REVIEW ARTICLE

STABILITY ANALYSIS OF PARALLEL CONNECTED DC-DC CONVERTERS

Abdulmajed O. Elbkosh

Department of Electrical and Computer Engineering, College of Engineering, Elmergib University, Libya.

*Corresponding Author email: [aalbkosh@elmergib.edu.ly](mailto:aalbosh@elmergib.edu.ly)

This is an open access article distributed under the Creative Commons Attribution License CC BY 4.0, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

ARTICLE DETAILS

Article History:

Received 02 August 2020
Accepted 04 September 2020
Available online 18 September 2020

ABSTRACT

Parallel controlled DC-DC converters are nonlinear and non-smooth systems, they show various nonlinear behaviour including smooth, non-smooth bifurcation, and chaos when they work under their design conditions. Usually, the Poincaré map approach is the most common method for studying the stability of those nonlinear systems. Stability is indicated using the eigenvalues of the Jacobian of the map computed at the fixed point. The other method is the monodromy matrix approach, where the stability can be concluded by computed the eigenvalues of the matrix. In this paper, the nonlinear dynamics of parallel connected DC-DC converters are investigated. It is shown that the concept of the monodromy matrix can be applied to determine the stability of the system as well as the Poincaré map approach.

KEYWORDS

Parallel DC-DC regulators, Poincaré map, Monodromy matrix, nonlinear dynamic.

1. INTRODUCTION

Parallel operation of DC-DC converters is widely used circuit in many applications (Elbkosh, 2009, Tse, 2003). The structure has many advantages compared to single DC-DC converters in terms of reliability, fault tolerance, and allowing high load currents to be distributed without using devices of high-power rating. Usually, it is desired to distribute the load equally among the converters (Ishikawa, and Saito, 2007, Lu and Tse, 2001). A lot of control methods have been proposed to do that (Banerjee and Verghese, 2001, Elbkosh, 2009, Elbkosh, et al., 2008, Tse, 2003). One widely used method for balancing the currents is the master-slave current sharing technique employed in this paper. Despite all the advantages, however, these converters are inherently nonlinear and non-smooth time varying systems, and one can expect nonlinear phenomena in such systems. The analysis of these phenomena is of great importance for a designer since small changes in the system's parameters (for example the supply voltage) can destabilize the system (Giaouris, et al., 2006, Giaouris, et al., 2007). However, to analyze the stability of these switching systems, one has to deal first with their discontinuity.

The most common method of studying the stability of periodic systems is the switching map. The system's stability is indicated by the eigenvalues of the map computed at the fixed point. However, in most power electronics circuits it is not easy to derive this map analytically in addition it is really complicated to derive this map as the switches elements in the system are increased. In this paper we study the stability and dynamic of parallel connected systems using two methods, the first one is the Poincaré map method and the other is the Monodromy matrix method. The first part of this paper shows briefly the operation of DC-DC parallel connected buck converter and its dynamic equations. The second section shows the smooth period doubling bifurcations that occur in the system

when some circuit parameters change. Next, the stability of the system is analyzed using the state transition matrix for one complete cycle (the monodromy matrix) and the Jacobian matrix method (the Poincaré map).

2. BASIC OPERATION OF THE PARALLEL CONNECTED DC-DC BUCK CONVERTER

Figure 1 shows a basic block diagram that describes the master-slave current sharing schemes for two DC-DC converters connected in parallel and feeding the same load.

Figure 1: Block diagram of two paralleled converters with master-slave current sharing

Quick Response Code

Access this article online

Website:
www.actaelectronicamalaysia.com

DOI:
10.26480/aem.02.2020.35.39

One of the converters (the master) has a simple feedback loop (proportional gain) to regulate the output voltage. The control signal of this converter is equal to:

$$V_{con1}(t) = V_{offset} - K_{p1}(V_{out}(t) - V_{ref}) \quad (1)$$

where V_{offset} is the dc offset voltage needed to give a steady state duty cycle, K_{p1} is the propositional gain, and V_{ref} is the desired output voltage. The other converter is the slave, which sets its current to be equal to that of the master via a loop involving a comparison of the currents of the two converters. The control signal of this converter can be written as:

$$V_{con2}(t) = V_{offset} - K_{p2}(V_{out}(t) - V_{ref}) - K_i(i_{L2} - m i_{L1}) \quad (2)$$

Where K_{p2} is the propositional gain of converter 2, K_i is the current feedback gain, and m is a current ratio to balance the current of the two converters. The switches S_1 and S_2 are controlled by comparators which compare the control signals V_{con1} and V_{con2} with a suitable periodic saw-tooth waveform V_{ramp} and a commutation occurs when these two signals become equal i.e., switch S is open if $V_{con} < V_{ramp}$, and closed otherwise. The ramp signal can be written as:

$$V_{ramp}(t) = V_L + (V_U - V_L) \left(\frac{t}{T} \bmod 1 \right) \quad (3)$$

where, V_U and V_L are the upper and lower limit values of the ramp signal respectively. When the converter is operated in continuous conduction mode, there are four switch states, namely (Elbkosh, 2009, Tse, 2003):

- (a) S_1 and S_2 are ON. (b) S_1 ON and S_2 OFF.
(c) S_1 OFF and S_2 ON. (d) S_1 and S_2 are OFF.

The state equations that represent these switch states during a switching cycle are:

$$\dot{\mathbf{x}} = \begin{cases} \mathbf{A}_1 \mathbf{x} + \mathbf{B}_1 E & S_1 \text{ ON and } S_2 \text{ ON} \\ \mathbf{A}_2 \mathbf{x} + \mathbf{B}_2 E & S_1 \text{ ON and } S_2 \text{ OFF} \\ \mathbf{A}_3 \mathbf{x} + \mathbf{B}_3 E & S_1 \text{ OFF and } S_2 \text{ ON} \\ \mathbf{A}_4 \mathbf{x} + \mathbf{B}_4 E & S_1 \text{ OFF and } S_2 \text{ OFF} \end{cases} \quad (4)$$

where $\mathbf{x}$ is the state vector defined as $\mathbf{x} = [v_{out} \quad i_{L1} \quad i_{L2}]^T = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \mathbf{x}_3]^T$, E is the input voltage and A, B are the system matrices involve the system's parameters. In the case of the buck converter, the system matrices A_1, A_2, A_3 and A_4 are equal to A_s , given by:

$$\mathbf{A}_s = \begin{bmatrix} \frac{-1}{CR} & \frac{1}{C} & \frac{1}{C} \\ \frac{-1}{L_1} & \frac{-r_{L1}}{L_1} & 0 \\ \frac{-1}{L_2} & 0 & \frac{-r_{L2}}{L_2} \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 0 & \frac{1}{L_1} & \frac{1}{L_2} \end{bmatrix}^T, \mathbf{B}_2 = \begin{bmatrix} 0 & \frac{1}{L_1} & 0 \end{bmatrix}^T, \mathbf{B}_3 = \begin{bmatrix} 0 & 0 & \frac{1}{L_2} \end{bmatrix}^T, \mathbf{B}_4 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \quad (5)$$

the circuit parameters are fixed at the following values: $V_{ref} = 24$ V, $K_{p1} = K_{p2} = 3.5$, $K_i = 5$, $V_{offset} = 5$ V, $L_1 = 0.02$ H, $r_{L1} = 0.05$ Ω , $L_2 = 0.04$ H, $r_{L2} = 0.2$ Ω , $C = 47$ μ F, $R = 10$ Ω , $T = 400$ μ s, $V_L = 2$ V, $V_U = 8$ V.

It is worth mentioning here that since r_{L2} is greater than r_{L1} , we expect the duty cycle of converter - 2 (d_2) to be always greater than the duty cycle of

converter - 1 (d_1). Hence, the second equation in (4) will be omitted from the analysis (there is no case for S_1 ON and S_2 OFF). The period-1 orbit of the system is shown in Figure 2, indicating the solution for one complete cycle is broken up into three intervals, for each interval the system is linear time invariant and can be solved directly using the exponential method.

Figure 2: Period one orbit of the system for

During the first interval, S_1 and S_2 are both ON, the solution of the system can be written as:

$$\begin{aligned} \mathbf{x}(d_1T) &= e^{A_s(d_1T)} \mathbf{x}(0) + \int_0^{d_1T} e^{A_s(d_1T-\tau)} \mathbf{B}_1 E d\tau \\ &= \Phi(d_1T, 0) \mathbf{x}(0) + \Gamma(d_1T, 0) \end{aligned} \quad (6)$$

where $\Phi(d_1T, 0)$, is the state transition matrix during the first interval.

During the second interval, S_1 OFF and S_2 ON, the state vector is given by:

$$\begin{aligned} \mathbf{x}(d_2T) &= e^{A_s(d_2T-d_1T)} \mathbf{x}(d_1T) + \int_{d_1T}^{d_2T} e^{A_s(d_2T-\tau)} \mathbf{B}_3 E d\tau \\ &= \Phi(d_2T, d_1T) \mathbf{x}(d_1T) + \Gamma(d_2T, d_1T) \end{aligned} \quad (7)$$

where $\Phi(d_2T, d_1T)$, is the state transition matrix during the second interval.

During the final interval, S_1 and S_2 are both OFF, the solution of the system is given by:

$$\begin{aligned} \mathbf{x}(T) &= e^{A_s(T-d_2T)} \mathbf{x}(d_2T) + \int_{d_2T}^T e^{A_s(T-\tau)} \mathbf{B}_4 E d\tau \\ &= \Phi(T, d_2T) \mathbf{x}(d_2T) + \Gamma(T, d_2T) \end{aligned} \quad (8)$$

where $\Phi(T, d_2T)$, is the state transition matrix during the final interval.

The Poincaré map, can be derived by stacking all the solutions (6 – 8) together for a switching cycle and can be simplified as:

$$\mathbf{x}(T) = \Phi(T, 0) \mathbf{x}(0) + \Gamma(T, d_2) \quad (9)$$

where

$$\begin{aligned} \Phi(T, 0) &= \Phi(T, d_2T) \times \Phi(d_2T, d_1T) \times \Phi(d_1T, 0) \text{ and} \\ \Gamma(T, d_2) &= \Phi(T, d_2T) [\Phi(d_2T, d_1T) \times \Gamma(d_1T, 0) + \Gamma(d_2T, d_1T)] \\ &\quad + \Gamma(T, d_2T) \end{aligned} \quad (10)$$

3. NONLINEAR BEHAVIOUR

Like many switched dynamical systems, parallel connected converters can exhibit a variety of nonlinear behaviour. Most standard bifurcations (such as period doubling bifurcation) are caused by the change of stability status without change in topological sequence. The stability of the system can be deduced by calculating the eigenvalues of the map, however, the movement of the eigenvalues out of the unit circle will determine whether the system has a smooth or non-smooth bifurcation i.e. for smooth bifurcation, one or more eigenvalues goes out from inside the unit circle to outside the unit circle smoothly whereas in non-smooth bifurcation, one or more eigenvalues of the map will jump out of the unit circle as a circuit parameter changes.

In the parallel connected buck converter, with variation in the supply voltage it is possible to have a smooth period doubling bifurcation as shown in the bifurcation diagram Fig. 3. The system loses stability to a new limit cycle with a period twice the period of the ramp signal. This bifurcation is very common in single DC-DC converters and is often considered as a standard type of bifurcation in nonlinear systems (Banerjee and Verghese, 2001, Elbkosh, 2009, Elbkosh et al., 2008, Tse, 2003, Giaouris et al., 2006).

Figure 3: Bifurcation diagram with input voltage as bifurcation parameter

4. STABILITY ANALYSIS OF THE SYSTEM

In this section, the stability of the standard period doubling bifurcation will be studied using two approaches, mainly, the monodromy matrix approach and the Poincare map approach

4.1 Stability analysis using monodromy matrix approach

The period-1 limit cycle of the parallel connected buck converter given in Figure 2 shows that the trajectory crosses the switching manifold three times at d_1T , d_2T and T . Therefore, the monodromy matrix of the system (M) for one complete cycle is obtained as:

$$M(T, 0) = e^{A_s(1-d_2)T} \times S_2 \times e^{A_s(d_2-d_1)T} \times S_1 \times e^{A_s d_1 T} \times S_3 \quad (11)$$

where S_3 , S_2 and S_1 are the state transition matrices at T , d_2T and d_1T respectively, calculated using the formula:

$$S = I + \frac{(f_+(x(t_\Sigma), t_\Sigma) - f_-(x(t_\Sigma), t_\Sigma)) n^T}{n^T f_-(x(t_\Sigma), t_\Sigma) + \frac{\partial h(x(t_\Sigma), t_\Sigma)}{\partial t}} \quad (12)$$

where t_Σ is the switching time (the time when the solution hits the switching manifold).

- Firstly, at switching instant d_1T , the switching condition is defined by:

$$h_1(x(d_1T)) = V_{offset} + [-K_{p_1} \ 0 \ 0]^T x(d_1T) - V_{ref} - V_{ramp} = 0 \quad (13)$$

Thus:

$$\frac{\partial h_1(x(d_1T))}{\partial t} = -\frac{V_U - V_L}{T} \quad (14)$$

The normal to the switching manifold is given by:

$$n_1 = \frac{\partial h_1(x(d_1T))}{\partial x(d_1T)} = [-K_{p_1} \ 0 \ 0]^T \quad (15)$$

The two smooth vector fields are:

$$f_-(x(d_1T)) = A_s x(d_1T) + B_1 E \quad f_+(x(d_1T)) = A_s x(d_1T) + B_3 E \quad (16)$$

Since $f_+ \neq f_-$, the system has a discontinuous vector field at the switching surface.

Substituting equations 14 – 16 into equation 12, the following expression for S_1 is obtained:

$$S_1 = I + \begin{bmatrix} 0 & 0 & 0 \\ S_{21} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (17)$$

where

$$S_{21} = \frac{\frac{E}{L_1}}{\frac{x_3(d_1T) + x_2(d_1T) - x_1(d_1T)}{C} \frac{1}{R} - \frac{V_U - V_L}{K_{p_1} T}} \quad (18)$$

- Secondly, at time $t_\Sigma = d_2T$, the indicator function is given by the equation:

$$h_2(x(d_2T)) = V_{offset} + [-K_{p_2} \ mK_i \ -K_i]^T x(d_2T) - V_{ref} - V_{ramp} = 0 \quad (19)$$

Therefore:

$$\frac{\partial h_2(x(d_2T))}{\partial t} = -\frac{V_U - V_L}{T} \quad (20)$$

The normal n is given by:

$$n_2 = \frac{\partial h_2(x(d_2T))}{\partial x(d_2T)} = [-K_{p_2} \ mK_i \ -K_i]^T \quad (21)$$

The two smooth vectors field before and after the switching, are given by:

$$f_-(x(d_2T)) = A_s x(d_2T) + B_3 E \quad f_+(x(d_2T)) = A_s x(d_2T) + B_4 E \quad (22)$$

Substituting equations 20 – 22 into equation 12, the following expression for S_2 is obtained:

$$S_2 = I + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \quad (23)$$

where

$$S_{31} = -\frac{K_{p_2} E}{G L_2}, \quad S_{32} = \frac{m K_i E}{G L_2} \quad \text{and} \quad S_{33} = -\frac{K_i E}{G L_2} \quad (24)$$

$$G = -K_{p_2} \left(\frac{x_3 + x_2 - \frac{x_1}{R}}{C} \right) + m K_i \left(\frac{-L_1 x_2 - x_1}{L_1} \right) - K_i \left(\frac{-L_1 x_3 - x_1 + E}{L_2} \right) - \frac{V_U - V_L}{T} \quad (25)$$

Finally, at $t_\Sigma = T$ the manifold is discontinuous (with respect to time) and the time derivative will be infinite. Therefore, the saltation matrix at this point will be the identity matrix (Elbkosh, 2009, Elbkosh, et al., 2008). The stability of the system can be determined by finding the eigenvalues of the fundamental solution matrix M . To calculate M and hence to check the stability of the system, we need first to find the values of state vectors at

the beginning of the cycle $\mathbf{x}(0)$, at the switching $\mathbf{x}(d_1T)$, $\mathbf{x}(d_2T)$ and also the duty cycles d_1, d_2 . This can be calculated numerically using similar methodology that has been used in the case of single DC-DC buck converter. Knowing the state vectors at the switching instants and the duty cycles, the saltation matrices and the monodromy matrix can be calculated. For instance, at $E = 55V$. The calculated switching instants are $d_1T = 0.439 \times T$ and $d_2T = 0.442 \times T$.

The state vectors at the switching instants are:

$\mathbf{x}(d_1T) = [24.10, 1.311, 1.301]^T$, $\mathbf{x}(d_2T) = [24.11, 1.310, 1.302]^T$, therefore the saltation matrices are:

$$S_1 = I + \begin{bmatrix} 0 & 0 & 0 \\ -0.3199 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, S_2 = I + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.1207 & 0.1724 & -0.1724 \end{bmatrix} \text{ and } S_3 = I \quad (26)$$

The state transition matrices are:

$$\Phi(T, d_2T) = \begin{bmatrix} 0.5932 & 3.7280 & 3.7269 \\ -0.0088 & 0.9769 & -0.0226 \\ -0.0044 & -0.0113 & 0.9876 \end{bmatrix}, \Phi(d_2T, d_1T) = \begin{bmatrix} 0.9971 & 0.0290 & 0.0290 \\ -0.0001 & 1.0000 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

$$\Phi(d_1T, 0) = \begin{bmatrix} 0.6691 & 3.0909 & 3.0902 \\ -0.0073 & 0.9851 & -0.0145 \\ -0.0036 & -0.0072 & 0.9919 \end{bmatrix} \quad (27)$$

Hence, the monodromy matrix is equal to:

$$M(T, 0) = \begin{bmatrix} -0.8840 & 0.4022 & -0.8841 \\ -0.2193 & -0.0223 & -1.0132 \\ -0.1198 & -0.3875 & 0.2703 \end{bmatrix} \quad (28)$$

The eigenvalues of (28) are $[-0.63 \pm 0.22i \text{ and } 0.78]$ implying the system is stable at the above input voltage.

The computed loci of the eigenvalues with varying input voltage are shown in Figure 4. It indicates that the system loses its stability through a smooth period doubling bifurcation at input voltage of around 56.7 V. This result is in very good agreement with the previous simulation results presented in Figure 3.

Figure 4: Loci of the eigenvalues of the monodromy matrix for different input voltage

4.2 Stability analysis using Poincaré map approach

In last section, the Poincaré map of the parallel connected buck converter has been derived (equation 9) for simplicity it can be written as:

$$\mathbf{x}(T) = f(\mathbf{x}(0), d_1, d_2) \quad (29)$$

The jacobian matrix $\frac{\partial \mathbf{x}(T)}{\partial \mathbf{x}(0)}$ can be calculated using the following formula [2]:

$$\frac{\partial \mathbf{x}(T)}{\partial \mathbf{x}(0)} = \frac{\partial f}{\partial \mathbf{x}(0)} - \frac{\partial f}{\partial d_1} \times \left(\frac{\partial h_1}{\partial d_1} \right)^{-1} \times \frac{\partial h_1}{\partial \mathbf{x}(0)} - \frac{\partial f}{\partial d_2} \times \left(\frac{\partial h_2}{\partial d_2} \right)^{-1} \left[\frac{\partial h_2}{\partial \mathbf{x}(0)} + \frac{\partial h_2}{\partial d_1} \times \left(\frac{\partial h_1}{\partial d_1} \right)^{-1} \times \frac{\partial h_1}{\partial \mathbf{x}(0)} \right] \quad (30)$$

This is a general form of the jacobian matrix for any switching systems with two switching instants.

Differentiation of equation (9) with respect to $\mathbf{x}(0)$ yields to:

$$\frac{\partial f}{\partial \mathbf{x}(0)} = \Phi(T, d_2T) \Phi(d_2T, d_1T) \Phi(d_1T, 0) \quad (31)$$

Likewise, differentiation of equation (9) with respect to d_1 and d_2 gives:

$$\frac{\partial f}{\partial d_1} = -T \Phi(T, d_2T) \Phi(d_2T, d_1T) (f_+(x(d_1T)) - f_-(x(d_1T))) \quad (32)$$

$$\frac{\partial f}{\partial d_2} = -T \Phi(T, d_2T) (f_+(x(d_2T)) - f_-(x(d_2T))) \quad (33)$$

Differentiating the switching condition equations (13) and (19), with respect to $\mathbf{x}(0)$ yields to:

$$\frac{\partial h_1}{\partial \mathbf{x}(0)} = n_1 \Phi(d_1T, 0) \quad (34)$$

$$\frac{\partial h_2}{\partial \mathbf{x}(0)} = n_2 \Phi(d_2T, d_1T) \Phi(d_1T, 0) \quad (35)$$

Likewise, differentiation of the same equations with respect to d_1 and d_2 gives:

$$\frac{\partial h_1}{\partial d_1} = T n_1 f_-(x(d_1T)) - (V_U - V_L) \quad (36)$$

$$\frac{\partial h_2}{\partial d_2} = T n_2 \Phi(d_2T, 0) (A_s x(0) + B_1 E) + T n_2 \Phi(d_2T, d_2T) (B_3 E - B_1 E) - (V_U - V_L) \quad (37)$$

$$\frac{\partial h_2}{\partial d_1} = -T n_2 \Phi(d_2T, d_1T) (B_3 E - B_1 E) \quad (38)$$

Finally, substituting all the derivatives into (30), the jacobian matrix can be obtained. In order to calculate the jacobian matrix and hence to check the stability of the system, we need to find the values of state vectors at the beginning of the cycle $\mathbf{x}(0)$, at the switching $\mathbf{x}(d_1T)$, $\mathbf{x}(d_2T)$ and also the duty cycles d_1, d_2 . This can be calculated numerically using similar methodology that has been used in last method.

The computed eigenvalues with varying input voltage are shown in Table 1. It indicates that the system loses its stability through a smooth period doubling bifurcation at input voltage of around 56.7 V. This result is in perfect agreement with the previous simulation results presented in Figure 3 as well as with those found in the monodromy matrix method.

Table 1: The eigenvalues of the jacobian matrix for different values of the input voltage

E	Eigenvalues	Stability
55	$[-0.63 \pm 0.22i, 0.78]$	Stable period-1
56.5	$[-0.992, -0.459, 0.771]$	Stable period-1
56.7	$[-1.000, -0.454, 0.7707]$	Period doubling
57	$[-1.018, -0.447, 0.7702]$	Unstable period-1

5. CONCLUSION

This paper outlined the basic topology of parallel connected DC-DC Buck converters. It is shown that the normal operating of the system losses stability through smooth period doubling bifurcations. The stability of the system is analyzed using the state transition matrix for one complete cycle (the monodromy matrix) and the Jacobean matrix method (the Poincare map). It is shown that the stability analysis using the monodromy matrix approach is simple compared to the other approach.

REFERENCES

- Banerjee, S. Verghese, G.C., 2001. *Nonlinear Phenomena in Power Electronics: Attractors, Bifurcations, Chaos, and Nonlinear Control*. New York, USA: IEEE Press.
- Elbkosh, A., Giaouris, D., Pickert, V., Zahawi, B., Banerjee, S., 2008. Stability analysis and control of bifurcations of parallel connected DC/DC converters using the monodromy matrix. *IEEE International Symposium on Circuits and Systems*, Pp. 556-559.
- Elbkosh, A.O., 2009. *Nonlinear analysis and control of dc-dc converters*, in school of electrical, electronic, and computer engineering. Newcastle University: Newcastle.
- Geetha, M., Suja, R., Ahilan, T., 2016. Current sharing in parallel connected boost converters, *Engineering*.
- Giaouris, D., Banerjee, S., Zahawi, B., Pickert, V., 2007, Stability Analysis of the Continuous Conduction Mode Buck Converter via Filippov's Method, *IEEE Transactions on Circuits and Systems – I*.
- Giaouris, D., Elbkosh, A., Banerjee, S., Zahawi, B., Pickert, V., 2006, Control of switching circuits using complete-cycle solution matrices. *IEEE International Conference on industrial Technology (ICIT)*, Mumbai, India.
- Ishikawa, Y., Saito, T., 2007. Synchronization and Chaos in Multiple-Input Parallel DC-DC Converters with WTA Switching, *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, E90-A, Pp. 1162-1169.
- Iu, H.H.C., Tse, C.K., 2001. Bifurcation behaviour of parallel – connected buck converter. *IEEE Trans. Circ. Syst - I*, 48 (2), Pp. 2333-240.
- Tse, C.K., 2003. *Complex Behavior of Switching Power Converters*. Boca Raton, USA: CRC Press.