ROBUST STABILITY OF FRACTIONAL-ORDER LTI SYSTEMS WITH INDEPENDENT STRUCTURE OF ELLIPSOIDAL PARAMETRIC UNCERTAINTY

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Abstract

This example-oriented contribution deals with the investigation of robust stability for fractional-order linear time-invariant (LTI) systems with the independent structure of parametric uncertainty and ellipsoid-shaped uncertainty bounding set. Even though the ellipsoidal uncertainty is dependent by definition, its combination with the independent uncertainty structure is studied in this paper. The robust stability is tested via the application of the value set concept in combination with the zero exclusion condition. An example is elaborated for the family of fractional-order polynomials with a three-dimensional ellipsoidal uncertainty bounding set in order to demonstrate the effectivity of the applied graphical method.

Keywords: Robust Stability; Fractional-Order Systems; Ellipsoidal Uncertainty; Parametric Uncertainty; Independent Uncertainty Structure.

1. Introduction

Automatic control is an inseparable part of our daily life [1], [2], [3], and robust stability is one of the most fundamental requirements of practical control systems. It means that the stability must be guaranteed not only for one (nominal) system but also for a whole family of systems. Such families are assumed to represent some neighborhood of the nominal system. A natural approach to defining this neighborhood is known as the parametric uncertainty [4], [5], i.e., the system's structure is fixed, but the parameters are supposed to possibly "slowly" change within given bounds. The mostly utilized bound has a shape of a box, but there are other useful alternatives, such as a sphere or an ellipsoid [5], [6], [7], [8].

Fractional-order control systems become more and more popular nowadays [9], [10], [11] due to their superior performance compared to the classical integer-order circuits. Thus, the robust stability of fractional-order systems under various conditions of uncertainty is of great importance, as well [12], [13].

This contribution is intended to present the example of robust stability analysis for fractional-order linear time-invariant (LTI) system with an independent structure of parametric uncertainty combined with an ellipsoid-shaped uncertainty bounding set.

Notice that the ellipsoidal (or spherical as a special case) parametric uncertainty is actually always dependent by its definition, but the case of its combination with the independent uncertainty structure is analyzed in this contribution. The well-known tool for robust stability of integer-order systems with independent uncertainty structure and box-shaped uncertainty bounding set (i.e., interval polynomials) is the Kharitonov theorem. Then their spherical/ellipsoidal versions can be analyzed by means of the Soh-Berger-Dabke theorem [5], [14]. Unfortunately, these techniques are not directly applicable to fractional-order systems [12]. In this contribution, the robust stability is tested via the numerical calculation and subsequent plot of the value sets, and the application of the zero exclusion principle [5], [15], [16], [17].

2. Ellipsoidal family of fractional-order polynomials with independent uncertainty structure

Consider the family of fractional-order polynomials with the independent uncertainty structure:

$$p(s, q) = s^{3.3} + q_2 s^{2.1} + q_1 s^{0.8} + q_0 \quad (1)$$

and with the uncertainty bounding set Q :

$$\begin{aligned} \|q - q^0\|_{2,W} &\leq r = 1 \\ q^0 &= [1, 2, 3] \\ W &= \begin{bmatrix} w_0^2 & 0 & 0 \\ 0 & w_1^2 & 0 \\ 0 & 0 & w_2^2 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \quad (2)$$

i.e., equivalently:

$$\begin{aligned} 4(q_0 - q_0^0)^2 + 2(q_1 - q_1^0)^2 + (q_2 - q_2^0)^2 &\leq r = 1 \\ q^0 &= [1, 2, 3] \end{aligned} \quad (3)$$

Obviously, this uncertainty bounding set constitutes a (three-dimensional) ellipsoid, which is shown in Fig. 1.

Fig. 1. Ellipsoidal uncertainty bounding set (2), (3)

3. Value-set-based robust stability analysis

The value set $p(j\omega, Q) = \{p(j\omega, q) : q \in Q\}$ [5] at frequency $\omega = 2$ is depicted in Figs. 2-4. In this specific case with 3 uncertain parameters, it can be obtained through gridding the surface area of the ellipsoid and subsequent direct calculation of the related value set points. The surface area can be parameterized e.g. by means of the equations:

$$\begin{aligned} q_0 &= \left(\frac{r}{w_0^2} \right) \cos \theta \cos \varphi + q_0^0 \\ q_1 &= \left(\frac{r}{w_1^2} \right) \cos \theta \sin \varphi + q_1^0 \\ q_2 &= \left(\frac{r}{w_2^2} \right) \sin \theta + q_2^0 \end{aligned} \quad (4)$$

where $\theta \in [0, 2\pi]$ and $\varphi \in [0, \pi]$.

Sampling both parameters θ and φ by 30 equidistant samples within the intervals mentioned above leads to the “sparse” value set (Fig. 2), sampling each parameter by 100 samples makes the value set more “dense” (Fig. 3), and finally, sampling by 300 samples produces the value set that represents almost continuous area visually (Fig. 4). Anyway, it can be clearly seen that the boundary of the uncertainty bounding set (the ellipsoid surface area from Fig. 1) is mapped not only to the boundary of the value set but also into the interior of the value set (Figs. 2-4). On the other hand, the boundary of the value set is mapped only from the boundary of the uncertainty bounding set, i.e., no internal point from the ellipsoid in Fig. 1 can be mapped to the boundary of the value set in Figs. 2-4). However, this holds true only for simple uncertainty structures (such as independent one, which is studied in this contribution, and affine linear one), not for more complicated ones [5], [15], [16], [17].

Fig. 2. Value set of (1), (2) for $\omega = 2$ and 30 samples of θ and φ

Fig. 3. Value set of (1), (2) for $\omega = 2$ and 100 samples of θ and φ

Fig. 4. Value set of (1), (2) for $\omega = 2$ and 300 samples of θ and φ

The value sets for the range of frequencies from 0 to 2 with step 0.1 are shown in Fig. 5. They would also enter the fourth quadrant for even higher values of ω . According to the zero exclusion condition [5], [17], the family of FO polynomials (1), (2) is robustly stable since the zero (complex plane origin) is excluded from the value sets and all necessary preconditions (invariant degree of the polynomial family, pathwise connected $\mathcal{Q}$, continuous coefficient functions, and at least one stable member of the family) are fulfilled.

Fig. 5. Value sets of (1), (2) for $\omega = 0:0.1:2$

4. Conclusion

The contribution was focused on presenting a brief example that illustrated the value-set-based method for testing the robust stability of the fractional-order LTI system with an independent structure of parametric uncertainty and an ellipsoid-shaped uncertainty bounding set. As could have been seen, the technique is straightforward, relatively easy-to-use, and tests the robust stability with necessary and sufficient condition. Contrarily, it suffers a considerable computational burden, especially if it is generalized for systems with higher dimensions of uncertainty bounding set and if a dense sampling is requested.

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6. References

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