ISBN: 978-1-948012-17-1

Ethics and Information Technology (ETIT)

DOI: <http://doi.org/10.26480/etit.02.2020.90.95>

CrossMark

A TWO STAGE OPTIMAL PID CONTROLLER TUNING STABILIZE HYDRODYNAMIC BEARING WITHOUT ROTOR INTERNAL DAMPING

Aayush Bhadani^a, Abhro Mukherjee^b, Patra Kusum Misra^b, Sayan Mondal^b, Abhijit Dey^c, Aditi Tokdar^c^a Department of Electrical Engineering, Shiv Nadar University, Greater Noida, India^b Department of ECE, Techno India University, Kolkata, India^c Department of ECE, Techno India Main Salt Lake Campus, Kolkata, India*Corresponding Author Email: markbishop2385@gmail.com

This is an open access article distributed under the Creative Commons Attribution License CC BY 4.0, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

ARTICLE DETAILS

Article History:

Received 25 October 2020

Accepted 26 November 2020

Available online 03 December 2020

ABSTRACT

Rotating machines are one of the most important and widely used machineries in modern engineering world. Now-a-days modern rotating equipments have a number of complicated accessories attached with it and they are also made extremely flexible. They are also required to run at higher speed, much higher than their first critical speed. Fluid-film bearings supporting the rotating shafts play an important role in the dynamic behaviour of the rotors because of their stiffness and damping properties. Hydrodynamic bearings are one of the most widely used bearings to support the rotating shafts. Rotor-bearing systems exhibit a wide variety of phenomena pertaining to its operations which if not properly addressed and rectified may lead to catastrophic failure of the system. One such rotor-bearing phenomenon is the self-acting rotor vibrations induced by the hydrodynamic bearings and popularly known as oil-whirl and oil-whip which have severe repercussion on the functioning of the rotors. Hydrodynamic bearing and integrated model tend to be unstable due to non-conservative nature of hydrodynamic forces and stiffness coefficients. The purpose of this work is to stabilize small mini-micro sized rotors, hydrodynamics bearing model and integrated model through certain elegant control strategies which can be practically deployable. Using the response function in the LQR and robust control algorithm, we are stabilizing the system. The final control structure is same as that of PID but here we are using the response function which is directly proportional to the velocity. The second stage tuning level is achieved by using the elements of Fractional control, but the realization of the final controller is like a normal PID easily acceptable for industrial environment.

© 2012 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of Global Science and Technology Forum Pte Ltd

KEYWORDS

internal damping, stiffness coefficients, oil-whirl, oil-whip, hydrodynamic bearing, optimal PID control.

1. INTRODUCTION

Rotor dynamics in modern day turbo machinery problems must be undertaken with absolutely greater depth as because very frequently operations of rotating machinery take place much above the sub-synchronous or critical speed (bou) (Newkirk et al., 1925; Salm et al., 1984; Sahimkaya et al., 1985; Schweiter, 1975). It is quite obvious that the rotating machines and its role and applications in industry affect directly the basic economic issues and deals very closely with the human life (Alfred et al., 1965; Knopse et al. 1997, Vitecek et al., 2008). Therefore, safe operation of this rotating machinery is an absolute necessity and dynamically to stabilize such machines is an absolute must for safe operations in industries (Muszynksa, 1986; Bentley et al., 1989; Tondl et al., 1991).

The dynamical systems subjected to such displacement dependent forces which cannot be easily be derived from slope or gradient of any potential function may become highly unstable. These self-induced instability is very common in tope's and pendulum like system (Or, 1994; Kirillov,

2007; Bou-Rabee et al., 2008; Samantaray et al., 2008). The shafts may draw energy from motor drives or energy from inertial parts causing instability. Some common examples of such engineering systems could be elastic structures subjected to aerodynamic forces, dissipative forces in couplings, hydrodynamic forces due to bearing supporting the rotor (Mukherjee et al., 1984; Mukherjee et al., 1994; Aurich et al., 2009; Glebov et al., 2005; Glebov et al., 2007).

This work is devoted to design a completely novel approach for well-tuned bi-stage 2-degree PID type controller using blends of optimal control in the initial stage and then improvising the controller further using the elements of fractional order (Burrows et al., 1989; Dimitri et al., 2010). This special type of tuning in two subsequent stages allows the designer to reach better tunable gains (Botte et al., 2002; Das et al., 2011; Bayat et al., 2009; Bayat et al., 2010). The beauty of the designed controller is that formidable damping levels are achieved by a bi-stage tuning technique using the elements of fractional order control, the actual representation of the controller still remains integer type like normal PID controller easily deployable in a real time processor unlike a conventional fractional order controller which imposes real time difficulties in practical deployment in

Quick Response Code	This paper was presented at	Access this article online	
	International Conference on Contemporary Issues in Computing (ICIC-2020) - Virtual IETE Sector V, Salt Lake, Kolkata From 25th-26th July 2020	Website: www.intelcomp-design.com	DOI: 10.26480/etit.02.2020.90.95

a chip due to programming complexities. This work also discusses the nature of non-potential force fields and how it is self-induced dynamics destabilize the shaft (Fittro et al., 2002; Gtorst, 2004; Doyle, 1985).

Our work by proposing embedded smart structure in rotating frame with attached piezo type actuator and velocity sensors and using well-tuned bi-stage 2-degree of freedom like PID controller could stabilize and control the shaft preferably of any size and description (Scho'nhoff et al., 2002; Scho'nhoff et al., 2003; Muszynska, 1986). Moreover, the novelty of the proposed controller differs from other fractional order controller as in conventional tuning of fractional order controller involves complex mathematics and yet even if the control gain parameters are achieved by programming the FOPID controller but then deploying it in a embedded chip always remains as engineer's nightmare (santos et al., 193; Li et al., 2001; Scho'nhoff et al., 2002; Zhang et al., 2002). Our improvised tuning of the controller employs all the useful benefits of a fractional controller and yet the real time implementation remains as simple and effective like any other integer like PID structures.

2. GENERAL DYNAMICS AND MODELLING OF JOURNAL BEARING WITHOUT ROTOR MATERIAL DAMPING

2.1 Dynamics of Short Hydro Dynamic Bearing

The global mass conservation equation in thin film flow is

$$\frac{\partial(\rho h)}{\partial t} + \frac{\partial M_x}{\partial x} + \frac{\partial M_z}{\partial z} = 0 \quad (1)$$

Or in terms of the mean flow velocities:

$$\frac{\partial}{\partial t} \{\rho h\} + \frac{\partial}{\partial x} \{\rho h \bar{V}_x\} + \frac{\partial}{\partial z} \{\rho h \bar{V}_z\} = 0 \quad (2)$$

The journal spins with angular speed (Ω). The journal center, denoted by O_j , may also perform translational motion within the bearing clearance. The bearing or housing is stationary (not moving) in most applications. Where clearance $C=R_B-R_j$

Figure 1: Schematic representation of cylindrical journal bearing with fixed (X, Y) coordinate system

Then, the Reynolds equation describing the flow in the journal bearing becomes:

$$\frac{1}{R^2} \frac{\partial}{\partial \theta} \left\{ \frac{\rho h^3}{12\mu} \frac{\partial P}{\partial \theta} \right\} + \frac{\partial}{\partial z} \left\{ \frac{\rho h^3}{12\mu} \frac{\partial P}{\partial z} \right\} = \frac{\partial}{\partial t} \{\rho h\} + \frac{\Omega}{2} \frac{\partial}{\partial \theta} \{\rho h\} \quad (3)$$

In the flow domain $\{0 \leq \theta \leq 2\pi, -1/2L \leq z \leq 1/2L\}$, where $h(\theta, z, t)$ is the film thickness, L is the bearing axial length and $\Omega = \Omega R_j$ is the journal surface speed.

Under static conditions, the reaction force F balances the applied external force W . Under dynamic load conditions, when the journal displaces in time, equations of motion that include the journal mass times its acceleration need be satisfied. The fluid reaction force can be written in terms of its components in either (X, Y) or (r, t) axes, i.e.

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{F_r^2 + F_t^2} \quad (4)$$

With reference to the (r, t) coordinate system, the radial and tangential components of the fluid film reaction force are

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = \int_{-L/2}^{L/2} \int_0^{2\pi} P(\theta, z, t) \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} R \cdot d\theta dz \quad (5)$$

With reference to the fixed (X, Y) system, the vertical and horizontal components of the (same) fluid film force are

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \int_{-L/2}^{L/2} \int_0^{2\pi} P(\theta, z, t) \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} R d\theta dz \quad (6)$$

The relationship between the components of the fluid film force in both coordinate systems is given by the linear transformation

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} F_r \\ F_t \end{bmatrix} \quad (7)$$

For small amplitude journal motions about the equilibrium position (e_0, ϕ_0) , the equation of motion of a rigid rotor supported on linear fluid bearing are:

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \Delta \ddot{X} \\ \Delta \ddot{Y} \end{bmatrix} + \begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix} \begin{bmatrix} \Delta \dot{X} \\ \Delta \dot{Y} \end{bmatrix} + \begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} = Mu\Omega^2 \begin{bmatrix} \sin\Omega t \\ \cos\Omega t \end{bmatrix} \quad (8)$$

3. A FRACTIONAL ORDER APPROACH FOR STABILIZING ROTORS WITH AND WITHOUT PARAMETRIC UNCERTAINTY

The control signal generated may be artificially given as the command parameter R_c . The role of R_c is to interact with the initial internal ramping 'Ri' of the rotor and modulate it through proper actuation system thus designed.

$$R_c = \alpha P + \beta \int_0^t P(\varepsilon) d\varepsilon + \sigma dP/dt \quad (9)$$

Some disadvantage of the proposed controller is that it is already a frustrating task to measure the actual shaft drawn power 'Ps'. The disadvantage of directly measuring or sensing 'Ps' is that certain amount of power will be used for running the additional load and also some power will be lost to the environment via external damping therefore to determine the amount of power which actually renders the shaft unstable is a tedious task. Instead of taking the shaft drawn power 'Ps' as the sensing element one may resort to the orbital speed of the rotor itself (Malwatkar et al., 2009; Saif et al., 1989; Fujii et al., 1982; Bradfield et al., 1987). This is a good design as because the orbital speed is proportional to the area of the orbit taken by the rotor which is indirectly proportional to the shaft drawn power 'Ps'. Therefore, the overall control structure could now be given as,

$$R'_c = (\alpha * V^2_{amp} + \beta * \int_0^t V^2_{amp} dt + \sigma * dV^2_{amp}/dt);$$

$$\text{where } V^2_{amp} = (\dot{X}_r * r/L)^2 + (\dot{Y}_r * r/L)^2 \quad (10)$$

To prevent this type of reverse whirl occurrence we have improvised the existing control by addition of a switch.

$$F_{orbes} = (\dot{X}_r^2 + \dot{Y}_r^2) * \eta; \quad (11)$$

3.1 LQR Design of the controller with Augmented State

The tracking error in the whirl orbit can be say taken as $e(t) = r(t) - y(t)$, where $r(t)$ and $y(t)$ are references and the output variables. The output equation can be then written as,

$$\begin{aligned} y(t) &= Hx(t), \\ e(t) &= Nr(t) - y(t), \\ e(t) &= Nr(t) - Hx(t), \end{aligned}$$

where $N \in R^{b \times b}$ and $H \in R^{b \times n}$ are the constant coefficient matrices.

Using the output equation $e(t) = Nr(t) - y(t)$ and let us try to only create augmentation just to realize a integral controller. Later the same algorithm could be generalized for a proportional and derivative type controller.

One may denote $e(t) = \dot{x}_{ref}(t)$, therefore the dynamics of the exogenous system then can be written as

$$\dot{X}_{ref}(t) = Nr - y = Nr - Hx, \quad (12)$$

Let us take a generalized system dynamic,

$$\begin{aligned} \dot{X} &= Ax + Bu, \\ x(t_0) &= x_0, \end{aligned}$$

Then the dynamic model before the state augmentation can be represented as,

$$\dot{x}(t) = Ax + Bu(t),$$

$$y = Hx, \text{ and } x_0(t_0) = x_0,$$

The first augmented state to realize the internal controller may be represented as,

$$\dot{x}_{ref}(t) = Nr - y = Nr - Hx,$$

That is,

$$\dot{X}_\Sigma = A_\Sigma X_\Sigma + B_\Sigma u + N_\Sigma r, \text{ where } X_\Sigma = \begin{bmatrix} x(t) \\ x_{ref}(t) \end{bmatrix} \in R^c \quad (13)$$

Then the augmented state vector will be,

$$A_\Sigma = \begin{bmatrix} A & 0 \\ -H & 0 \end{bmatrix} \in R^{c \times c},$$

$$B_\Sigma = \begin{bmatrix} B \\ 0 \end{bmatrix} \in R^{c \times m},$$

$$N_\Sigma = \begin{bmatrix} 0 \\ N \end{bmatrix} \in R^{c \times b}$$

The quadratic performance functional is given as,

$$J \left[\begin{matrix} x(\cdot) \\ x_{ref}(\cdot) \end{matrix} \right] = \frac{1}{2} \int_{t_0}^{t_f} \left(\begin{bmatrix} X \\ x_{ref} \end{bmatrix}^T Q \begin{bmatrix} X \\ x_{ref} \end{bmatrix} + U^T C_l U \right) dt \quad (14)$$

Here $Q \in R^{c \times c}$ is the positive definite constant coefficient matrix.

Making use of the quadratic performance functional, one obtains the hermitian function as,

$$H(X_\Sigma, u, r, \frac{\partial v}{\partial x_\Sigma}) = \frac{1}{2} (X_\Sigma^T Q X_\Sigma + U^T C_l U) + \left(\frac{\partial v}{\partial x_\Sigma} \right)^T \times A_\Sigma X_\Sigma + B_\Sigma u + N_\Sigma r \quad (15)$$

$$\frac{\partial H(x, u, r, \frac{\partial v}{\partial x_\Sigma})}{\partial u} = U^T C_l + \left(\frac{\partial v}{\partial x_\Sigma} \right)^T B_\Sigma \quad (16)$$

One may get the following form of optimal control algorithm

$$u = C_l^{-1} B_\Sigma^T \frac{\partial v(x_\Sigma)}{\partial x_\Sigma} = -C_l^{-1} \cdot \begin{bmatrix} B \\ 0 \end{bmatrix}^T \cdot \frac{\partial v \left(\begin{bmatrix} x \\ x_{ref} \end{bmatrix} \right)}{\partial \begin{bmatrix} x \\ x_{ref} \end{bmatrix}} \quad (17)$$

The solution of the Hermitian - Jacobi Bellman partial differential equation,

$$-\frac{\partial v}{\partial t} = \left\{ \frac{1}{2} x_\Sigma^T Q x_\Sigma + \left(\frac{\partial v}{\partial x_\Sigma} \right)^T A x_\Sigma - \frac{1}{2} \left(\frac{\partial v}{\partial x_\Sigma} \right)^T B x_\Sigma C_l^{-1} B_\Sigma^T \frac{\partial v}{\partial x_\Sigma} \right\} \quad (18)$$

This equation is satisfied by the quadratic return function, $V(x_\Sigma) = \frac{1}{2} x_\Sigma^T K(t) x_\Sigma$, where $K \in R^{c \times c}$ is the symmetric matrix. One finds the Riccati equation, $-\dot{K} = Q + A_\Sigma^T K + K A_\Sigma - K B_\Sigma C_l^{-1} B_\Sigma^T K$, $K(t_f) = K_f$

Which gives the unknown symmetric matrix 'K', the above riccati equation needs to be solved

$$u = -C_l^{-1} B_\Sigma^T K x_\Sigma = -C_l^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix}^T K \begin{bmatrix} x \\ x_{ref} \end{bmatrix} \quad (19)$$

From $\dot{x}_{ref}(t) = e(t)$ one has $x_{ref} = \int e \cdot dt$

Therefore, the optimal tuning parameter for the augmented state for the integral control is

$$u(t) = -C_l^{-1} B_\Sigma^T K x_\Sigma(t) = -C_l^{-1} \begin{bmatrix} B \\ 0 \end{bmatrix}^T K \left[\begin{matrix} x(t) \\ \int e(t) \cdot dt \end{matrix} \right] \quad (20)$$

Further to create the structure of a 2 degree of freedom PID structure enhanced or augmented states were artificially created as

$$\dot{x}_{ref1} = e \Rightarrow x_{ref1} = \int e \cdot dt \quad (21)$$

$$\dot{x}_{ref2} = \frac{de}{dt} \Rightarrow x_{ref2} = e \quad (22)$$

$$\dot{x}_{ref3} = \frac{d^2 e}{dt^2} \Rightarrow x_{ref3} = \frac{de}{dt} \quad (23)$$

The error 'e' can be considered as the error in the orbital speed. One may define the orbital speed with a special function $V_{am}^2 = (\dot{x}_r^2 + \dot{y}_r^2)$, then

the error function can be taken as, $e = ref - V_{am}^2$.

3.2 General Features of State Regulated Optimal Control using Riccati Engine

The following assumption should hold:

- 1) The entire state vector $x(t)$ is available for feedback.
- 2) $[A, B]$ is stabilizable and $[A, C]$ is detectable.
- 3) $R = R^T > 0$.

Then,

- 1) The linear quadratic controller is unique, optimal and full state feedback control law, $U(t) = -Kx(t)$ with $K = R^{-1}B^T S$ that minimum cost function 'J' is subjected to the dynamic constraints imposed by the open-loop dynamics.
- 2) 'S' is the unique, symmetric positive semi definite notation to the algebraic riccati equation, $SA + A^T S + C^T C - SBR^{-1}B^T S = 0$
- 3) The closed loop dynamics will give $\dot{X}(t) = [A - BK]X(t)$ are guaranteed to be asymptotically stable.
- 4) The minimum value of the cost function 'J' is given by $= X_0^T S X_0$.

3.3 Features of min-max and H-∞ State Regulated Optimal Control Based on Riccati Engine

We then define the min-max quadratic cost function as

$$J(U, d) = \frac{1}{2} \int_0^\infty [Z^T(t)Z(t)\rho U^T(t)U(t) - r^2 d^T(t)d(t)] \cdot dt \text{ where } \rho > 0 \quad (24)$$

In which 'r' and 'ρ' are user specified design variables that weight the relative influence of the control and the disturbance. Under the following assumptions:

- 1) The entire state vector $X(t)$ is available for feedback.
- 2) $d(t)$ is a deterministic bounded energy signal with $\int_0^\infty d^T(t)d(t) \cdot dt < \infty$.
- 3) Both $[A, B]$ and $[A, L]$ are stabilizable and $[A, C]$ is detectable.

If then the optimum value of the cost $J(U, d)$ constrained by the system dynamics exists, it is a unique saddle point of $J(U, d)$ where

- 1) The optimal min-max control law $U(t) = -KX(t)$ with $K = -\frac{1}{s} B^T S$
- 2) The optimal worst-case disturbance is $d(t) = \frac{1}{r^2} L^T s X(t)$.
- 3) 's' is the unique symmetric, positive semi-definite solution of the matrix riccati equation $sA + A^T s + C^T C - s \left(\frac{1}{r^2} BB^T - \frac{1}{r^2} LL^T \right) = 0$
- 4) The closed loop dynamics then becomes $\dot{X}(t) = (A - BK)X(t) + Ld(t)$

3.4 Algorithm and Calculations of System Matrices of short journal bearing in Presence of Parametric Uncertainty

The general configuration can be depicted as

Figure 2.1: Block diagram for parametric uncertainty

The interconnection transfer function matrix 'M' is then partitioned as,

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$$

where the dimensions of M_{11} conform with three of 'Δ'. By routine manipulation it can be derived that $Z = [M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}]$, if $(I - M_{11}\Delta)$ is invertible. When the inverse exists, one may define

$$F(M, \Delta) = (M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12}) \quad (25)$$

$F(M, \Delta)$ is called the Linear Fractional Transformation (LFT) of M and Δ .

Because the upper loop of M is closed by the block Δ , this kind of linear fractional transform is called as Upper Linear Fractional Transformation (ULFT) and denoted with subscript 'u', i.e. $F_\mu(M, \Delta)$ to show the way of connection.

Similarly, there is Lower Fractional Transformation (LFT) that are usually used to indicate the incorporation of a controller 'K' into the system. Such a lower LFT can be depicted and defined by

$$F_l(M, \Delta) = (M_{11} + M_{12}K(I - M_{22}\Delta)^{-1}M_{21}) \quad (26)$$

Figure 3: Model hydro dynamic bearing in presence of uncertainty

3.5 Realization of the Final Tuning Algorithm using Conformal maps

The fractional order PID structure is given as,

$$\tilde{C}(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \quad (27)$$

From the above section it has been clear that the number of variables could be reduced with integral order (λ) and the derivative order (μ) which are essentially set to the same value 'q' i.e. $\lambda = \mu = q$. Therefore, now the transfer function of the PID controller becomes

$$K_p + \frac{K_i}{s^q} + K_d s^q = \frac{K_d s^{2q} + K_p s^q + K_i}{s^q}, \quad (28)$$

The initial level or the first stage of tuning was done by the LQR controller and through fractional calculus. The improvised tuning parameters depending only on the value of 'q' is given as,

$$\begin{aligned} \tilde{K}_d &= (K_d)^{1/q}, \\ \tilde{K}_p &= -2(K_i K_d)^{1/2q} \cos\left(\frac{\pi}{q}\right) \\ \tilde{K}_i &= (K_i)^{1/q} \end{aligned} \quad (29)$$

If the Bearing transfer function is 2nd order of the form $\frac{\omega_\eta^2}{(s^2 + 2\xi\omega_\eta s + 1)} = P_1(s)$ and the controller transfer function $C_i(s) = K_p + \frac{K_i}{s^q} + K_d s^q$, then the combined transfer function that is with controller can be given as,

$$\eta(s) = \left\{ \left[\frac{\omega_\eta^2}{(s^2 + 2\xi\omega_\eta s + 1)} \right] \left[\tilde{K}_p + \frac{\tilde{K}_i}{s^q} + \tilde{K}_d s^q \right] \right\} \quad (30)$$

The only parameter to tune to reach the sub-optimal level of damping is 'q'. Now with the variation in the controller differ-integral order 'q' of FOPID different set of controller parameters is obtained for the equivalent PID controller. One can observe that with the increase/decrease in the differ-integral order of FOPID (q) changes the position of the open loop controller zeros and closed loop system poles in the root-locus plot while the location of the open loop system poles the same (Hagg et al., 1953; Kaya et al., 1984; Morrison et al., 1964).

Controller Gains:

Table 1: Gain values K_{sh} HDB Model for $e=0.4$				
O_1	O_2	O_{ref1}	O_{ref2}	O_{ref3}
-0.19536	-1.9428	0.8306	-0.25109	-0.49704

Table 2: Gain values K_{sh} HDB Model for $e=0.5$				
O_1	O_2	O_{ref1}	O_{ref2}	O_{ref3}
-0.19479	-1.8059	0.93376	-0.33453	-0.12722

Table 3: Gain values K_{sh} HDB Model for $e=0.7$				
O_1	O_2	O_{ref1}	O_{ref2}	O_{ref3}
-0.20104	-1.866	0.67721	-0.73087	-0.084927

Figure 4: Root locus for HDB for $q=1.615$, $e=0.4$ and $q=1.614$ for $e=0.7$

Controller gains after 2nd stage of Tuning:

Table 4: HDB for $e=0.4$		
Modified K_p	Modified K_d	Modified K_i
0.37153	-0.054314	-0.2223

Table 5: HDB for $e=0.5$		
Modified K_p	Modified K_d	Modified K_i
-0.009846	-0.077651	-0.0848

Table 6: HDB for $e=0.7$		
Modified K_p	Modified K_d	Modified K_i
0.44641	-0.099533	-0.055984

Robust Optimal Tuning after first Stage:

Table 7: HDB for $e=0.4$; Gamma value = 2.1614 ; $e_{max} = 0.59675$;				
O_1	O_2	O_{ref1}	O_{ref2}	O_{ref3}
0	0.1197	26.8465	-0.1861	-17.6194

Table 8: HDB for $e=0.5$; Gamma value = 2.2319; $e_{max} = 0.69672$;				
O_1	O_2	O_{ref1}	O_{ref2}	O_{ref3}
0	0.0268	25.6752	-0.0907	-17.2233

Table 9: HDB for $e=0.7$; Gamma value = 2.2328; $e_{max} = 0.6752$;				
O_1	O_2	O_{ref1}	O_{ref2}	O_{ref3}
0	0.0220	27.1259	-0.0781	-17.1837

Controller gains after 2nd stage of Tuning keeping controller robustness:

Table 10: HDB for $e=0.4$

Modified K_p	Modified K_d	Modified K_i
21.375	-2.0866	-14.028

Table 10: HDB for $e=0.5$

Modified K_p	Modified K_d	Modified K_i
4.5523	-0.77548	-3.0538

Table 10: HDB for $e=0.7$

Modified K_p	Modified K_d	Modified K_i
5.8969	-0.53169	-3.7356

4. SIMULATION

4.1 Brief Study of the Overall Control Strategy

4.1.1 Dynamics of Uncontrolled Hydrodynamic Bearing System

Figure 5: The instability is clearly seen in 'X' and 'Y' displacement of the journal bearing.

Figure 6: The Unstable phase plot of the journal bearing in both fixed and rotating plane

4.2 Dynamics of Hydrodynamic bearing with a blend of optimal LQR control and Fractional control (Conformal mapping) without parametric uncertainty

Figure 7: The improvement of the control stabilization is clearly visible (right) in terms of amplitude due to second stage Fractional control for eccentricity $e=0.5$

Figure 8: The improvement of the control stabilization is clearly visible (right) in terms of amplitude due to second stage Fractional control for eccentricity $e=0.7$

4.3 Dynamics of Hydrodynamic bearing with a blend of Robust optimal control and Fractional control (Conformal mapping) with parametric uncertainty

Figure 9: The improvement of the control stabilization is clearly visible (right) in terms of amplitude due to second stage Fractional control for eccentricity $e=0.5$

Figure 10: The improvement of the control stabilization is clearly visible (right) in terms of amplitude due to second stage Fractional control for eccentricity $e=0.7$

CONCLUSION

This work relates us to stabilize short journal bearing with two stage PID like controller with orbital states as principle feedback to the controller. The optimal controller thus designed gives stabilization both with and without parametric uncertainty. However, the second stage of tuning using fractional controller using conformal maps gives an improved response in terms of both stabilizing amplitude and in terms of transients. Although fractional controller is used in the second stage the final controller still acts as a normal integer order PID controller readily acceptable in Industries.

REFERENCES

- Alfred, J.S. 1965, Protecting turbo machinery from self excited rotor whirl. Journal of Engineering for Power, Transaction of the ASME, Series A. 333-34.
- Aurich A, Bierman D., Blumtt, Brecher C, Carstensen C and Denkena B. 2009, "Modeling and simulation of process, Machine interaction in grinding, production engineering research and development, 3, 111-120.
- Bayat F.M and Karimi M- Ghartemani. 2010. Method for designing PID stabilizer for minimum phase fractional order systems, IET Control Theory and applications, 4(1), 61-67.
- Bayat F.M, Afshar M. and Karimi M – Ghartemani. 2009. Extension of the root locus method to a certain class of fractional order system", ISA Transactions, 48(1), 48-53.
- Bentley D.E and Muszynska A, 1989. Fluid generated instabilities of rotors. Orbit, 10(1).

- Botte J, Wang Q.G and Lee T.H, 2002. PI/PID controller via LQR approach, Chemical Engg. Science, 55(13), 2429-2439.
- Bou Rabee NM., Marsden JE and Romero LA. 2008, Dissipation-induced hetclinic orbits in tippe top's, SIAM Revicco, 50(2), 325-344.
- Bradfield, CD., Roberts, J.B and Karawirdiran, R. 1987. Performance of electropmagnetoc bearing for the vibration control of super-critical shaft, Proc Inst Mech Engr 201, 201-212.
- C Burrows, M Sahinkoya, 1989. Active Vibration control of flexible rotors: an experimental and theoretical study, Proceedings of the Royal Society of London, A 422, 123-146.
- Das S. 2011. Functional Fractional Calculus. springer, Berlin.
- Dimitri A., El-Shafei, A, 2010. Instability control and unbalance compensation of flexible rotors supported on journal bearing using magnetic bearings, Proceedings of the Eight IFTOMM International Conference on Rotor Dynamics, Seol, Korea, 657-664.
- Doyle J, 1985. Structured Uncertainty in Control System Design, Proceedings of the 24th Conference on Decision and Control, Fort Lauderdale, FL USA 1985.
- Fittro R, Knopse C, 2002. Rotor compliance minimization via μ control of active magnetic bearing, IEEE Transaction on Control System Technology, 10(2), 238-249.
- Fujii T. 1982. A new approach to the LQ design from the view point of the linear regulator problem", IEEE Transactions on Automatic Control, 32(11), 289-294.
- Glebov VV, Sorokin VM, Pichkov VP and Ignat'ev Da, 2005, Analysis and improvement of grinding wheels with vibration damper for brittle materials", In proceeding of 5th International Scientific-Engineering Conference on Assuming and Improving the Quality of machines in the life cycle stages. Bryansk A, 2, 63-66.
- Glebov VV, Sorokin, VM Puchkov VP and Ignat'ev DA. 2007, Eliminating vibrations in the face grinding of glass, Russian Engg. Research, 27, 305-308.
- Gttorst H., 2004. Active Vibration reduction of an elastic rotor by piezo electric actuators", PhD Thesis, TU Darmstadt (in German).
- Hagg, A.C and Warner, P.C. 1953, Oil whip of flexible rotors. Trans Am Soc Mech Engrs 75, 1339-1344.
- Kaya, F and Roberts, J.B. 1984, Optimum Vibration control of flexible transmission shafts. In I Mech Engrs Third Int Conf. Vibs in Rotating Machinery, New York, paper no. C398/84, 271-279.
- Knopse, C R Tamer, S.M and Fedigan, 1997. Robustness of adaptive rotor vibration control to structured uncertainty, ASME, Journal of Dynamic Systems, Measurement and Control.
- Li G, Lin Z, Allaire PE, Huang B, Jiang W, Zorzi E.S, Barlott R.O, 2001. Stabilization of a high speed rotor with active magnetic bearing by a piece wise μ -synthesis controller, Proceedings of the sixth international symposium on magnetic suspension Technology, Italy, 416-421.
- Malwatkar G.M, Sonawane S.H and Waghmare L.M. 2009. Tuning PID Controller for higher order oscillating systems with improved performanace, ISA Transactions, 48(3), 347-353.
- Morrison, D and Patterson, A.N. 1964. Criteria for unstable oil whirl of flexible rotors., Proc Instn Mech Engrs. 179, 45-55.
- Mukherjee A, Bhattacharya R and Rao Desari Am. 1994, A theoretical study of stabilization of a rigid rotor under the influence of dilute Viscoelastic lubricants., Journal of Tribology, 107(1), 75-81.
- Mukherjee A. 1984, Effects of biphasic lubricants on dynamics of rigid rotors, Journal of Lubrication Technology. 105, 29-38.
- Muszynska A, 1986. Whirl and whip rotor/bearing stability problems., Journal of sound and vibration, 110(3), 443-462.
- Muszynska A, 1986. Whirl and whip rotor/bearing stability problems. Journal of sound and vibration, 110(3), 443-462.
- Newkirk, B.L and Taylor, H.D, 1925. Shaft whipping due to oil action in journal bearings, General Elec. 28, 559-568.
- Or AC. 1994, The dynamics of tippe top, SIAM, Journal of applied mathematics, 54, 597-604.
- Sahimkaya, M.N and Burrows, C.R. 1985. Control of stability and synchronous vibrations of flexible supported on oil film bearing., Am. Soc. Mech.
- Saif M. 1989. Optimal linear regulator pole placement by weight selection, International Journal of Control, 50(1), 399-414.
- Salm, J and Schweiter, G. 1984. Modelling and control of flexible rotor with magnetic bearing, In I Mech E. Third Int Conf. Vibs in Rotating Machinery, New York, paper no. C277/84, 553-561.
- Samantaray AK, Bhattacharya R and Mukherjee A. 2008, On stability of Crandall gyro pendulum., Physics Letters, A372, 238-243.
- Santos, H Ulbrich, 1993. Application of control Strategies for active titting pad bearings, Zeitschrift für Angewandte Mathematik und Mechanik, 73.
- Schönhoff U., 2003. Practical Robust Control of Mechatronics System with structural flexibilities, PhD Thesis, TU Darmstadt 2003.
- Schönhoff U., Ehmann E, Nordmann R, 2002. Design of a robust PID like μ synthesis controller for position control, Proceedings of the 2002 international symposium an active control of sound and vibration.
- Schönhoff U., Nordmann R, 2002. A H_{∞} weighting scheme for PID like motion control, IEEE Conf. on control Applications (CCA), Glasgow, Scotland 2002.
- Schweiter, G. 1975. Stabilization of self excited rotor vibrations by an active damper. In Dynamics of rotors (ed. FI Niorelson), 472-493., Berlin, Springer-Verlag.
- Tolle GC and Muster D. 1975, Effect of a biphasic lubricant on half - frequency whirl in a full journal bearing, ASME Journal of Engineering for industry, 97(4), 1345-1353.
- Tondl A., 1991. Quenching of self excited vibrations, Academia, Prague.
- Vitecek A., Tima J and Viteckova M, 2008, Stability of rigid rotors in journal bearings, Transactions of the VSB-Technical University of Ostrava. Mechanical Series, LIV articles 1638(2), 159-164.
- Zhang Y, Wang Q.Q and Astram K.J. 2002. Dominant Pole Placement for multi-loop control systems. Automatica, 38(7), 1213-1220.