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COMPARISON OF ERRORS IN CALCULATING THE JOURNAL FORCE OF A SHORT CYLINDRICAL OIL FILM BEARING

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ABSTRACT

In the present paper, a new model is developed for analysis of the errors for calculating journal force of a short cylindrical oil film journal bearing. The pressure distribution in oil film short journal bearing has been determined with the help of Reynold's equation. Fluid is assumed incompressible and steady. In this paper three different computation methods FDM conventional, FDM matrix formation and FEM using MATLAB PDE solver are investigated for calculating journal force. From the results it has been observed that the absolute error percentage for force calculation in short cylindrical bearing is minimum by using the FEM MATLAB PDE solver compared to other two methods. Also a vivid statistical analysis is carried with the help of ANOVA tool to understand the dependency of method, step or element size and eccentricity on computational time and errors.

KEYWORDS

FDM, sparse matrix, FEM, MATLAB PDE, error, computational cost, ANOVA.

1. INTRODUCTION

Kramer presented the derivation of classical Reynolds equation in a very clear and distinct manner (Kramer, 1993). The short bearing approximation for solving Reynolds equation was clearly reported with the formulation and calculation of damping and stiffness coefficients for various types of short cylindrical bearings. The procedure to calculate the static equilibrium position was also clearly mentioned. The Somerfield's solutions and boundary conditions were also investigated. The empirical relation between the attitude angle and the eccentricity was studied. Sahoo presented the derivation of Navier stokes equation and reduced it to Reynolds equation using certain assumptions (Sahoo, 2005). Different types of bearings such as hydrostatic bearing, squeeze film bearing, elasto hydrodynamic bearing were studied with different boundary conditions. By finite difference method Machado and Calvacla presented the solution of Reynolds equation for pressure field using finite difference method (Machado and Cavalca, 2009). They reported pressure distribution for different types of bearings such as cylindrical bearing, elliptical bearing, 3 lobed bearing etc. They also found the locus of shaft journal center in the bearing and the dynamic coefficients in different types of bearings. Squeeze effects were also investigated in the above all bearings. Faria presented the application of finite element method in the development of computer procedures for hydrodynamic elliptical journal bearing (Faria, 2015). The finite element formulation for the classical Reynolds equation was made and the pressure field was calculated. The method used here for

the calculation of load capacity and the stability formulation were a bit unique. The procedure for finding the static equilibrium position was not clearly stated. How discretization affects the solution of Reynolds equation was also analyzed which was called mesh sensitivity analysis. The results obtained for finite element method was validated with other researchers. In the last step performance analysis of elliptical journal bearing was made. Rowe and Chang used finite difference method for calculating pressure field from Reynolds equation based on non-dimensional linearized dynamic coefficients (Rowe and Chong, 1986). Two techniques, Finite difference and perturbation technique were used for prediction of dynamic coefficients. This paper gave a clear picture of dynamic coefficients and detailed flowchart of the same, but the determination of static equilibrium position was not clearly mentioned.

From different literatures stated above the basic idea about the solution of the Reynolds equation was gained. Different numerical methods like FDM, FEM, and analytical methods were understood. It was evident from the literature that for finding pressure distribution, solving of the Reynolds equation is necessary.

A rotor system is always supported by bearings that may be sliding or ball or roller bearing. To study the pressure and force characteristics, a general idea about the bearings is of utmost importance. In most of the heavy duty works the sliding bearings are used of which fluid film bearing is of the most important category.

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2. JOURNAL FORCE FROM REYNOLDS EQUATION

$$\frac{\partial}{\partial s} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial s} \right) + \frac{\partial}{\partial z} \left(\frac{h^3}{\mu} \frac{\partial p}{\partial z} \right) = 6U \frac{\partial h}{\partial s} + 12 \frac{\partial h}{\partial t} \quad (1)$$
$$\frac{\partial}{\partial \theta} \left(H^3 \frac{\partial \bar{P}}{\partial \theta} \right) + \alpha^2 \frac{\partial}{\partial \bar{z}} \left(H^3 \frac{\partial \bar{P}}{\partial \bar{z}} \right) = \frac{\partial H}{\partial \theta} + \frac{2}{\omega} \frac{dH}{dt} \quad (2)$$
$$\bar{P} = \frac{p}{P_{\text{ref}}}, P_{\text{ref}} = \frac{6U\mu R}{(R-r)^2}, \bar{Z} = \frac{z}{L}, \alpha = \frac{R}{L}, \varepsilon = \frac{e}{(R-r)}, \omega = \frac{U}{R} \quad (3)$$

$$H = 1 - \varepsilon \cos(\theta - \phi) \quad (4)$$

$$\frac{\partial H}{\partial \theta} = \varepsilon \sin(\theta - \phi), \quad (5)$$

$$\frac{dH}{dt} = -\varepsilon \sin(\theta - \phi) \dot{\phi} - \dot{\varepsilon} \cos(\theta - \phi) \quad (6)$$

$$\frac{\partial}{\partial \theta} \left(H^3 \frac{\partial \bar{P}}{\partial \theta} \right) + \alpha^2 \frac{\partial}{\partial \bar{z}} \left(H^3 \frac{\partial \bar{P}}{\partial \bar{z}} \right) = \frac{\partial H}{\partial \theta} \quad (7)$$
$$\alpha^2 \frac{\partial}{\partial \bar{z}} \left(H^3 \frac{\partial \bar{P}}{\partial \bar{z}} \right) = \frac{\partial H}{\partial \theta} \quad (8)$$
$$\bar{P} = \frac{\epsilon}{2 \times a^2 \times H^3} \times (\bar{Z}^2 - \bar{Z}) \times \sin(\theta - \phi) \quad (9)$$
$$\bar{F} = \left(\frac{\pi}{24\alpha^2}\right) \times \frac{\varepsilon}{(1-\varepsilon^2)^2} \times \sqrt{1 - \varepsilon^2 + \left(\frac{4}{\pi}\varepsilon\right)^2} \quad (10)$$

3. FDM APPROACH

$$A_i \bar{P}_{i-1,j} + B_i \bar{P}_{i,j-1} + C_i \bar{P}_{i,j} + D_i \bar{P}_{i+1,j} + E_i \bar{P}_{i,j+1} = F_i \quad (11)$$
$$A_i = H_{i-\frac{1}{2}}^3, B_i = \alpha^2 H_i^3, D_i = H_{i+\frac{1}{2}}^3, C_i = -(A_i + D_i + 2B_i), E_i = B_i \quad (12)$$

$$F_i = s^2 \left[\varepsilon \sin(\theta_i - \varphi) - \frac{2}{\omega} \{ \varepsilon \dot{\varphi} \sin(\theta_i - \varphi) + \dot{\varepsilon} \cos(\theta_i - \varphi) \} \right] \quad (13)$$

$$\bar{P}_{i,j} = \frac{F_i - A_i \bar{P}_{i-1,j} + B_i \bar{P}_{i,j-1} + D_i \bar{P}_{i+1,j} + E_i \bar{P}_{i,j+1}}{C_i} \quad (14)$$
$$[LC] \times [Nodal\ pressure] = [RC] \quad (15)$$
[illegible]

$$[RC] = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ \vdots \\ F_{N-1} \\ F_N \\ F_1 \\ F_2 \\ \vdots \\ \vdots \\ \vdots \\ \begin{bmatrix} F_{N-1} \\ F_N \end{bmatrix}_{MN \times 1} \end{bmatrix} \quad (17)$$

$$[\text{Nodal pressure}] = \begin{bmatrix} \bar{P}_{1,1} \\ \bar{P}_{2,1} \\ \vdots \\ \bar{P}_{N-1,1} \\ \bar{P}_{N,1} \\ \bar{P}_{1,2} \\ \bar{P}_{2,2} \\ \vdots \\ \vdots \\ \bar{P}_{N-1,M} \\ \bar{P}_{N,M} \end{bmatrix}_{MN \times 1} \quad (18)$$

Here along the circumferential axis N numbers of nodes and along the length of the journal M numbers of nodes are considered. Hence MN numbers of equations are formed in square matrix format. As relative pressure outside these nodes along journal length is zero the following relation is used for boundary condition.

$$\bar{P}_{1 \dots N,0} = 0 \quad (19)$$

$$\bar{P}_{1 \dots N,M+1} = 0 \quad (21)$$

After calculating nodal pressure, same standard conventional algorithm [6] is used to calculate journal force. For short bearing A_i , D_i will be zero in eqn. (11), (14) and (16) when eqn. (8) is referred to FDM method.

4. FEM APPROACH

In this proposed method MATLAB PDE tool for solving eqn. (7) and (8) can be used. Triangular mesh elements are created in this method as shown in Fig 1.

Figure 1: Triangular FEM 2D mesh generation of bearing oil film area

For short fluid film bearing eqn. (8) is solved using MATLAB PDE tool function to calculate pressure at all nodal points. Author has derived pressure at the centroids of each triangular mesh using linear interpolation of nodal pressure of each triangle. After that journal force components and resultant journal force are calculated using eqn. (22), (23) and (24).

$$F_x = \sum_{i=1}^T \bar{P}_i \times S_i \times \cos x_i \quad (22)$$

$$F_y = \sum_{i=1}^T \bar{P}_i \times S_i \times \sin x_i \quad (23)$$

$$F = \sqrt{(F_x^2 + F_y^2)} \quad (24)$$

Where F is resultant journal force and F_x , and F_y are the journal force orthogonal components along radial plane perpendicular to journal rotational axis. S_i and x_i are area and circumference coordinate of each mesh accordingly. T is total number of mesh element.

5. INPUT PARAMETERS

The input data considered for analysis is mentioned in the table 1. But for calculating non-dimensional journal force, α and ϵ are sufficient for short bearing.

Table 1: Data used for journal force calculation

Input parameters	Values (In SI)
Coefficient of viscosity	0.25
Relative pressure outside oil film	0
Journal radius	0.025
Bearing radius	0.026
Length of the bearing	0.050
eccentricity taken	0.1, 0.3, 0.5, 0.7,
Angular velocity of journal	60000 (rpm)

6. RESULTS AND STATISTICAL ANALYSIS

The analytical solution of journal force from eqn. (10) is shown in table 2. These solutions have been used for calculating absolute errors in different method of approaches.

Table 2. Analytical solution of non-dimensional journal forces of short bearing

Eccentricity	0.1	0.3	0.5	0.7	0.9
Force	0.0495	0.1802	0.4625	1.4879	14.7967

6.1 Computational time and errors in different method

At different eccentricity and element size, solutions were found and computational time is recorded in seconds. Absolute error is calculated in percentage with respect to force data as available in table 2. Relative error also calculated in percentage with respect to previous results where element or grid size is decreasing in geometric progression with a factor of multiplication 0.5. The results of above three methods are shown in table 3, 4 and 5 respectively.

Table 3: Output data of computation time and errors of journal force using FDM with iteration

Eccentricity	Element size	Computation cost (seconds)	Absolute error (%)	Relative error (%)
0.1	0.112	0.50	0.93	49.43
0.1	0.056	2.32	1.88	0.94
0.1	0.028	10.50	2.17	0.29
0.1	0.014	82.19	-1.54	-3.63
0.1	0.007	338.77	76.70	79.46
0.3	0.112	0.27	0.88	49.62
0.3	0.056	1.35	1.87	0.98
0.3	0.028	7.53	2.17	0.30
0.3	0.014	86.09	-1.72	-3.80
0.3	0.007	322.22	30.86	33.15
0.5	0.112	0.13	0.77	50.13
0.5	0.056	0.76	1.84	1.06
0.5	0.028	6.72	2.16	0.31
0.5	0.014	85.13	-1.76	-3.83
0.5	0.007	318.66	2.39	4.22
0.7	0.112	0.12	0.49	51.44
0.7	0.056	0.73	1.76	1.27
0.7	0.028	8.03	2.13	0.36
0.7	0.014	84.38	-1.79	-3.84
0.7	0.007	330.29	-8.28	-6.61
0.9	0.112	0.12	-0.96	59.41
0.9	0.056	0.75	1.41	2.38
0.9	0.028	9.13	2.04	0.62
0.9	0.014	89.89	-1.83	-3.79
0.9	0.007	320.24	-10.87	-9.21

Table 4: Output data of computation time and errors of journal force using FDM matrix formation

Eccentricity	Element size	Computation cost (seconds)	Absolute error (%)	Relative error (%)
0.1	0.112	0.01	85.03	-21.73
0.1	0.056	0.03	39.83	-24.43
0.1	0.028	0.39	20.16	-14.06
0.1	0.014	4.33	6.77	-11.14
0.1	0.007	179.75	4.61	-1.37
0.3	0.112	0.00	84.95	-21.63
0.3	0.056	0.04	39.81	-24.40
0.3	0.028	0.28	20.16	-14.06
0.3	0.014	4.10	6.77	-11.14
0.3	0.007	177.07	4.61	-1.36
0.5	0.112	0.00	84.74	-21.36
0.5	0.056	0.03	39.78	-24.34
0.5	0.028	0.25	20.15	-14.04
0.5	0.014	4.26	6.77	-11.14
0.5	0.007	175.75	4.61	-1.36
0.7	0.112	0.01	84.23	-20.68
0.7	0.056	0.02	39.68	-24.18
0.7	0.028	0.24	20.13	-13.99
0.7	0.014	4.20	6.77	-11.13
0.7	0.007	175.66	4.61	-1.36
0.9	0.112	0.01	81.58	-16.50
0.9	0.056	0.03	39.19	-23.35
0.9	0.028	0.25	20.03	-13.77
0.9	0.014	4.29	6.74	-11.07
0.9	0.007	174.29	4.60	-1.36

Table 5: Output data of computation time and errors of journal force using MATLAB PDE tool

Eccentricity	Element size	Computation cost (seconds)	Absolute error (%)	Relative error (%)
0.1	0.112	2.24	0.34	-0.96
0.1	0.056	6.73	0.08	-0.25
0.1	0.028	24.02	0.02	-0.06
0.1	0.014	86.08	0.01	-0.02
0.1	0.007	274.50	0.00	0.00
0.3	0.112	1.84	0.35	-0.96
0.3	0.056	6.65	0.09	-0.26
0.3	0.028	25.67	0.02	-0.06
0.3	0.014	88.85	0.01	-0.02
0.3	0.007	276.77	0.00	0.00
0.5	0.112	1.84	0.38	-0.96
0.5	0.056	6.58	0.10	-0.28
0.5	0.028	23.92	0.03	-0.07
0.5	0.014	85.99	0.01	-0.02
0.5	0.007	276.30	0.00	0.00
0.7	0.112	1.96	0.43	-0.86
0.7	0.056	6.79	0.11	-0.31
0.7	0.028	24.53	0.03	-0.08
0.7	0.014	86.18	0.01	-0.02
0.7	0.007	280.44	0.00	-0.01
0.9	0.112	2.05	0.61	0.40
0.9	0.056	6.74	0.18	-0.42
0.9	0.028	24.20	0.05	-0.13
0.9	0.014	86.37	0.01	-0.03
0.9	0.007	273.09	0.00	-0.01

6.2 Analysis of results

To understand the effect of method, grid or mesh size and eccentricity on the error and computation time the data of table 3, table 4 and table 5 is analysed using analysis of variance (ANOVA) statistical tool. Three groups of source variables as method, eccentricity and element size are considered as shown in table 6.

Table 6: Grouping variables with their levels

Method	Eccentricity	element size
FDM with iteration	0.1	0.112
FDM matrix	0.3	0.056
MATLAB PDE tool	0.5	0.028
	0.7	0.014
	0.9	0.007

Three ways ANOVA with two factors interaction test have been conducted on three response vectors of computation cost, absolute error and relative error separately. The ANOVA output tables are shown in table 7, table 8 and table 9 for response vectors of computation cost, absolute error and relative error accordingly.

Table 7: ANOVA table for computation cost

Source	Sum Square	Degree of freedom	Mean Square	F critical	Probability> F critical
method	34871.80	2	17435.90	3328.44	0.00
eccentricity	27.74	4	6.93	1.32	0.28
element size	732287.38	4	183071.85	34947.62	0.00
method* eccentricity	43.45	8	5.43	1.04	0.43
method*element size	47108.51	8	5888.56	1124.10	0.00
eccentricity* element	142.66	16	8.92	1.70	0.10
Error	167.63	32	5.24		
Total	814649.17	74			

In table 7, the probability value of 0.28 indicates that the mean responses of computation cost for different levels of eccentricity are not that significant. But methods and element sizes both have strong evidence of effect on computation cost as corresponding probability values are zero in two decimals round off. Last three entries of probability column of table 7 are the probability value of null hypothesis for the two way interactions. The interaction between methods and element sizes is significant as probability value is also zero in two decimals round off. The interaction between method and eccentricity is not significant as probability value 0.43, which is comparatively much higher than others. The interaction between eccentricity and element sizes is very little significant as probability value shows 0.10.

Table 8: ANOVA table for absolute error

Source	Sum Square	Degree of freedom	Mean Square	F critical	Probability> F critical
method	14144.22	2	7072.11	79.07	0.00
eccentricity	393.37	4	98.34	1.10	0.37
element size	6241.58	4	1560.39	17.45	0.00
method* eccentricity	728.66	8	91.08	1.02	0.44
method* element size	16518.70	8	2064.84	23.09	0.00
eccentricity* element	1405.98	16	87.87	0.98	0.50
Error	2862.14	32	89.44		
Total	42294.65	74			

The probability value of 0.37 in table 8 indicates that the mean responses of absolute error for levels of eccentricity are not significantly different. But methods and element sizes both have strong influence on absolute error as corresponding probability values are zero in two decimals round off. The interaction between method and element size is significant as probability value is also zero in two decimals round off. Also found that the interaction between method and eccentricity is not significant as probability value 0.44. Similarly, the interaction between eccentricity and element size is also significant as probability value shows 0.50 which is comparatively higher than others.

Table 9: ANOVA table for relative error

Source	Sum Square	Degree of freedom	Mean Square	F critical	Probability > F critical
method	9957.97	2	4978.99	52.49	0.00
eccentricity	295.10	4	73.78	0.78	0.55
element size	3761.65	4	940.41	9.91	0.00
method* eccentricity	670.76	8	83.85	0.88	0.54
method* element size	8522.24	8	1065.28	11.23	0.00
eccentricity* element	1611.30	16	100.71	1.06	0.43
Error	3035.64	32	94.86		
Total	27854.67	74			

Similarly, from table 9, it can be interpreted that methods and element sizes both have strong evidence of effect on relative error while eccentricity remains dormant. The interaction between method and element size is significant as probability value is close to zero. The interactions in method-eccentricity and eccentricity-element size are insignificant as probability values are higher.

The levels of minimum absolute errors in different methods at different eccentricity are presented in table 10. The table 10 is prepared with bold-faced rows from table 3, 4 and 5 which are at minimum value of absolute errors at different eccentricities. For all cases computational cost is considered to be less than approximately 300 seconds.

Table 10: Consolidated data of minimum absolute errors in different methods

Method	Eccentricity	Grid/element length	Computation cost (seconds)	Absolute error (%)	Relative error (%)
FDM with iteration	0.1	0.112	0.50	0.93	49.43
FDM with iteration	0.3	0.112	0.27	0.88	49.62
FDM with iteration	0.5	0.112	0.13	0.77	50.13
FDM with iteration	0.7	0.112	0.12	0.49	51.44
FDM with iteration	0.9	0.112	0.12	-0.96	59.41
FDM matrix formation	0.1	0.007	179.75	4.61	-1.37
FDM matrix formation	0.3	0.007	177.07	4.61	-1.36
FDM matrix formation	0.5	0.007	175.75	4.61	-1.36
FDM matrix formation	0.7	0.007	175.66	4.61	-1.36
FDM matrix formation	0.9	0.007	174.29	4.60	-1.36
MATLAB PDE tool	0.1	0.007	274.50	0.00	0.00
MATLAB PDE tool	0.3	0.007	276.77	0.00	0.00
MATLAB PDE tool	0.5	0.007	276.30	0.00	0.00
MATLAB PDE tool	0.7	0.007	280.44	0.00	-0.01
MATLAB PDE tool	0.9	0.007	273.09	0.00	-0.01

The absolute error data of table 10 is rounded off upto two decimal points. The same data is plotted in Fig 2.

Figure 2: Minimum absolute error comparisons plot for different computation method

It is seen that method using MATLAB PDE tool shows minimum error and it is almost zero after rounding off up to two decimal points and also remain unchanged with variation of eccentricity.

7. CONCLUSIONS

This paper presents statistical and graphical comparisons of errors and computational time on non-dimensional journal force estimation for short cylindrical bearing. It is concluded with more than 99.99% confidence that the absolute error, relative error and computational time are affected strongly with the changes of methods and element or grid sizes. It is also seen that the eccentricity has much less contribution on errors and computational time compared to methods and element sizes. FDM Matrix method shows a steady and consistent trend on absolute error with higher computational time (refer table 4). So FDM Matrix method may be a better choice over the conventional FDM on account of consistency and stability. But on account of absolute error in very short computational time and typical larger step size consideration, FDM conventional is better than FDM Matrix method. MATLAB PDE tool method is much more accurate, consistent and stable on estimation of journal force when the problem is allowed moderate computational time. In this method Computational time and accuracy both increase further as element size decreases. This paper shows a way to choose a method and subsequent step or element size depending on degree of error margin and constraint on computational time.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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