STRUCTURES OF AUTOMATIC SYSTEMS WITH UNSTABLE PARAMETERS (PART II)

A.Potapov, S.Stazhkov, J.Dalyaev*, M.Ohochinsky

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Abstract

In the first part of the research work it was offered to apply the desirable structures of system with the maximum degree of stability for automatic systems with unstable parameters. An example was given to show the effectiveness of the approach. Recommendations on the synthesis of the 4th order structures were given. In this paper the structures of $5 \div 8$ orders are presented and a general method for the synthesis of the arbitrary order structures are given. It is shown that to build the desirable structure with real roots a non-stiff system of equations can be obtained and solved. Also, a method for selecting a structure based on the required parameters is proposed.

Keywords: automatic systems; standard rated characteristic equations; unstable parameters; stability; quasi optimal structures

1. Introduction

In the first part of the research work [1] recommendations on the synthesis of the 4th order structures were given. In this paper the synthesis of the higher orders equations is presented. In Table 1, the structures of orders $n = 5-8$ (two equations each) for a nominal load ($z = 1.0$) are given. The structures with the maximum degree of stability are described below.

It is known that in the synthesis of almost any complex systems it is necessary to solve the system of nonlinear algebraic equations with unknown initial conditions. This is the problem of optimization or quasi-optimization [1, 2, 3, 4]. The process of searching for the adjustable parameters $a_1, \dots, a_n$ is often hampered by the concavity (non-convex) of the level surface structure of the multidimensional objective function for $n \geq 3$ [2], which determines the trajectory of descent to the minimum point in the stiff systems of differential equations. It is shown in [2] that the "concave" situation is typical in almost all problems of parametric optimization. Known constraints for the roots λ_i in a stiff system (1)

$$\dot{x} = Ax \tag{1}$$

for real roots $\lambda_i = \text{Re} \lambda_i$; $\text{Im} \lambda_i = 0$ и $\text{Re} \lambda_i < 0$, $i = \overline{1, n}$.

The concept of the spectral condition number of the system (1)– $K(A)$ is introduced. If $K(A) = \frac{\max |\lambda_i|}{\min |\lambda_i|}$ for the SRCE-DS (case with real roots (RR)), then it is always obtained $K(A) = 1,0$ and the system is non-stiff. The degree of concavity is characterized by the number $S = \frac{\lambda_i}{|\min \lambda_i|} = 1$ for $K(A) = 1$. The foregoing is applicable to gradient methods. Thus, SRCE-DS structures of order $n = 9-16$ with equal real roots for $c = 1,0$ (table 2) and $\theta_{no} = 0$ can be recommended.

SRCE / Parameters	A_7	A_6	A_5	A_4	A_3	A_2	A_1	η	t_{2H}	θ_{no}
DS–51 (all RR)	–	–	–	5,0	10,0	10,0	5,0	1,0	9,15	–
DS –52 (4CR+1RR)	–	–	–	3,25	6,3	6,79	4,0	0,65	6,86	–
DS –61 (all RR)	–	–	6,0	15,0	20,0	15,0	6,0	1,0	10,0	–
DS –62 (4CR+2RR) Equal roots	–	–	3,9	8,57	11,3	9,58	4,77	0,65	8,01	–
DS –71 (all RR)	–	7,0	21,0	35,0	35,0	21,0	7,0	1,0	11,84	–
DS –72 (4CR+3RR) equal roots	–	4,9	15,75	31,1	35,15	22,2	7,38	0,7	13,25	–
DS –81 (all RR)	8,0	28,0	56,0	70,0	56,0	28,0	8,0	1,0	13,14	–
DS –82 (4CR+4BK)	4,94	16,0	29,6	40,0	33,5	18,9	6,52	0,246	9,60	0,3

Table 1. Parameters of the SRCE

The account of instability of the structures should be conducted similarly as considered earlier in the first part of the research [1] for $n = 4$ structure through the coefficients Z . The results are similar, but there are some peculiarities. In those systems where the load (the moment of inertia) decreases significantly the application of SRCE-B and SRCE-GL is unacceptable. It is necessary to use SRCE-DS with RR.

The SRCE-DS with the MDS on the order $0,7 \div 0,8$ or with RR should be applied in more stable systems. In fact, the structures of SRCE-B and SRCE-GL work fine only with an accurate nominal setting. In other modes and cases, systems of any order are preferably represented by the structures with the maximum degree of stability.

Parameter s/n	A_{15}	A_{14}	A_{13}	A_{12}	A_{11}	A_{10}	A_9	A_8	A_7	A_6	A_5	A_4	A_3	A_2	A_1	t_{2H}
9	–	–	–	–	–	–	–	9	36	84	126	126	84	36	9	14.4 5
10	–	–	–	–	–	–	10	45	120	210	252	210	120	45	10	15.7 0
11	–	–	–	–	–	11	55	165	330	462	462	330	165	55	11	16.9 6
12	–	–	–	–	12	66	220	495	792	924	792	495	220	66	12	18.2 0
13	–	–	–	13	78	286	715	1287	715	286	1287	715	286	78	13	19.4 4
14	–	–	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	20.6 6
15	–	15	105	455	1365	3003	5005	6432	6432	5005	3003	1365	455	105	15	26.8 7
16	16	120	560	1820	4368	8008	11443	12873	11443	8008	11308	1820	560	120	16	23.0 8

Table 2. Parameters of the high order structures

At the design stage of the system, the structures can be identified, which have the required characteristics, without any calculations. The process of selecting the structures is shown in Figure 1 and Figure 2.

For example, there are restrictions on the overshoot and transient time (figure 2) or, on contrary, there can be the restrictions (figure 1) of the operating modes (the maximum deviations from the nominal values are indicated, i.e. Z_i).

Assume that it is necessary to synthesize the system with $\theta_{no\ max} \leq 10\%$ and $t_{2H} \leq 7,5$ ms. The structure of SRCE-B is unacceptable with such restrictions. However, this system can be considered the optimal SRCE-DS under "considerable overload".

SRCE with real roots are the optimal selection for systems with a wide range of load changes. We recall the notations in the figures: θ – steady-state error, t – settling time, ΔL и μ – amplitude margin and phase margin, H_{n-1} – value of the highest Hurwitz determinant.

It is known that H_{n-1} characterizes the stability of the system indirectly ("the more the better"). Let us recall another useful functional connection. Often the transitional errors are of greater importance than, that in the stable state. According to O.M. Kryzhanovsky, if the system is acted upon by i-disturbance $F_i(t)$, then the integral minimized criterion can be written

$$J_i = \int_0^{\infty} \frac{F_i(t)}{H_{n-1}} dt \quad (2)$$

Note that for all expressions (2) the denominator is the same - H_{n-1} . If at setting it is possible to increase the value of H_{n-1} , then the transient errors should decrease accordingly.

Let us cite one more method of forming the high-speed response SRCE-DS of high and very high order.

Fig. 1. SRCE-DS (restrictions of a overshoot and a transient time)

Fig. 2. SRCE-DS (restrictions of the operating modes)

Such tasks are encountered in navigation, astronomy and other fields. Let the output coordinate be

$$\alpha_{om}(t) = \sum_{k=0}^n R_k, \quad (3)$$

where $R_0 = 1$,

$$R_{2k-1} = (-1)^{n/2} \frac{\prod_{i=1}^{n/2-1} (a^2 + b_i^2) - B_2}{\prod_{i=1}^{n/2-1} (b_k^2 + b_i^2)} e^{-at} \cos b_k t,$$

or

$$R_{2k-1} = (-1)^{n/2} \frac{\prod_{i=1}^{n/2-1} (a^2 + b_i^2) - B_1 + B_2 a}{b_k \prod_{i=1}^{n/2-1} (b_k^2 + b_i^2)} e^{-at} \sin b_k t,$$

where a, b_k and b_i - are the absolute values of the real and imaginary parts of the root; B_1 and B_2 – are forcing signals;

$$i \neq k; k = i, \frac{n}{2}; k \in Z; i \in Z.$$

Here the structure of SRCE-DS is formed - all real parts of the roots are equal to each other. The system (3) gives the exact analytic solution of the equation of motion of even orders for any n , hence the algorithm is:

1. Starting from the given overshoot θ_{no} in %, we determine the first imaginary part of the root

$$b_1 = \frac{a\pi}{\ln \frac{\theta_{no}}{\theta_{00}}}$$

2. Find the values of the remaining b_k

$$b_k = b_1(2k + 1)$$

3. If there are forcing signals $B_1 \neq 0$ and $B_2 \neq 0$, we search for them by the system:

$$\begin{cases} \alpha_0(t_1 \max) = 1 + \theta_{no} \\ \alpha_0(t_2 \min) = 1 - \theta_{no} \end{cases}$$

where $\alpha_0(t_1 \max)$ and $\alpha_0(t_2 \min)$ are typical extremum. For odd orders, system of equations is:

$$R_0 = 1,0$$

$$R_1 = -\frac{\prod_{i=1}^{(n-1)/2} (a^2 + b_i^2) - B_1 + B_2 a}{\prod_{i=1}^{(n-1)/2} (b_i)^2} e^{-at} \cos b_k t$$

$$R_{2k} = (-1)^{(n-1)/2} \frac{a^2 \prod_{i=1}^{(n-3)/2} (a^2 + b_i^2) - B_1 + B_2 a - at}{b_k^2 \prod_{i=1}^{(n-3)/2} (b_k^2 - b_i^2)} e^{-at} \cos b_k t$$

$$R_{2k-1} = (-1)^{(n-1)/2} \frac{-a \prod_{i=1}^{(n-3)/2} (a^2 + b_i^2) + B_2 - at}{b_k \prod_{i=1}^{(n-3)/2} (b_k^2 - b_i^2)} e^{-at} \sin b_k t$$

Here $K = (1 \div \frac{n-1}{2})$. The steps of the algorithm are the same. Further

$$\alpha_0(t) \approx 1 \rightarrow \alpha_0(t_1 \max) = f_1(B_1)$$

$$\alpha_0(t) \approx 1 \rightarrow 2\theta_{no} = f_2(B_2)$$

After obtaining the roots by Vieta's formulas, the coefficients are found and the equation is normalized. Further, we give the equation of the 40th order with $\theta_{no} = 0,5\%$ after normalizing.

$A_{40}' \div A_{35}'$											
0	1,000e+00	0	4,247e+00	1	5,205e+0,0	2	1,864e+0,0	3	1,206e+0,0	3	3,701e+0,0
$A_{34}' \div A_{29}'$											
4	1,657e+0,0	4	4,408e+0,0	5	1,515e+0,0	5	3,520e+0,0	5	9,782e+0,0	6	1,995e+0,0
$A_{28}' \div A_{23}'$											
6	4,617e+0,0	6	8,292e+0,0	7	1,626e+0,0	7	2,573e+0,0	7	4,319e+0,0	7	6,018e+0,0
$A_{22}' \div A_{17}'$											
7	8,693e+0,0	8	1,063e+0,0	8	1,324e+0,0	8	1,415e+0,0	8	1,514e+0,0	8	1,404e+0,0
$A_{16}' \div A_{11}'$											
8	1,285e+0,0	8	1,025e+0,0	7	7,943e+0,0	7	5,372e+0,0	7	3,477e+0,0	7	1,958e+0,0
$A_{10}' \div A_5'$											
7	1,037e+0,0	6	4,736e+0,0	6	1,989e+0,0	5	7,088e+0,0	5	2,251e+0,0	4	5,872e+0,0
$A_4' \div A_0'$											
1,297e+0.07		2,206e+0.03		2,852e+0.02		2,374e+0.01		1,000e+000			

It should be noted that SRCE-DS are written in the form of logarithmic-frequency characteristics [5].

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2. Conclusion

For structures of the 5-8 orders, the values of the parameters of the SRCE-DS at the nominal load ($z = 1.0$) are given. For structures of 9-16 orders, the values of the parameters of the SRCE-DS with equal real roots for $c = 1,0$ (table 2) and $\theta_{no} = 0$ are presented. A general method for the synthesis of the arbitrary order structures are given. It is shown that to build the desirable structure with real roots a non-stiff system of equations can be obtained and solved. Also, a method for selecting a structure based on the required parameters is proposed.

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