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CONFORMAL MAPPING BASED CONTROL APPROACH FOR ROTOR SUPPORTED ON JOURNAL BEARING

Aayush Bhadani^a, Abhiro Mukherjee^b, Satyabrata Das^c^aDepartment of Electrical Engineering, Shiv Nadar University, Greater Noida, India^bDepartment of Electrical Engineering, Techno India University, Kolkata, India^cDepartment of Electrical Engineering, NIST Behrampur, Behrampur, India

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ABSTRACT

Rotors supported on journal bearing are widely used throughout the modern industrial world. Above a certain value of rotor speed, these rotor-bearing systems become unstable due to self-excited vibrations. To ensure stability and guarantee reliable operation, the vibrations of the rotor bearing system has to be controlled. This work focuses on a 2-stage controller tuning methodology for stabilization of rotor supported on a journal bearing. The tuning strategy uses LQR based tuning and elements of fractional order control to give an approximated integer order PID controller. This approach enables us to achieve the digital/ analog realization of Fractional Order PID (FOPID) controller.

KEYWORDS

FOPID, LQR, Conformal Mapping, 2-stage tuning, threshold speed, rotor-bearing system.

1. INTRODUCTION

Rotor instability at high operating speeds is one of the prime concern of rotors-supported on journal bearing. One of the main reasons of this instability is the journal bearing oil film which causes the fluid-induced instability problem of oil whirl and whip as suggested in (Muszynska, 2005; Jiri, 2011). Apart from the self-excited vibrations caused by the hydrodynamic forces of journal bearing, rotors above threshold speed are also susceptible to rotating damping forces produced by dissipation in rotor material, couplings, or due to friction in tooltips and splines as in (Mukherjee, 2010). These non-conservative forces can drive the system to an unstable condition. Practical stabilization of rotor bearing system is achieved by active vibrational control with the use of embedded piezo actuators which can be adjusted to suit different loading conditions and machine configurations. This work is devoted towards designing a controller for stabilization of rotor supported on journal bearing. The tuning methodology first assumes LQR based PID controller in stage 1 of tuning and then improves upon these stage 1 gains by utilizing conformal mapping based fractional order approach in stage 2 of tuning. The resultant controller has a PID structure and is thus easy to implement compared to a standard fractional order controller. As studied in, this process of tuning in two successive stages enables us to achieve better tunable gains with lesser initial controller effort and lesser control cost compared to conventional LQR based PID (Suman et al., 2012). The performance of this suboptimal two stage PID tuning strategy involving fractional order approach has been shown and brought out by the stimulation results and the analysis presented.

2. DYNAMICS OF ROTOR-BEARING SYSTEM

The fluid film of the journal bearing acts as a set of springs and dampers. This introduces a resultant fluid force which acts on the shaft. The same can be expressed by damping D and stiffness K_B of the bearing fluid. This fluid force in stationary frame (x-y) can be written as,

$$F_r = [D(\dot{o} - j\lambda\Omega o) + K_B o]e^{-\lambda\Omega t} \quad (1)$$

$$[o = x + jy, \quad \dot{o} = re^{j\lambda\Omega t}]$$

Where, r is rotor lateral displacement and $\lambda\Omega$ is fluid average circumferential angular velocity.

One may consider an isotropic rotor with stiffness K_s , mass M and external damping D_s while momentarily ignoring bearing fluid force. So then, the dynamic model in stationary frame (x-y) is expressed as,

$$\begin{aligned} M\ddot{x} + D_s\dot{x} + K_s x &= 0 \\ M\ddot{y} + D_s\dot{y} + K_s y &= 0 \end{aligned} \quad (2)$$

It can be further expressed in coupled rotor state (in complex form) as,

$$M\ddot{o} + D_s\dot{o} + K_s o = 0 \quad (3)$$

Now, by combining the bearing fluid model to rest of the rotor system

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dynamics, the rotor-bearing model in stationary frame (x-y) as in becomes (Chen-Chao et al., 2011),

$$M\ddot{o} + D_s\dot{o} + K_s o + D(\dot{o} - j\lambda\Omega o) + K_b o = 0 \quad (4)$$

The threshold speed beyond which the rotor becomes unstable is given as, $\Omega_{th} = \frac{1}{\lambda} \sqrt{\frac{K}{M}}$.

The practical way of increasing the threshold is by changing the value of 'K', by modulating it through additional stiffness.

3. SOURCES OF INSTABILITY

Rotor internal damping (RID) could be problematic as it can initiate a series of interactions of different self-induced instabilities, such as those resulting from rubs and oil whip. Thus, it is imperative to look into the force component which is associated with the rotor internal damping and creates instability in rotor.

The forces acting on the rotor due to its viscoelastic nature in two perpendicular directions X and Y as in [3] is given by,

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = -K \begin{bmatrix} X \\ Y \end{bmatrix} + R_i \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} + \begin{bmatrix} 0 & -\omega R_i \\ \omega R_i & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} \quad (5)$$

The first two components are stiffness dependent force and ordinary damping force. But the last component of force has a special significance because of its regenerative nature. This last component of force is non-potential which can be shown by its non-vanishing curl and thus responsible for creating instability. This is shown by the following equations.

$$\bar{F}_c = -\omega R_i Y \hat{i} + \omega R_i X \hat{j} \quad (6)$$

Here $\bar{F}_c$ is the last force component in vectorized form. The curl of this force component is given as,

$$\begin{aligned} \nabla \times \bar{F}_c &= \left(\frac{\partial}{\partial X_i} \hat{i} + \frac{\partial}{\partial Y_j} \hat{j} \right) \times (-\omega R_i Y \hat{i} + \omega R_i X \hat{j}) \\ &= 2\omega R_i \hat{k}, i.e. \neq 0 \end{aligned} \quad (7)$$

If we now look at the equation related to journal bearing fluid forces given in,

$$\begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{bmatrix} \ddot{X} \\ \ddot{Y} \end{bmatrix} + \begin{bmatrix} C_{XX} & C_{XY} \\ C_{YX} & C_{YY} \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} + \begin{bmatrix} K_{XX} & K_{XY} \\ K_{YX} & K_{YY} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = Mu\Omega^2 \begin{pmatrix} \sin \Omega t \\ \cos \Omega t \end{pmatrix} \quad (8)$$

It is seen from this equation that there is one stiffness coefficient matrix with elements $(K_{ij})_{ij=X,Y}$ and a damping coefficient matrix having elements $(C_{ij})_{ij=X,Y}$. These stiffness and damping coefficients are function of eccentricity ratio. So, the fluid force of the bearing has a damping component and a stiffness component. The stiffness co-efficient matrix is anti-symmetric due to K_{YX} being negative. Thus, this component of fluid-bearing force is non-conservative and circulating in nature which makes it a potential cause for making system unstable.

4. OPTIMAL CONTROL WITH AUGMENTED STATE BASED ON LQR

We create an augmented state space model written in terms of error ('e') of the response function ('o') with reference taken as zero in order to realize a PID feedback structure as in [8].

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{1}{2M} \left[\frac{2}{C\epsilon} - (\omega R_i - K) \right] \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ -\frac{\mu R L^3 \pi}{C^3} + R_i \\ 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$e_1 = e, \quad e_2 = \frac{de}{dt}, \quad e_3 = \int e \cdot dt \quad (9)$$

The LQR objective is to minimize the cost given below, where $Q_{3 \times 3}$ and $R_{1 \times 1}$

are the weight matrices.

$$J = \int_0^\infty (e^T Q e + u^T R u) dt \quad (10)$$

Table 1: Stage 1 Controller Gains		
K_p	K_d	K_i
0.0010	0.0001	0.3162

5. FRACTIONAL ORDER PID DESIGN BASED ON CONFORMAL MAPPING

Structure of Fractional Order PID (FOPID) is given by,

$$\tilde{C}(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \quad (11)$$

The number of variables can be reduced by keeping $(\lambda = \mu = q)$ where (λ) is integral order and (μ) is differential order. With initial tuning done using LQR, the modified gains of approximated PID controller given through fractional calculus as a function of 'q' is given as in [8]:

$$\bar{K}_d = (K_d)^{1/q} \quad (12)$$

$$\bar{K}_p = -2(K_i K_d)^{1/2q} \left(\cos \frac{\varphi}{q} \right) \quad (13)$$

$$\bar{K}_i = (K_i)^{1/q} \quad (14)$$

$$\varphi = \pi - \tan^{-1} \left(\frac{\sqrt{4K_i K_d - K_p^2}}{K_p} \right), \quad r = \sqrt{\frac{K_i}{K_d}} \quad (15)$$

Now, as the integral-differential order 'q' of FOPID controller is varied, different set of controller parameters are derived for the equivalent PID controller. The values of 'q' are subject to primary Riemann sheet condition.

Table 2: Stage 2 Controller Gains			
q	Modified K_p	Modified K_d	Modified K_i
0.91	0.001796718	4.99962E-05	0.282195087
0.93	0.00170396	6.18633E-05	0.28997836
0.95	0.001562897	7.58638E-05	0.297635143
0.97	0.001369437	9.22534E-05	0.305165955
0.99	0.001119709	0.000111301	0.312571584

A suitable choice of 'q' has to be made in order to achieve the desired closed loop performance.

Figure 1: Root locus for different values of 'q'. 'q' varied in increments of 0.01 from 0.51 to 1.

Obtained $\varphi = 1.6492$ rad ; $r = 50.9312$

5. SIMULATION RESULTS

States for Stage 2 (FOPID) Tuning at 'q'=0.9 in fixed frame:

Figure 2: (a) X1 (displacement in X) in Y)

Figure 2: (b) Y1 (displacement in Y)

Fixed frame phase portrait of displacement for stage 2 (FOPID) controller:

Figure 3: X1_Y1 (fixed frame displacement phase portrait)

Fixed frame phase portrait of velocity for stage 2 (FOPID) controller:

Figure 4: X2_Y2 (fixed frame velocity phase portrait)

Fixed frame phase portrait of displacement for stage 2 (FOPID) controller:

Figure 6: Xr_Yr (rotating frame phase portrait)

6. CONCLUSION

An approximated FOPID controller having equivalent integer order PID structure has been designed and successfully implemented for controlling the rotor-supported on journal bearing. The controller is tuned in two successive stages and it operates on the vibrational feedback taken in terms of the orbital states of the system. Different sets of controller parameters for this equivalent PID controller is obtained by varying the FOPID controller differential-integral order 'q', some of which are mentioned in Table.2. The value of 'q' thus has to be suitably chosen in order to achieve the desired closed loop performance expectation.

It is observed from Fig.1 that the root locus plot shifts towards the left side

with decreasing FOPID order 'q'. This change in 'q' pushes open loop zeros and subsequently the closed loop poles of the system towards greater damping regions, achieving desired damping levels for the approximated integer order PID controller. The phase portraits of Fig.3 and Fig.4, are bounded and suggest that the control action is working as desired. The vibrations are suppressed and maintained in the range of micrometers. This is evident from the amplitudes of the state displacement plots of X & Y as given in Fig.2. Also, the shaft position is maintained well within the clearance level (~220um) suggesting that the whirling motion has been limited. Results obtained after stage 2 of tuning for FOPID controller turns out to be better than LQR based PID of stage 1 in terms of transient & stabilizing response with improved stability, reduced vibrational displacement and lesser control cost.

This conformal mapping based FOPID controller however retains its simplicity because of being integer type just like a normal PID controller. As a result, the deployment of this controller in real-time processor is much more convenient than a standard fractional order controller which poses practical challenges due to its inherent complexities. Therefore, exploiting true potential of the FOPID controller in a relatively simpler manner to achieve better stabilization and practical solution in the rotor-bearing system.

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