau_p)}]}{\mu_2^{1,0} - \mu_1^{1,0}}, & \tau_p \leq t \leq \tau_B \\ \frac{1}{(\mu_2^{1,0} - \mu_1^{1,0})(\lambda_2^{1,2} - \lambda_1^{1,2})} \{ \mu_1^{1,0} \mu_2^{1,0} [1 - e^{-(\mu_1^{1,0} + \lambda_2^{1,2})t + \mu_1^{1,0}\tau_B + \lambda_1^{1,2}\tau_p}] - \\ - \mu_2^{1,0} \mu_1^{1,0} [1 - e^{-(\mu_2^{1,0} + \lambda_1^{1,2})t + \mu_2^{1,0}\tau_B + \lambda_2^{1,2}\tau_p}] - \\ - \mu_1^{1,0} \lambda_2^{1,2} [1 - e^{-(\mu_2^{1,0} + \lambda_1^{1,2})t + \mu_1^{1,0}\tau_B + \lambda_1^{1,2}\tau_p}] - \\ - \mu_1^{1,0} \lambda_1^{1,2} [1 - e^{-(\mu_1^{1,0} + \lambda_2^{1,2})t + \mu_2^{1,0}\tau_B + \lambda_2^{1,2}\tau_p}] \}, & t \geq \tau_p, t \geq \tau_B \end{cases} \quad (22)$$

Average time $\overline{t_{\xi_1}}$ of remaining in state S_1 :

$$\begin{aligned}
\overline{t_{\xi 1}} = & \frac{\mu_1^{1,0} \mu_2^{1,0}}{\mu_2^{1,0} - \mu_1^{1,0}} \left\{ -\left[\frac{\tau_P}{\mu_1^{1,0}} + (\mu_1^{1,0})^{-2} \right] e^{-\mu_1^{1,0}(\tau_P - \tau_B)} - \left[\frac{\tau_B}{\mu_1^{1,0}} + (\mu_1^{1,0})^{-2} \right] + \right. \\
& + \left[\frac{\tau_P}{\mu_2^{1,0}} + (\mu_2^{1,0})^{-2} \right] e^{-\mu_2^{1,0}(\tau_P - \tau_B)} - \left[\frac{\tau_B}{\mu_2^{1,0}} + (\mu_2^{1,0})^{-2} \right] \left. \right\} + \\
& + \frac{1}{(\lambda_2^{1,2} - \lambda_1^{1,2})(\mu_2^{1,0} - \mu_1^{1,0})} \left[\mu_2^{1,0} \lambda_2^{1,2} (\tau_P + (\mu_1^{1,0} + \lambda_2^{1,2})^{-1}) e^{-\mu_1^{1,0}(\tau_P - \tau_B)} - \right. \\
& - \mu_2^{1,0} \lambda_1^{1,2} (\tau_P + (\mu_1^{1,0} + \lambda_2^{1,2})^{-1}) e^{-\mu_1^{1,0}(\tau_P - \tau_B)} - \\
& - \mu_1^{1,0} \lambda_2^{1,2} (\tau_P + (\mu_2^{1,0} + \lambda_1^{1,2})^{-1}) e^{-\mu_2^{1,0}(\tau_P - \tau_B)} + \\
& + \mu_1^{1,0} \lambda_1^{1,2} (\tau_P + (\mu_2^{1,0} + \lambda_1^{1,2})^{-1}) e^{-\mu_2^{1,0}(\tau_P - \tau_B)} \left. \right], \quad \tau_B \leq \tau_P
\end{aligned} \tag{23}$$

$$\begin{aligned}
\overline{t_{\xi 1}} = & \frac{\mu_1^{1,0} \mu_2^{1,0}}{\mu_2^{1,0} - \mu_1^{1,0}} \left\{ -\left[\frac{\tau_P}{\mu_1^{1,0}} + (\mu_1^{1,0})^{-2} \right] e^{-\mu_1^{1,0}(\tau_B - \tau_P)} + \left[\frac{\tau_P}{\mu_1^{1,0}} + (\mu_1^{1,0})^{-2} \right] + \right. \\
& + \left[\frac{\tau_B}{\mu_2^{1,0}} + (\mu_2^{1,0})^{-2} \right] e^{-\mu_2^{1,0}(\tau_B - \tau_P)} - \left[\frac{\tau_P}{\mu_2^{1,0}} + (\mu_2^{1,0})^{-2} \right] \left. \right\} + \\
& + \frac{1}{(\lambda_2^{1,2} - \lambda_1^{1,2})(\mu_2^{1,0} - \mu_1^{1,0})} \left[\mu_2^{1,0} \lambda_2^{1,2} (\tau_P + (\mu_1^{1,0} + \lambda_2^{1,2})^{-1}) e^{-\mu_1^{1,0}(\tau_P - \tau_B)} - \right. \\
& - \mu_2^{1,0} \lambda_1^{1,2} (\tau_P + (\mu_1^{1,0} + \lambda_2^{1,2})^{-1}) e^{-\mu_1^{1,0}(\tau_P - \tau_B)} - \\
& - \mu_1^{1,0} \lambda_2^{1,2} (\tau_P + (\mu_2^{1,0} + \lambda_1^{1,2})^{-1}) e^{-\mu_2^{1,0}(\tau_P - \tau_B)} + \\
& + \mu_1^{1,0} \lambda_1^{1,2} (\tau_P + (\mu_2^{1,0} + \lambda_1^{1,2})^{-1}) e^{-\mu_2^{1,0}(\tau_P - \tau_B)} \left. \right], \quad \tau_B > \tau_P
\end{aligned} \tag{24}$$

Average time $\overline{\tau_i}$ when the line section remains in the sub-group of the states of readiness to work $E_p = \{S_0, S_i\}$ is defined by the equation which has the following form:

$$\overline{\tau_i} = \overline{t_{\xi i}} + \sum_{S_j \in E_p} P_{ij} \overline{\tau_j}, \quad S_i \in E_p \tag{25}$$

where: $\overline{\tau_i}$ - average time when the line section remains in E_p state sub-group till the first exit from the sub-group on the condition that at the initial time the section was in state S_i .

Time of remaining in the states of readiness to work $\tau_i (i = 0, 1)$:

$$\overline{\tau_0} = \overline{t_{\xi 0}} + P_{01} \overline{\tau_1} \tag{26}$$

$$\overline{\tau_1} = \overline{t_{\xi 1}} + P_{10} \overline{\tau_0} \tag{27}$$

Final $\overline{\tau_0}$:

$$\overline{\tau_0} = \frac{\overline{t_{\xi 0}} + P_{01} \overline{t_{\xi 1}}}{1 - P_{01} P_{10}} = \frac{\overline{t_{\xi 0}} + \overline{t_{\xi 1}}}{1 - P_{10}} \tag{28}$$

where: probability transition from state S_0 to S_1 - $P_{01} = 1$, τ_0 - time work to failure.

When times of remaining in specific work states are calculated, it is necessary to define the form of K_G readiness coefficient. For this purpose, it is necessary to use the so-called cyclical Markov chain which is the combination of the work states: $E_1 \rightarrow E_2 \rightarrow E_3 \rightarrow \dots E_n \rightarrow E_1$.

In discrete phase space the distribution parameters are recorded as:

$$\Pi = (\Pi_k, \quad k \in E) \tag{29}$$

$$\Pi_k = \frac{\rho_k \overline{t_{\xi k}}}{\overline{t_{\xi k}}} \tag{30}$$

$$\overline{t_{\xi k}} = \sum_{k=0}^2 \rho_k \overline{t_{\xi k}} \tag{31}$$

Where: $\overline{t_{\xi k}}$ - average time remain in state k .

System of algebraic equations which describe steady state:

$$\begin{cases} \rho_1 = \rho_0 + P_{01} \\ \rho_2 = \rho_1 + P_{12} \\ \rho_0 + \rho_1 + \rho_2 = 1 \\ \rho_0 = \rho_1 P_{10} + \rho_2 P_{20} \end{cases} \quad (32)$$

Entering vectors of the stationary distribution of the Markow chain:

$$\begin{aligned} \rho_0 &= \frac{1}{1 + P_{01} + P_{01} + P_{12}} \\ \rho_1 &= \frac{P_{01}}{1 + P_{01} + P_{01} P_{12}} \\ \rho_2 &= \frac{P_{01} P_{12}}{1 + P_{01} + P_{01} P_{12}} \end{aligned} \quad (33)$$

where: $P_{01}=1$.

Value $\overline{t_{\xi_k}}$ and parameters of the stationary distribution Π :

$$\begin{aligned} \overline{t_{\xi}} &= \frac{\overline{t_{\xi_0}} + P_{01} \overline{t_{\xi_1}} + P_{01} P_{12} \overline{t_{\xi_2}}}{1 + P_{01} + P_{01} P_{12}} \\ \Pi_0 &= \frac{\overline{t_{\xi_0}}}{\overline{t_{\xi_0}} + P_{01} \overline{t_{\xi_1}} + P_{01} P_{12} \overline{t_{\xi_2}}} \\ \Pi_1 &= \frac{P_{01} \overline{t_{\xi_1}}}{\overline{t_{\xi_0}} + P_{01} \overline{t_{\xi_1}} + P_{01} P_{12} \overline{t_{\xi_2}}} \\ \Pi_2 &= \frac{P_{01} P_{12} \overline{t_{\xi_2}}}{\overline{t_{\xi_0}} + P_{01} \overline{t_{\xi_1}} + P_{01} P_{12} \overline{t_{\xi_2}}} \end{aligned} \quad (34)$$

Using the Markow chains we obtain the expression which defines K_G readiness coefficient for the line section including a reliable dispenser:

$$K_G = \Pi_0 + \Pi_1 = \frac{\overline{t_{\xi_0}} + P_{01} \overline{t_{\xi_1}}}{\overline{t_{\xi_0}} + P_{01} \overline{t_{\xi_1}} + P_{01} P_{12} \overline{t_{\xi_2}}} \quad (35)$$

Description of the reactions between the assembly cell (AC) and H dispenser in case of which failures occur, was described as the functioning of the system which is being returned to operation and corresponding to the superpositions of two independent and alternative processes of returning to operation (Figure 2, Table 2).

Figure 2: Diagram of the reactions between AC and H dispenser in case of which failures occur

Table 2: Identification concerning marking of operation states

State	Assembly cell	Dispenser
S_0	Efficient	Efficient
S_1	Inefficient	Efficient
S_2	Inefficient	Blocked
S_3	<i>Inefficient</i>	<i>Inefficient</i>

K_G readiness coefficient which has to be found is defined as:

$$K_G = K_G^{KM} K_G^H \quad (36)$$

The system presented on Figure 2 can be replaced with the two-state system – Figure 3.

Figure 3: Two-state system

The readiness coefficient K_G of the assembly cell (AC) is defined according to (35), whereas the readiness coefficient of H dispenser:

$$K_G^H = \frac{\int_0^{\infty} (1 - F_{03}(t)) dt}{\int_0^{\infty} (1 - F_{03}(t)) dt + \int_0^{\infty} (1 - F_{30}(t)) dt} = \frac{M(t_{\xi 3})}{M(t_{\xi 3}) + M(t_{\eta 3})} \quad (37)$$

where: $t_{\xi 3}$ - time of transition from state 0 to state 3, $t_{\eta 3}$ - time of transition from state 3 to state 0.

Final value of the dispenser readiness coefficient has the form:

$$K_G^H = \frac{(\lambda_1^{0,3} + \lambda_2^{0,3} + \lambda_1^{0,3} \lambda_2^{0,3} \tau_p) \mu_1^{3,0} \mu_2^{3,0}}{\mu_1^{3,0} \mu_2^{3,0} (\lambda_1^{0,3} + \lambda_2^{0,3}) + \lambda_1^{0,3} \lambda_2^{0,3} (\mu_1^{3,0} + \mu_2^{3,0}) + \mu_1^{3,0} \mu_2^{3,0} \lambda_1^{0,3} \lambda_2^{0,3} (\tau_B + \tau_p)} \quad (38)$$

The readiness coefficient K_G^{Σ} for the whole AL can be calculated using the general formula:

$$K_G^{\Sigma} = \prod_{k=1}^n K_{Gk} \quad (39)$$

where: n - number of the assembly cells (AC) connected in series in the assembly line (AL).

Unit capacity of the assembly line (AL) as a whole:

$$Q = \frac{1}{t_{\max}} K_G^{\Sigma} \quad \overline{t_{\max}} = \min(\overline{t_i}, i = 1, n) \quad (40)$$

Where: $t_{\max}$ - time of performing an assembling operation in the cell having the minimum capacity,
 i - cell number.

4. CONCLUSIONS

Specific characteristics of the tasks carried out in the assembly cells result in diversified structure of the cells with regard to quantity and type of equipment applied. Complexity and diversity of the tasks necessitate making general record concerning the relationships in the assembly cell (AC) while maintaining the possibility of the record application with regard to a specific case analysed. Also, the work process in the assembly cell must not be analysed separately from the cell environment. Even if the assembly cell is an autonomous unit, its working time is influenced by the flow of the attachment workpieces. Series of the assembly line (AL) and the assembly cell (AC) models, which were constructed with the application of the simplifications aimed at assuring their common usage, did not include the influence of the consequences caused by simultaneous functioning of AC and workpiece dispenser H (as well as other components). Such modelling, although already applied in subject literature, is based on the application of the exponential distribution and, in some cases, such modelling during verification on the actual model does not correspond fully to the model. In this article the author eliminates the above presented disadvantages while maintaining the general methodology concerning the construction of AL models.

Development of the assembly systems theory is based on the reliability theory, the mass servicing theory and the queue networks theory. However, in many cases the research apparatus cannot be utilized in a comprehensive way directly for modelling and optimization of AL parameters due to the following reasons:

1. Mathematical models are required for solving the problems connected with parametric optimization which occur during designing and operation of AL. Such mathematic models enable evaluating not only the reliability and capacity of the line but also the size of intro-production reserves. Currently existing models do not provide the answers for the questions concerning production reserves in the dispensers.
2. In the research works concerning modelling of the multi-phase systems with the feeders (dispensers) the approximation of actual distribution laws is not satisfactorily utilized as a result of the mathematic description complexity. It has been proved in many research works that the application of the exponential laws which describe functioning of the actual systems with failures enables obtaining low accuracy.

In the above article, by limiting the mathematical description to one assembly cell connected with the workpiece dispenser, the considerations concerning the time redundancy are partially eliminated and the preferred material redundancy, in the form of the workpiece reserves in the dispensers, is achieved thanks to the reserve of the assembly cell productivity. Additionally, it has been assumed in the article that all random variables have the Erlang distribution of 2 grade. Two cases were presented in the model: when the workpiece dispenser fails and when the dispenser has absolute reliability. Such a division is justified by the industrial practice because it is often assumed that the vibrating dispensers or robocars are the failure-free devices.

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