ANALYSIS OF ASPHALT PAVEMENT MECHANICAL BEHAVIOUR BY USING A TIRE-PAVEMENT COUPLING MODEL

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Abstract

The tire operation condition in highway transportation exerts a significant influence on pavement mechanical behaviour. However, in-depth analysis of pavement mechanical behaviour under complicated tire operation conditions is limited. This study employed the finite element method to analyse the influence of truck tires on pavements in expressway freight transportation. A tire-pavement coupling model was established, and a simulation analysis was conducted for pavement stress under different parameters. Results indicate that the σ_1 of the upper layer is significantly affected by the horizontal contact stress between the tire and pavement. The horizontal contact stress should therefore be fully considered when conducting a refined mechanical analysis of pavement surface layers. Tresca stress is mainly concentrated in the pavement surface layer and the upper base, and the high Tresca stress in the two layers is related to unstable rutting and top-down cracking. Moderate tire load and pneumatic tire pressure effectively improve the operation status of the pavement. On the contrary, excessive tire load or pneumatic tire pressure increases the road deterioration, and should be forbidden in expressway transportation. With the established tire-pavement coupling model, this study revealed the influence of tire operation condition on pavement mechanical behaviour. The conclusions obtained in this study provide a reference for pavement design and highway transportation management.

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Key Words: Tire-Pavement Coupling Model, Tire Force, Pavement Mechanical Response

1. INTRODUCTION

China has established a large-scale highway network to meet the increasing demand for transportation. In 2015, highway mileage reached 4.58 million kilometers. China will continue constructing a series of highway infrastructure in the future. Highway mileage is estimated to reach 5 million kilometers in 2020 [1]. A pavement is affected by repeated actions of traffic load, rainfall, and temperature, and these repeated actions result in rapid pavement deterioration. Against such a background, pavement construction and maintenance in China face significant challenges. Since the construction quality and maintenance level of pavements have direct influence on the service life of pavements, it is urgent to have further research on pavement design theory and acknowledgement of pavement deterioration mechanisms [2].

The mechanical behaviour of pavements under vehicle load is the foundation of pavement structure and material design and is one of the basic problems in pavement engineering [3]. Vehicle load is applied on a pavement by tires. Therefore, a reasonable description of the contact stress between tires and a pavement (i.e., tire force) is the key factor that influences the correct analysis of pavement responses [4]. Pavement mechanical behaviour analysis has always regarded the pavement under uniformly distributed constant vertical stress as the target and has revealed the mechanical characteristics of pavement to a certain degree [5].

Tire force is non-uniform, and its spatial distribution is much more complicated than that of uniformly distributed stress [6]. In addition, when vehicles move on a pavement at a certain speed, the load undertaken by the pavement is dynamic [7]. Therefore, reflecting the actual status of pavement mechanical behaviours by simplifying dynamic tire force into uniformly distributed constant vertical stress is difficult [6]. With the development of computer technology, the finite element method has become an effective means of analysing pavement mechanical behaviours under complicated tire force [8]. The tire-pavement coupling finite model proposed in recent years directly realizes the interaction between the tire model and pavement model and lays the foundation for pavement mechanical simulation under various tire conditions [9]. This study established a tire-pavement coupling model by employing the finite element method. A simulation analysis of pavement stress under different parameters (pavement friction coefficient, tire load, and pneumatic tire pressure) was also conducted to analyse the influence of truck tires on pavements in Chinese expressway freight transportation.

2. STATE OF THE ART

The complexity of tire force has been proven through experimentation and simulation. De Beer et al. [6, 10] developed the “stress-in-motion” system and applied this system in tire force measurement. Al-Qadi and Wang [11] and Hernandez and Al-Qadi [12] established a tire finite element model to perform a simulation analysis of tire force; the model reflects the spatial distribution characteristics of tire force. These studies showed that three non-uniform stresses exist in vertical, longitudinal, and transverse directions between the tire and pavement. In addition, the spatial distribution of tire force is significantly affected by tire type, pneumatic tire pressure, tire load, and rolling conditions.

Scholars in pavement engineering possess a full understanding of the complexity of tire force and have performed simulations of pavement response under 3D contact stress by improving traditional pavement models. De Beer et al. [6] measured non-uniform tire force and compared the differences in pavement response under two loads: 3D non-uniform stress and uniform stress. Their research showed that 3D non-uniform stress exerts a significant influence on pavements. Casey et al. [13] conducted a similar study and proposed that 3D contact stress must be considered to understand the deterioration of a pavement surface accurately.

In the studies of De Beer et al. and Casey et al., tire force was considered a constant stress. In the pavement mechanical problem, the complexity of tire force also originates from its moving characteristics. Scholars in pavement engineering have studied pavement responses under dynamic stress. In the models established by Khavassefat et al. [14] and Beskou et al. [15], moving contact stress is applied on a pavement. The two studies revealed the significant influence of tire force movement speed on pavements in a certain degree. However, these models do not reflect the actual characteristics of 3D tire force because tire load is simplified into uniform vertical stress.

The pavement finite element model can be further improved by applying moving 3D contact stress on a pavement. Gungor et al. [16] verified the accuracy of this type of model through a field experiment. Wang and Al-Qadi [17] analysed the influence of the nonlinear anisotropic characteristic of a granular base layer on pavement viscoelasticity mechanical behaviour. Cho et al. [18] compared the damaging effect of a wide-base tire and traditional tire on a pavement.

Tire force is the result of the interaction between the tire and pavement. Since there were abundant successful experience in the modelling methods of tires and pavements separately, it was proposed that the tire force in a pavement model could be replaced by the tire model to further improve the pavement model. Wang et al. [19] established a static 2D tire-pavement

coupling model to analyse the influence of rutting on tire and pavement mechanical behaviours. Wollny et al. [20] adopted the arbitrary Lagrange-Euler finite element method to establish a tire-pavement coupling model, and Hernandez and Al-Qadi [21] built a semi-coupling model between the tire and pavement. In the studies conducted by Wollny et al. and Hernandez and Al-Qadi, the interaction between the rolling tire and pavement was conducted by 3D models.

These previous studies showed that the finite element method is an effective means to analyse pavement behaviour under complicated tire force. Scholars have conducted continuous improvement on the description of tire force in pavement models. The tire-pavement coupling model is a breakthrough in traditional modelling methods of pavements and tires. Although the tire-pavement coupling model laid the foundation for the analysis of pavement behaviour under complicated tire operation conditions, in-depth analysis of pavement behaviour for actual freight transportation remains limited.

This study established a tire-pavement coupling finite element model to analyse the influence of truck tires on a pavement in Chinese expressway freight transportation. A simulation analysis of pavement behaviour was also conducted. The remainder of this study is organized as follows. Section 3 introduces the tire-pavement coupling model and discusses the methodology. Section 4 presents a simulation analysis of pavement behaviour and reveals the influence of parameters (pavement friction coefficient, tire load, and pneumatic tire pressure) on the pavement. The conclusions are summarized in Section 5.

3. METHODOLOGY

3.1 Computational domain and domain discretion

Regarding a section of the G45 expressway in China as prototype, a pavement model was built with ABAQUS software (Fig. 1). The X-axis direction is the driving direction, the Y-axis direction is the transverse direction of the pavement, and the negative direction on the Z-axis is the depth direction. The length of the model in X, Y, and Z directions is 20, 4, and 5 m, respectively. The computational domain of the pavement model is refined near the tire contact zone, and the refined grid on the YOZ plane is shown in Fig. 1 b.

a) Pavement model

b) Refined computational grid on YOZ plane

Figure 1: Pavement modelled in ABAQUS software.

The tire model used an 11.00 R20 tire (section width of 293 mm and external diameter of 1085 mm) as prototype. The tire model, which is shown in Fig. 2, was established with ABAQUS software. The rolling of the tire consisted of angular velocity ω and linear velocity v . The rotation axis of ω is parallel to the Y -axis, and the direction of v is parallel to the X -axis.

The material and structural parameters of the pavement and tire models were adopted from literature [9]. According to the wheelset form of the rear axle of a common truck, the wheelset in the coupling model is a dual wheel with a distance of 346 mm from the tire centres. The established tire-pavement coupling model is shown in Fig. 3.

Figure 2: 11.00 R20 tire modelled in ABAQUS software.

Figure 3: Tire-pavement coupling model in ABAQUS software.

3.2 Model solution

The finite element equation of the tire-pavement coupling model can be expressed as:

$$\mathbf{M}\mathbf{a} = \mathbf{f}^{ext} - \mathbf{f}^{int} \quad (1)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{a}$ is the node acceleration, $\mathbf{f}^{ext}$ is the external load of the model, and $\mathbf{f}^{int}$ is the element internal force.

Eq. (1) is the ordinary differential equation with respect to the time variable, and the central difference method is an effective means to solve this equation [22]. Simulation time t can be divided into N time steps. With the n^{th} time step as t^n and the corresponding displacement being $\mathbf{d}^n$, the time increment can be defined as:

$$\Delta t^{n+\frac{1}{2}} = t^{n+1} - t^n, \quad t^{n+\frac{1}{2}} = \frac{1}{2}(t^{n+1} + t^n), \quad \Delta t^n = t^{n+\frac{1}{2}} - t^{n-\frac{1}{2}} \quad (2)$$

Velocity $\mathbf{v}$ in the midpoint of the time interval is defined; thus, the central difference equation for $\mathbf{v}$ is

$$\dot{\mathbf{d}}^{n+\frac{1}{2}} \equiv \mathbf{v}^{n+\frac{1}{2}} = \frac{\mathbf{d}^{n+1} - \mathbf{d}^n}{t^{n+1} - t^n} = \frac{1}{\Delta t^{n+\frac{1}{2}}}(\mathbf{d}^{n+1} - \mathbf{d}^n) \quad (3)$$

Eq. (3) can be converted into an integration formula for $\mathbf{v}$ as follows:

$$\mathbf{d}^{n+1} = \mathbf{d}^n + \Delta t^{n+\frac{1}{2}} \mathbf{v}^{n+\frac{1}{2}} \quad (4)$$

$\mathbf{a}$ can be expressed as:

$$\ddot{d}^n \equiv a^n = \frac{v^{n+\frac{1}{2}} - v^{n-\frac{1}{2}}}{t^{n+\frac{1}{2}} - t^{n-\frac{1}{2}}} \quad (5)$$

The integration formula for a is:

$$v^{n+\frac{1}{2}} = v^{n-\frac{1}{2}} + \Delta t^n a^n \quad (6)$$

According to these equations, by substituting the related variables in Eq. (5), a can be resolved as:

$$\ddot{d}^n \equiv a^n = \frac{\Delta t^{n-\frac{1}{2}} (d^{n+1} - d^n) - \Delta t^{n+\frac{1}{2}} (d^n - d^{n+1})}{\Delta t^{n+\frac{1}{2}} \Delta t^n \Delta t^{n-\frac{1}{2}}} \quad (7)$$

Given that the node force of the n^{th} time step is f^n ,

$$f^n = f^{\text{ext}}(d^n, t^n) - f^{\text{int}}(d^n, t^n) \quad (8)$$

According to Eq. (1),

$$a^n = M^{-1} f^n \quad (9)$$

Substituting Eq. (9) into Eq. (6) results in:

$$v^{n+\frac{1}{2}} = v^{n-\frac{1}{2}} + \Delta t^n M^{-1} f^n \quad (10)$$

3.3 Simulation process and characteristic section for result outputting

The tire was assumed to be operated under a free-rolling condition with a linear velocity of 1.2 m/s, and the simulation process is shown in Fig. 4. Four operation conditions were simulated to reveal the influence of the tire on the pavement, and the parameters of the operation conditions are shown in Table I.

Figure 4: Simulation process.

Table I: Parameters of the operation conditions.

Operation condition	Friction coefficient f	Tire load W (kN)	Pneumatic tire pressure P (MPa)
Standard	0.9	25	0.72
Non-standard 1	0.0	25	0.72
Non-standard 2	0.9	45	0.72
Non-standard 3	0.9	25	0.92

Figure 5: Characteristic section for result outputting.

4. RESULT ANALYSIS AND DISCUSSION

4.1 Pavement response under standard operation conditions

The σ_1 and Tresca stress of the characteristic sections under standard operation condition are shown in Figs. 6 and 7, respectively.

Fig. 6 shows that the spatial distribution of σ_1 presented the following characteristics. The main form of σ_1 showed compressive stress in the surface layer, and the compressive stress reached the maximum in the tire-pavement contact zone. Below the contact zone, the amplitude of σ_1 rapidly decreased with the increase in depth. In the base and sub-base layers, the main form of σ_1 showed tensile stress, and the tensile stress reached the maximum at the bottom of the base layer. The tensile stress at the bottom of the sub-base layer was slightly lower than that at the bottom of the base layer. The change of tensile stress in depth was smaller than that of compressive stress.

According to Fig. 7, Tresca stress was mainly concentrated in the surface and upper base layers. The maximum value of Tresca stress appeared in the intermediate layer below the tire-pavement contact zone. The A-A section shows that Tresca stress had relatively large amplitude close to the tire shoulder, and the Tresca stress on the external side of the tire contact zone was slightly larger than that on the internal side. The Tresca stress rapidly decreased in the layers beneath the surface layer and was negligible in the subgrade.

The surface material instability caused by shear stress is the main cause of the rutting of asphalt pavements. The spatial distribution characteristics of Tresca stress showed that increasing the shearing resistance capacity of the intermediate layer is important. The amplitude of Tresca stress close to the tire shoulder was approximately 0.30 MPa, which approximates the shear strength of the asphalt mixture. Hence, Tresca stress will lead to top-down crack (TDC) formation.

Figure 6: σ_1 under a standard operation condition.

Figure 7: Tresca stress under a standard operation condition.

4.2 Influence of f on pavement response

The σ_1 and Tresca stress of the characteristic sections under non-standard operation condition 1 are shown in Figs. 8 and 9, respectively.

In non-standard operation condition 1, no horizontal stress existed between tires and the pavement. Comparison of the situations in the standard operation condition and non-standard operation condition 1 revealed the influence of horizontal contact stress on the pavement response. Horizontal stress increased σ_1 in the upper layer. The closer the element was to the pavement surface, the larger the influence of horizontal stress on σ_1 was. In the standard operation condition and non-standard operation condition 1, the maximum values of σ_1 were -0.71 and -0.55 MPa, respectively. In the other layers of the pavement, horizontal stress exerted no significant influence on the value of σ_1 . In addition, horizontal stress also increased the maximum value of Tresca stress. In the standard operation condition and non-standard operation condition 1, the maximum values of Tresca stress were 0.46 and 0.42 MPa, respectively. Therefore, the influence of horizontal contact stress on Tresca stress was less than that on σ_1 .

Traditional pavement finite element analysis always regards uniformly distributed vertical stress as the simplified tire force without considering the influence of horizontal contact stress. Comparison of the simulation results of the standard operation condition and non-standard operation condition 1 showed that ignoring the horizontal tire force resulted in a significant error during the forecasting of the mechanical behaviour of the pavement surface

layer. Therefore, in a refined pavement mechanical response analysis, horizontal contact stress should be applied to the pavement.

Figure 8: σ_1 under non-standard operation condition 1.

Figure 9: Tresca stress under non-standard operation condition 1.

4.3 Influence of W on pavement response

The σ_1 and Tresca stress of the characteristic sections under non-standard operation condition 2 are shown in Figs. 10 and 11, respectively.

Figure 10: σ_1 under non-standard operation condition 2.

Figure 11: Tresca stress under non-standard operation condition 2.

With a fixed P and an increased W , the stiffness of the tire crown unchanged, and the stiffness of the tire shoulder became larger. Therefore, the contact stress of the tire crown presented minimal change. However, the contact stress of the tire shoulder increased. Under the over-load condition, the spatial distribution characteristic of tire force determined that of pavement stress. Comparison of the simulation results in the standard operation condition and non-standard operation condition 2 showed that W significantly influenced the pavement response.

The difference in σ_1 close to the tire crown in both the standard operation condition and non-standard operation condition 2 was small because no obvious change in contact stress occurred on the tire crown. However, in the over-load condition, the high contact stress of the tire shoulder increased σ_1 , and the maximum value of σ_1 (-1.5 MPa) in non-standard operation condition 2 appeared below the tire shoulder. The maximum value of σ_1 is 2.1 times that in the standard operation condition. Furthermore, the increase in W always increased the tensile stress in the base layer. In the standard operation condition and non-standard operation condition 2, the maximum values of tensile stress were 77 and 118 kPa, respectively.

The influence of tire load on Tresca stress in the surface layer was minimal. However, the high contact stress of the tire shoulder in non-standard operation condition 2 directly resulted in a hazardous stress condition for the pavement surface. As shown in Fig. 11, the maximum value of Tresca stress in non-standard operation condition 2, which was 0.74 MPa, appeared below the tire shoulder and was higher than the shear strength of the asphalt mixture in a general condition. Overloading appeared to be the main factor that caused pavement damage; hence, forbidding excessive tire load in highway transportation is necessary.

4.4 Influence of P on pavement response

The σ_1 and Tresca stress of the characteristic sections under non-standard operation condition 3 are shown in Figs. 12 and 13, respectively.

With other parameters fixed and P increasing, the contact stress of the tire crown increased, and the tire-pavement contact area was reduced. Compared with that in the standard operation condition, the change in spatial distribution of tire force caused a change in pavement response in non-standard operation condition 3.

Comparison of the simulation results in Figs. 6 and 12 indicated that increasing the tire pressure induced an increase in σ_1 , and the maximum value increased by 40 % from -0.71 MPa to -1.00 MPa. However, the difference in tensile stress extreme values at the bottom of the semi-rigid material layer in Figs. 6 and 12 was not obvious (0.77 and 0.78 MPa, respectively). Therefore, the extreme value of σ_1 at the bottom of the semi-rigid material layer was unaffected by tire pressure.

Figure 12: σ_1 under non-standard operation condition 3.

Comparison of the simulation results in Figs. 7 and 13 showed that high tire pressure influenced Tresca stress. Under a high tire pressure, the amplitude of Tresca stress in the surface layer was high (0.64 MPa). Therefore, high tire pressure can easily induce pavement deterioration, and implementing measures to forbid trucks from adopting excessive pneumatic tire pressure in highway transportation is necessary.

Figure 13: Tresca stress under non-standard operation condition 3.

5. CONCLUSIONS

Taking the G45 expressway in China and 11.00 R20 tire as prototypes, a tire-pavement coupling finite element model was established in this study. Simulation of pavement response was conducted under different parameters (pavement friction coefficient, tire load, and pneumatic tire pressure) to analyse the influence of truck tires on pavements in Chinese expressway freight transportation. The main conclusions are shown as follows.

(1) The main form of σ_1 in the surface layer is compressive stress, and the maximum value of compressive stress is on the pavement surface. In the base and sub-base layers, the main form of σ_1 is tensile stress, and the maximum value of tensile stress is at the bottom of the base layer. Tresca stress is mainly concentrated on the surface and upper base layers, and the maximum value appears in the intermediate layer. At the layers beneath the surface layer, Tresca stress rapidly decreases, and can be ignored in the subgrade.

(2) The formation mechanism of rutting and TDC can be explained by the third strength theory. Because the Tresca stress reaches the shear strength of the asphalt mixture, rutting is mainly produced in the intermediate layer, and TDC is produced close to the tire shoulder.

(3) If the tensile stress at the bottom of the semi-rigid material layer is regarded as a design indicator, then horizontal contact stress can be ignored. However, horizontal contact stress exerts a significant influence on the σ_1 of the upper layer. In the element that is close to the pavement surface, σ_1 is significantly affected by horizontal contact stress. If a refined mechanical analysis is to be conducted for the surface layer, then the influence of horizontal contact stress should be considered.

(4) An increase in tire load or pneumatic tire pressure increases the pavement response, and moderate tire load or pneumatic tire pressure can effectively improve the operation status of a pavement. Excessive tire load and tire pressure should be forbidden in expressway transportation to improve the service life of pavements.

With the proposed tire-pavement coupling model, this study implemented a mechanical behaviour analysis in different operation conditions. The research conclusions can provide a reference for pavement design and expressway management. In this study, a skidding tire model was employed; however, it cannot reflect the influence of the tread pattern on the pavement. In future studies, the tread pattern should be modelled in the tire-pavement coupling model to conduct further pavement mechanical analyses.

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