A MODEL FOR SIMULATING THE SPRING FLOW AND GROUNDWATER LEVEL IN A DEPLETED KARST AQUIFER - A CASE STUDY OF THE JINCI SPRING

Yuqiao Long^{1,2}, Tingting Cui^{1,2}, Wei Li^{1,2}, Zhongping Yang³, Yongwei Gai⁴

1. Nanjing Hydraulic Research Institute, Nanjing, China;

2. State Key Laboratory of Hydrology-Water Resources and Hydraulic Engineering, Nanjing, China;

3. College of civil engineering, Chongqing University, Chongqing, China;

4. Water Resources Service Center of Jiangsu Province, Nanjing, China

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ABSTRACT

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A reservoir model is developed to characterize the functioning of the karst aquifer of the Jinci Spring. It is based on the relationship of the groundwater storage deficit and the groundwater level. The relationship is derived from the monitoring data of the aquifer. The model is verified with the monitoring data (1981-2006). The results show that the reservoir model can describe the character of karst development and the heterogeneity in vertical direction. The model takes account of the artificial recharge, pumping and coal mine drainage, and can simulate the spring discharge, karst groundwater level and the storage deficit accurately at an annual time scale. The model is suitable for macroscopic groundwater management.

1. INTRODUCTION

At present, China has 164 groundwater overexploitation zones of which the total area is about 181291 km² [1]. The karst groundwater overexploitation zones with a total area about 7393 km² mainly locate in the Liao River Basin (1941 km²), the Yellow River Basin (4216 km²) and the Huai River Basin (1144 km²) [1]. Long term overexploitation leads to the depletion of the karst aquifers in some parts of Shanxi Province and Shandong Province (they locate in the Yellow River Basin), where some famous springs (Jinci Spring, Lan Spring, Nangziguang Spring and Baotu Spring) dry up. It is concerned about the time span and the exploitation schemes to recover the groundwater level and spring flows in these districts, so the model for karst aquifer management is needed to answer these questions.

Long term depletion-type extraction of groundwater in a karst aquifer provides the reservoir model required data with which the reservoir model can resemble the karst aquifer in a better way. Under a depletion status, the discharge of the karst groundwater exceeds its recharge, the storage is consumed continuously, and the karst groundwater level drops gradually. So the storage and the relationship of the storage and the groundwater level can be derived from the monitoring data. The change of storage reflects the change of the volume of the karst voids which can yield water, and exemplifies the heterogeneity of the karst aquifer in the vertical direction. A better similarity between the karst aquifer and the reservoir model can be obtained, if we take the groundwater storage as the reservoir volume.

This paper is aimed at predicting the recovery of spring flows and karst groundwater level in the Jinci Spring basin. A reservoir model is built to characterize the functioning of the karst aquifer of the Jinci Spring based on the data of recharge, discharge and groundwater level during the long term depletion-type extraction. The model has three inlets (rainfall infiltration, river seepage and artificial recharge) and five outlets (well withdrawal, coal mine drainage, Jinci Spring, Xuan spring and lateral discharge). The discharge, the recharge and the deficit (or storage variation) compose a water balance equation which is a nonlinear equation with a single variable and is solved by the bisection method. The model is verified with the monitoring data.

2. Method

2.1 Reservoir model and water balance equation

A reservoir model is built to characterize the functioning of the karst aquifer of the Jinci Spring. The model has three inlets (rainfall infiltration, river seepage and artificial recharge) and five outlets (well withdrawal, coal mine drainage, Jinci Spring, Xuan spring and lateral discharge) (Figure 1). The artificial recharge, the well withdrawal and the coal mine drainage can be controlled by exploitation schemes, while the spring flow and the lateral discharge are the unknown variables to be evaluated by the model.

The volume of reservoir is defined as the volume of groundwater yielded from the karst aquifer between the highest groundwater level and the lowest groundwater level during the monitoring time span. The empty volume of the reservoir at the time point t is the deficit at the same time point, noted as Q^t . The groundwater level is highest and the deficit is zero at the beginning of the monitoring time span, the groundwater level is lowest and the deficit reaches the maximum (the reservoir volume) at the end of the monitoring time span, when a karst aquifer is depleting. The difference of the discharge and recharge at the time point t is the variation of deficit at the time point t , noted as ΔQ^t . We have

$$Q^t = \sum_{i=1}^t \Delta Q^i \quad (1)$$

The ΔQ^t can be expressed as the difference between deficit at the time point t and the deficit at the time point $t-1$. Hence, we have

$$\Delta Q^t = Q^t - Q^{t-1} \quad (2)$$

The water balance equation of karst aquifer of Jinci Spring is

$$Q_{i1}^t + Q_{i2}^t + Q_{i3}^t - (Q_{o1}^t + Q_{o2}^t + Q_{o3}^t + Q_{o4}^t + Q_{o5}^t) = \Delta Q^t \quad (3)$$

Where Q_{i1}^t , Q_{i2}^t and Q_{i3}^t represent the rainfall infiltration, the Fen River leakage and the artificial recharge at the time point t , Q_{o1}^t , Q_{o2}^t , Q_{o3}^t , Q_{o4}^t and Q_{o5}^t are the well withdrawal, coal mine drainage, Jinci Spring flow, Xuan Spring flow and lateral discharge at the time point t , and ΔQ^t is the

variation of deficit at the time point t .

Figure 1 General architecture of the reservoir model

2.2 Deficit and karst groundwater level

When a karst aquifer is depleting, h^t , the groundwater level at the time point t , equals the difference of the h^{t-1} , the groundwater level at time point $t-1$, and Δh^t , the groundwater level variation at the time point t , i.e. $h^t = h^{t-1} - \Delta h^t$. The quotient of ΔQ^t divided by Δh^t is the average area of the karst voids yielding water between h^t and h^{t-1} , noted as A^{t-1} .

The variation of deficit could be rewritten as $\Delta Q^t = (h^{t-1} - h^t) \cdot A^{t-1}$. Here, we suppose that the variation of deficit between each two adjacent monitoring time point could be described as a linear function of the karst groundwater level. So the ΔQ^t could be written as $\Delta Q^t \approx (h^{t-1} - h^t) \cdot A^{t-1}$, when h^t falls in $[h^{t-1}, h^t]$.

According to equation (1), Q^t , the deficit at the time point j ($j \in [t-1, t]$), could be expressed as $Q^t \approx (h^{t-1} - h^t) \cdot A^{t-1} + (h^{t-1} - h^t) \cdot A^{t-1}$. Where h^j is the independent variable, Q^j is the dependent variable, and the others are constants which could be represented by C . So the deficit between each two adjacent monitoring time point would be expressed as a linear function of the karst groundwater level approximately (Figure 2).

$$Q^j \approx f(h^j) = -A^{t-1} \cdot h^j + C, j \in [t-1, t] \quad (4)$$

Figure 2 Deficit and average area of void karst voids in vertical direction

2.3 Spring flow and karst groundwater level

Predicting the spring flow is difficult, because of the complex flow in karst conduits, heterogeneity of karst aquifers, and the turbulent spring flow. Hydraulic and statistical methods are usually used to predict spring flows^[2]. Sepúlveda (2009)^[3] studied the effects of the hydraulic methods in spring flow predicting. When they are used in karst aquifer, several of the assumptions of the hydraulic equations are violated, the parameters in the equations lose some of their physical basis^[3]. Other hydraulic methods need detailed information about the karst aquifer (e.g. boundary conditions, porosity, hydraulic conductivity, etc.). However, these

information are unavailable for such a large karst groundwater basin^[4]. Therefore, these methods are not suitable for the well-developed karst aquifer.

In this work, we choose statistical methods to describe the spring flows. The MLR and ANNs are the common statistical methods, but they are lack of the physical bases. A nonlinear regression is used to model the spring flow in the reservoir model. The karst groundwater level is taken as the only direct determinant of the spring flows, because the spring flows have good correlations with the groundwater level. In fact, the groundwater level control the spring flow physically, that provides the statistical method some physical bases, but the method is still an empirical one. Other factors, such as rainfall infiltration, well withdrawal, and coal mine drainage, would influence the spring flow indirectly by affecting the groundwater level. The effects of these factors would be embodied in the iteration equations (9~11).

Gaussian functions are employed to build the nonlinear regression models for the Jinci Spring and the Xuan Spring (equation 5, 6, and Table1). And the nonlinear regression models have good agreement with the monitoring data (Figure 3). It shows that Q_{03}^t and Q_{04}^t are monotone increasing with h^t in [802.32, 804.00], and [795.94, 804.00], respectively.

$$Q_{03}^t = a_1 \cdot \exp(-((h^t - b_1)/c_1)^2) + a_2 \cdot \exp(-((h^t - b_2)/c_2)^2) \quad (5)$$

$$Q_{04}^t = a_3 \cdot \exp(-((h^t - b_3)/c_3)^2) + a_4 \cdot \exp(-((h^t - b_4)/c_4)^2) \quad (6)$$

Table 1 Parameters of the nonlinear regression

Springs	Id	a	b	c
Jinci Springs	1	5.116	805.3	0.7315
	2	0.5	803.7	0.6808
Xuan Springs	3	0.413	804	0.6128
	4	0.255	825.6	25.09

Figure 3 Rating curves of the Jinci Spring and the Xuan Spring

2.4 Lateral discharge and groundwater level

The lateral discharge is calculated with the Darcy's Law, i.e.

$$Q_{04}^t = \sum_{i=1}^n q_i^t = \sum_{i=1}^n T_i \cdot L_i \cdot I_i^t \quad (7)$$

Where n is the number of the cross-sections, q_i^t is the lateral discharge perpendicular to the cross-section i at time point t . q_i^t is expressed as $q_i^t = T_i \cdot L_i \cdot I_i^t$. Where T_i is the average transmissivity of porous aquifer on the cross-section i , L_i is the length of the cross-section i , and I_i^t is the average hydraulic gradient on the cross-section i at the time point t . I_i^t is calculated by $I_i^t = (h^t - h_0^t)/L_0^t$. Where h^t is the groundwater level in karst aquifer, h_0^t is the groundwater level in porous media (we use the same value on each cross-section), L_0^t is the length between h^t and h_0^t along the groundwater flow direction on each cross-section. When we rewrite equation (7) as equation (8), we could find that Q_{04}^t is a linear function of h^t , and monotone increasing with h^t .

$$Q_{04}^t = (h^t - h_0^t) \cdot \sum_{i=1}^n T_i \cdot L_i \cdot I_i^t \quad (8)$$

Seven cross-sections along the southeast boundary of the Jinci Spring basin is used to evaluate the later discharge. The cross-section C-D is coincident with the cross-section C'-D' in a plain view. The C-D is a cross-section of the first and second confine aquifers, while the C'-D' corresponds to the third and fourth confined aquifers. So are the cross-section D-E and D'-E'. T_i , L_i , and I_i are shown in Table 2. h_0 is a variable which could be decided with the help of a porous media model.

Table 2 Parameters of the lateral discharges

Cross-sections	L(m)	T(m ² /d)	L ⁰ (m)
A-B	6000.00	69.33	517.24
B-C	9000.00	8.64	500.00
C-D	7500.00	115.20	1500.00
C'-D'	7500.00	122.00	1363.64
D-E	4500.00	76.80	1500.00
D'-E'	4500.00	180.00	1875.00
E-F	8000.00	216.00	1500.00

2.5 Solving water balance equation

We substitute equation (5), (6) and (8) into equation (3), in order to obtain the karst groundwater level h^t at the time point t . Then we have iteration equation (9), (10) and (11).

When h^t is in [781.12, 795.94], both of the Jinci Spring and Xuan Spring dry up. No spring flow should be considered. Then we have

$$Q'_{i1} + Q'_{i2} + Q'_{i3} - Q'_{o1} - Q'_{o2} - (h' - h_0) \cdot \sum_{i=1}^n T_i \cdot L_i / L_i^0 \approx f(h^{t-1}) - f(h') \quad (9)$$

When h^t is in [795.94, 802.32], the Xuan Spring flows out. Then we have

$$Q'_{i1} + Q'_{i2} + Q'_{i3} - Q'_{o1} - Q'_{o2} - a_3 \cdot \exp\left(-\left((h' - b_3)/c_3\right)^2\right) - a_4 \cdot \exp\left(-\left((h' - b_4)/c_4\right)^2\right) - (h' - h_0) \cdot \sum_{i=1}^n T_i \cdot L_i / L_i^0 \approx f(h^{t-1}) - f(h') \quad (10)$$

When h^t is in [802.32, 804.00], both the Jinci Spring and the Xuan Spring flow out. Then we have

$$Q'_{i1} + Q'_{i2} + Q'_{i3} - Q'_{o1} - Q'_{o2} - a_1 \cdot \exp\left(-\left((h' - b_1)/c_1\right)^2\right) - a_2 \cdot \exp\left(-\left((h' - b_2)/c_2\right)^2\right) - a_3 \cdot \exp\left(-\left((h' - b_3)/c_3\right)^2\right) - a_4 \cdot \exp\left(-\left((h' - b_4)/c_4\right)^2\right) - (h' - h_0) \cdot \sum_{i=1}^n T_i \cdot L_i / L_i^0 \approx f(h^{t-1}) - f(h') \quad (11)$$

In equation (9), (10) and (11), Q'_{i1} and Q'_{i2} could be determined by the future rainfall data provided by the National Climate Center (NCC) and Pearson III density probability plot, respectively. Q'_{i3} , Q'_{o1} and Q'_{o2} could be determined by the exploitation schemes. We have not a verified porous model for porous groundwater in the Taiyuan basin, so we suppose that the h_0 is a constant. h^{t-1} is calculated in $t-1$ iteration. Therefore h^t is the only unknown quantity in equation (9), (10) and (11).

In order to solve equation (9), (10) and (11), we rewrite them as

$$g(h') = Q'_{i1} + Q'_{i2} + Q'_{i3} - Q'_{o1} - Q'_{o2} - (h' - h_0) \cdot \sum_{i=1}^n T_i \cdot L_i / L_i^0 - f(h^{t-1}) + f(h'), h' \in [781.12, 795.94] \quad (12)$$

$$g(h') = Q'_{i1} + Q'_{i2} + Q'_{i3} - Q'_{o1} - Q'_{o2} - a_3 \cdot \exp\left(-\left((h' - b_3)/c_3\right)^2\right) - a_4 \cdot \exp\left(-\left((h' - b_4)/c_4\right)^2\right) - (h' - h_0) \cdot \sum_{i=1}^n T_i \cdot L_i / L_i^0 - f(h^{t-1}) + f(h'), h' \in [795.94, 802.32] \quad (13)$$

$$g(h') = Q'_{i1} + Q'_{i2} + Q'_{i3} - Q'_{o1} - Q'_{o2} - a_1 \cdot \exp\left(-\left((h' - b_1)/c_1\right)^2\right) - a_2 \cdot \exp\left(-\left((h' - b_2)/c_2\right)^2\right) - a_3 \cdot \exp\left(-\left((h' - b_3)/c_3\right)^2\right) - a_4 \cdot \exp\left(-\left((h' - b_4)/c_4\right)^2\right) - (h' - h_0) \cdot \sum_{i=1}^n T_i \cdot L_i / L_i^0 - f(h^{t-1}) + f(h'), h' \in [802.32, 804.00] \quad (14)$$

Then the original problems are transferred into finding zero point of equation (12), (13) and (14). The bisection method[5] could be applied to these problems, because equation (13) and (14) are nonlinear equations in $[h^{t-1}, \sigma, h^{t-1}, \sigma]$. The radius of neighborhood of h^t , σ , is 5, because the annual variation of karst groundwater level of the Jinci Spring basin is within 5 m.

2.6 Verification of the model

The model is verified with rainfall infiltration, Fen River leakage, well withdrawal, coal mine drainage, spring flow and lateral discharge, karst groundwater level and deficit from 1981 to 2006. Only modeling results from 1982 to 2006 are analyzed, because 1981 is the initial time point. Data used to verify the model are obtained from the Taiyuan Hydraulic Research Institute.

3. Discussion

3.1 Error

Error is used to evaluate the model. Here we define error, e , as $e = |u - u'|$, because the error is so small that semi-log coordinates are needed to make the error plot clearly. Where u is the calculated by the model, u' is measured data. Errors of groundwater level, deficit, spring flow, and lateral discharge are used to analyze the accuracy of the reservoir model. Satisfactory errors are got by the model using the verification data. All errors are less than 0.5, and most of them is smaller than 0.01 (Figure 4). The errors have a trend that they go up with the increment of time steps. These errors vary in a same tendency roughly.

Karst groundwater level increases with the increment of time steps, and reaches the maximum (0.4) at the end of the time span (Figure 4). The model can only find the approximate solution with bisection methods, so errors would occur at each time step and affect the results at the next time step. These influences could be accumulated resulting in the trend that the errors go up with the increment of time steps.

Spring flows make a contribution to the errors. The nonlinear regression models of the two springs have errors inherently, so these errors are induced in the solving process and influence the modeling results. That explains that the peaks of spring flow error curve correspond to the peaks of the groundwater level error curve and the deficit error curve (Figure 4). The error of deficit corresponds to the error of lateral discharge. Deficit is obtained by equation (4) after karst groundwater level is obtained from equation (12), (13) and (14). The deficit error curve has the same variation with karst groundwater level error curve, because the deficit is a linear function of karst groundwater level. Moreover, the lateral discharge is a linear function of karst groundwater level (equation (8)), so the lateral discharge error has the same tendency with the error of karst groundwater level.

Figure 4 Errors of groundwater level, spring flows and lateral discharge

3.2 Water balance

The $g(h')$ does not only show the solving effect, but reflects the mass conservation. The closer $g(h')$ approximates to zero, the better the solving effect is. Moreover, the $g(h')$ getting close to zero means that the deficit is close to the difference of recharge and discharge, so there is no water being created or consumed in the reservoir model. In the verification and prediction periods, $g(h')$ is less than 10^{-3} , so the model follows the mass conservation approximately

3.3 Heterogeneity of the kast aquifer

The deficit and the average area of the karst voids show the heterogeneity of the kast aquifer to some extent. Equation (2) shows that the deficit

corresponding to Δh^t equals to the volume of water yield by karst voids. The proportion of deficit to the volume of karst aquifer corresponding to Δh^t approximates the effective porosity which can be expressed as a proportion of the average area of the karst voids to the average area of the karst aquifer corresponding to Δh^t approximately. So the deficit or the average area of the karst voids could show the vertical heterogeneity of the karst aquifer. And that makes the reservoir model characterized as a lumped model be equipped with some character of distributed model. The karst becomes undeveloped gradually with the increment of the depth (Figure 2). The developed extent could be divided into 5 degrees (a~e in Figure 2). The degree a is inaccurate, because the average area of karst voids exceeds the area of the Jinci Spring basin (about 2430km²).

4. Conclusion

This paper is aimed at building a reservoir model is built to characterize the functioning of the karst aquifer of the Jinci Spring based on the data of recharge, discharge and groundwater level during the long term depletion-type extraction. The model is verified with rainfall infiltration, Fen River leakage, well withdrawal, coal mine drainage, spring flow and lateral discharge, karst groundwater level and deficit from 1981 to 2006. The vital step in this work is building the relationship between deficit and the karst groundwater level. In the step, the vertical heterogeneity of karst aquifer is detected by the deficit increment which varies with the karst groundwater level. The volume of the reservoir model based on the deficit has similarity with the volume of the voids in the karst aquifer. That endows the lumped reservoir model characterized with some characters of distributed model in vertical direction, so the model could give satisfactory results within the measure data. However, if the karst groundwater level exceeds the measure data, the model can only give a result by extrapolation which could not be accurate.

The reservoir model could be used to model the recovery of spring flow, karst groundwater level and deficit, at an annual time scale. Well withdrawal, coal mine drainage, and artificial recharge are set in the model, so it could be used in macroscopic groundwater resources management. The modeling results at a daily time scale are based on more detail information of the epikarstic zone and the physical process of the Fen River leakage. The information is not available at present, so it is not tested that the performance of the model at a smaller time scale.

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