MATHEMATICAL MODEL OF A CONTINUOUS-DISCRETE GUIDANCE SYSTEM FOR A ROBOT MANIPULATOR

Vadim Kramar

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Abstract

The article considers the approach to the construction of a mathematical model in the field of the Laplace transform of the robot arm's guidance system when it approaches the object. Non-stationary systems with a singular point or τ -systems are considered. In contrast to the well-known approach of continuous conversion of measured angles in the process of approaching an object into control signals, the paper deals with the construction of a mathematical model of a continuous-discrete guidance control system according to the angular orientation data with discrete control.

Keywords: Guidance control system; Continuous-discrete control system; Non-stationary systems with a singular point; Mathematical model; Laplace transformation.

1. Introduction

One of the main tasks of the manipulators of anthropomorphic complex robotic systems is to perform some operations with various items according to flow charts. The first step of the operation is to take the desired item from a given point. The task is reduced to moving the gripper of the manipulator to a given point associated with the object in a given time.

The solution to this problem is carried out in several ways. The first approach is based on the Denavit-Hartenberg method [1], [2], [3], [4], which allows solving the direct kinematics problem of determining the position function in the manipulator space and gripper. This approach does not reflect the requirements for the dynamics of the manipulator. For example, in solving the inverse problem of kinematics, uncertainty may arise, since a gripper can get into the same point of space in different ways. The second approach, currently under active review, is "Programming by Demonstration" [5], [6]. The approach is based on training the robot to perform various operations on the results of the operator (teacher). The approach relies entirely on the dynamics of the operator and does not contain an analytical approach to the construction of control.

The paper considers an approach based on an analytical approach to build a manipulator grip control system to bring it to a given point [7]. In contrast to the well-known approach based on continuous control systems [8], the paper proposes to consider an approach to constructing a mathematical model of a continuous-discrete approach control system using angular orientation data with discrete control.

2. Non-stationary systems with a singular point

The class of systems considered in the work refers to non-stationary systems with a singular point or τ - systems [8]. The mathematical model of these essentially non-stationary systems is represented as a non-linear stationary block of the static type covered by intensive non-stationary feedback of the form $(t-t_0)\xi = x$, where t_0 is the approach time, $0 \leq t < t_0$.

Consider the guidance task of a managed object A, which is a robotic arm manipulator, with an object B (Fig. 1).

Fig. 1. The scheme of the guidance task

The guidance control is carried out as a result of angle measurement ψ, q, ϵ . Let be $\eta(\varphi, \psi, \dots)$ the state vector of the system, which does not contain, in addition to φ and ψ , other variables, indicated in Fig.1.

For the considered control systems of continuous type, the equations of motion are as follows

$$\left. \begin{aligned} \dot{x} &= -u \cos \varphi \\ \dot{h} &= u \sin \varphi - w \\ \dot{\eta} &= F(\eta, \epsilon, t) \\ \operatorname{tg} \epsilon &= \frac{h}{x} \end{aligned} \right\} \quad (1)$$

where $u = \text{const}$ the speed of movement of the manipulator, w - the speed of movement of the object. In general, $\frac{w}{u} \ll 1$. If the object is not mobile, then $w = 0$.

When considering the control object and system elements as linear, we obtain the widespread [8] model of a weakly perturbed guidance process in the form

$$\left. \begin{aligned} \dot{y} &= Ay + a\xi + f(t) \\ \xi &= \frac{h}{t-t_0} = \frac{h}{\tau} \\ 0 \leq t < t_0 = \frac{x_0}{u}, y(0) &= y_0 \end{aligned} \right\} \quad (2)$$

where $y(h, \varphi, \psi, \dots)$ is the state vector of the system, A is the constant matrix, a is the constant vector, ξ is the input action proportional to the “bearing” (the angle between the northern part of the observer’s meridian and the direction from the observation point to the object, clockwise), $f(t)$ is the vector of effects, t_0 is the approach time.

Under the action of control signals, according to the measured angles ψ, q, ε , the control system must bring the object (point A) to a moment t_0 in a state close to the unperturbed [8].

Equations (2) can be considered as linearized with respect to unperturbed motion ($y \equiv 0$ with $f(t) \equiv 0, w \equiv 0$) equation (1), and

$$\dot{x} = -u, \quad x = x_0 - ut = u(t - t_0), \quad \varepsilon = -\frac{1}{u}\xi \quad (3)$$

As mentioned above, the considered system model refers to τ - systems [8].

3. Continuous-discrete guidance system

We construct a mathematical model of the system under consideration for the continuous-discrete case, under the assumption that the conversion of angles into signals is performed by a discrete device.

The equation describing continuously discrete τ - the system has the form

$$\dot{y} = Ay + Bz(kT) + f(t) \quad (4)$$

$$z(kT) = Cz(k - 1T) + \sum_{i=0}^N D_i y(k - iT) + \sum_{i=0}^N G_i \xi(k - iT) + \zeta(kT) \quad (5)$$

$$(t - t_0)\xi = h \quad (6)$$

$$y(0) = y_0, \quad y(-iT) = y_{-i}, \quad i = 1, \dots, N, \quad z(-T) = z_{-1} \quad (7)$$

The vector $y(h, \varphi, \psi, \dots)$ as before describes the continuous part of the system, and the vector z - the discrete control part, working with the discretization cycle T . Angles ε and q are converted into signals by a discrete device. Consideration of system (4) - (7) will be conducted in discrete time $kT = 0, T, 2T, \dots$.

4. The equation of a continuous-discrete system in the Laplace transform

To use the Laplace transform apparatus, we use the idea of analytic continuation with respect to the parameter [8] and replace t_0 with some complex number $\bar{t}_0$ in the system (4) - (7). The system thus formed as a system with coefficients that are bounded on the positive semiaxis has exponentially bounded solutions and allows the use of the Laplace transform.

Applying a continuous Laplace transform to equation (4) and a discrete Laplace transform to equation (5), one can obtain equations in the form

$$y(p) = W_z(p)z^*(pT) + M_y(p), \quad (8)$$

$$z^*(p) = W_y^*(pT)y^*(pT) + W_\xi^*(pT)\xi^*(pT) + M_z^*(pT), \quad (9)$$

where

$$W_z(p) = \frac{1 - e^{-Tp}}{p} (pI - A)^{-1} B, \quad M_y(p) = (pI - A)^{-1} [f(p) + y_0], \quad (10)$$

$$W_y^*(pT) = (I - Ce^{-pT})^{-1} \sum_{i=0}^N D_i e^{-ipT}, \quad W_\xi^*(pT) = (I - Ce^{-pT})^{-1} \sum_{i=0}^N G_i e^{-ipT}, \quad (11)$$

$$M_z^*(pT) = (I - Ce^{-pT})^{-1} \{ Cz(-T) + \sum_{i=1}^N [D_i \sum_{\nu=1}^i e^{-(i-\nu)pT} y(-\nu T) + G_i \sum_{\nu=1}^i e^{-(i-\nu)pT} \xi(-\nu T)] \}. \quad (12)$$

Let the vector $y(t)$ be observed at discrete points in time $0, T, 2T, \dots$. Discrete Laplace transforms of a vector $y(kT)$ can be represented using the relation (8) in the form

$$y^*(pT) = W_z^*(pT)z^*(pT) + M_y^*(pT). \quad (13)$$

Next, we find

$$y^*(pT) = \Phi_{\xi y}^*(pT)\xi^*(pT) + R_y^*(pT), \quad (14)$$

$$z^*(pT) = \Phi_{\xi z}^*(pT)\xi^*(pT) + R_z^*(pT), \quad (15)$$

where

$$\Phi_{\xi y}^*(pT) = (I - W_z^*W_y^*)^{-1}W_z^*W_{\xi}^*, \quad (16)$$

$$\Phi_{\xi z}^*(pT) = (I - W_y^*W_z^*)^{-1}W_{\xi}^*, \quad (17)$$

$$R_y^*(pT) = (I - W_z^*W_y^*)^{-1}(W_z^*M_z^* + M_y^*), \quad (18)$$

$$R_z^*(pT) = (I - W_y^*W_z^*)^{-1}(W_y^*M_y^* + M_z^*). \quad (19)$$

It is convenient to combine the vectors y^* and z^* into one vector (we denote the combined vector y^*) and consider relations (14), (15) componentwise

$$y_m^*(pT) = \Phi_m^*(pT)\xi^*(pT) + R_m^*(pT), \quad m = 1, 2, \dots \quad (20)$$

Accept that $y_1 = h$.

$$\Phi_1^*(pT) = \frac{h_0^*(pT)}{\xi_0^*(pT)} = T\Phi^*(pT), \quad (21)$$

$$R_1^*(pT) = TR^*(pT). \quad (22)$$

In relation (21), subscript 0 marks the images of variables with zero initial and external disturbances of the system. Function $\Phi^*(pT)$ - the transfer function of the system.

Equations (20) describe the stationary part of the guidance system in the form of the discrete Laplace transform. The second terms in (20) are the forms of Laplace transform of the reaction of the stationary part of the system to the initial and external perturbations.

In order to obtain the equations of a closed system, equation (6) should be transformed. To this end, we set

$$t = nT, \quad \bar{t}_0 = \bar{n}_0T, \quad (23)$$

where $\bar{n}_0$ is a complex number.

We write the relation (6) in the form

$$(nT - \bar{n}_0T)\xi(nT) = h(nT), \quad n = 0, 1, \dots \quad (24)$$

Applying the discrete Laplace transform, we find the equations in the images of a closed τ - system

$$\frac{d}{dq}\xi^*(q) + [\bar{n}_0 + \Phi^*(q)]\xi^*(q) = -R^*(q), \quad (25)$$

$$y_m^*(q) = \Phi_m^*(q)\xi^*(q) + R_m^*(q), \quad m = 1, 2, \dots, \quad (26)$$

where $q = pT$.

Equations (25) and (26) allow us to determine $\xi^*(q)$ and $y_m^*(q)$. After this, $y_m(p)$ can also be defined.

5. Conclusion

The article discusses the construction of mathematical models of non-stationary systems with a special point - τ - systems, under the assumption that the conversion of the guidance angles into signals is performed by a discrete device. As a result, the task is reduced to the construction of a mathematical model of a continuous-discrete control system for guidance a manipulator of a robotic system with a given object. The construction of a mathematical model in the form of the Laplace transform will make it possible in the future to solve the problem of analyzing the stability of such systems. This is due to the fact that the concept of asymptotic stability for the class of systems under consideration is of no interest since these systems are usually considered on a finite time interval.

The proposed models open a new approach to planning the trajectory of robot manipulators to intercept a rapidly maneuvering object. In contrast to previously known methods, the guidance law is used to control the manipulator.

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