DYNAMIC BEHAVIOUR COMPARISON OF THREE DIFFERENT MATHEMATICAL MODEL COMPLEXITIES

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Abstract

Dynamic behaviour comparison of three different mathematical descriptions complexity for first 3 joints of 6 degrees of freedom robotic structure is presented in this article. Firstly, 3D CAD model is designed in SolidWorks, which is used as the basis for a physical and mathematical model. The CAD model is exported directly from SolidWorks to SimMechanics as a physical model which is considered as the most accurate replacement for a real model in this work. The first type of a mathematical model is the most precise but also the most complex; it is based on SolidWorks inertia matrices and matrix form of Lagrange's motion equations of the second kind. The second type of a mathematical model is created by each part replacement with a suitable simplified shape; classical integration approach with Lagrange's motion equations of the second kind is used. The third type of a mathematical model is based on the same approach as the second type, but all the objects are replaced by mass points. At the end, all the results of dynamic behaviour are compared with the physical model, for utilization in controller design.

Keywords: Model-Based-Design; Mathematical model; Simulation; Motion equation; SimMechanics

1. Introduction

Nowadays, a wide range of different support software tools are available; it provides many possibilities and ways to determine mathematical or physical model [4, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19, 20]. Accurate detection of a mathematical model is complicated in the case of a complex mechanical structure with many parts, and it should not be used for simulation purposes and controller determination at the same time. The chosen way is to split the problem between 2 models is: the most accurate model for a simulation as a real (physics) model substitution - physical model [4, 9, 14, 18] and appropriately simplified model purely for a controller determination - mathematical model [7, 19, 20].

For a mathematical and physical model, a CAD model is very helpful. SolidWorks is used for a CAD model design [12, 14, 15, 16, 17, 18]. Only the shape and the material properties of an object are needed to know for a full dynamic description; these parameters are included in the CAD model. This means that the CAD model should always be the first step in an accurate dynamic simulation of any system.

The physical model is exported directly from SolidWorks to the SimMechanics Second Generation [14, 15]. It includes all dimensions, joints and material properties of the CAD model. It is the most accurate model that includes all important shapes (inertia matrices) and mass details for dynamic simulation purposes [16, 17].

The first type of mathematical model is the most accurate but also the most complex. It is based on inertia matrices [4, 11], transformation matrices [1, 3, 14, 17, 20], centres of mass position vectors [2, 16] and model masses. All of these parameters are taken from the SolidWorks CAD model. In this case, the matrix form of Lagrange's motion equations of the second kind is used [2, 16].

The second type of mathematical model uses shape simplification. Every link is replaced by a mass rod, every actuator is superseded by a mass point and significant cylindrical part is replaced by a mass cylindrical plate. The whole assembly is divided into many essential parts whose contribution of energy sum is the energy of the entire assembly.

The third type of mathematical model is similar to the second type, but all stationary objects are replaced by mass points. This mathematical model is the simplest, but its accuracy is the lowest.

In the results of this work, the differences between the above mentioned mathematical descriptions during dynamic simulation should be visible. These results should help to understand when it is better to use a more precise mathematical model and when the simplest model is sufficient as a basis for design control law.

2. CAD model

The SolidWorks CAD model consists of 9 parts as shown in Fig.1. Each part has an associated local coordinate system, the material and joins each other by structural bonds to the final fully bonded assembly [12, 14, 15, 16].

Fig. 1. The CAD model of the analysed system

The CAD model has 6 degrees of freedom (DOF) [3, 18] generally, but the first 3 joints have the most interesting dynamic behaviour and will be studied in this paper. Therefore, the last 3 DOF (parts 7, 8 and 9 in Fig. 1.) will be fixedly connected to each other and with the link 6.

3. Physical model

The SolidWorks model is exported directly into the SimMechanics Second Generation diagram along with all the linkages, dimensions and material properties [14, 15].

Fig. 2. SimMechanics Second Generation physical model

A physical model is considered to be the most accurate substitute for a real physics system in case of precise CAD model which is the SimMechanics model based on. All mathematical models dynamic behaviours are compared with the physical model to see how accurate the mathematical models are.

4. Mathematical models

Fig. 3. Graphic preview of mathematical model simplifications

Three different CAD model simplifications are shown in Fig. 3. and their mathematical descriptions are derived below.

4.1 Transformation of coordinate systems

The transformation matrices are used to determine the position and orientation of a local coordinate system firmly connected with the component in the overall global coordinate system [8, 15]. They are an integral part of any complex mathematical model determination.

Fig. 4. Simplified models with the selected coordinate system placement

$$T_1 = {}^0T_2 = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 & 0 \\ 0 & 0 & 1 & a \\ -\sin(\alpha) & -\cos(\alpha) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$T_2 = {}^0T_4 = \begin{bmatrix} \cos(\beta)\cos(\alpha) & \cos(\alpha)\sin(\beta) & -\sin(\alpha) & b \cdot \cos(\alpha) - d \cdot \sin(\alpha) \\ \sin(\beta) & -\cos(\beta) & 0 & a + c \\ -\cos(\beta)\sin(\alpha) & -\sin(\alpha)\sin(\beta) & -\cos(\alpha) & -d \cdot \cos(\alpha) - b \cdot \sin(\alpha) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$T_3 = {}^0T_6 = \begin{bmatrix} \cos(\beta + \varphi)\cos(\alpha) & \cos(\alpha)\sin(\beta + \varphi) & -\sin(\alpha) & b \cdot \cos(\alpha) - d \cdot \sin(\alpha) + l \cdot \cos(\alpha)\cos(\beta) \\ \sin(\beta + \varphi) & -\cos(\beta + \varphi) & 0 & a + c + l \cdot \sin(\beta) \\ -\cos(\beta + \varphi)\sin(\alpha) & -\sin(\alpha)\sin(\beta + \varphi) & -\cos(\alpha) & -d \cdot \cos(\alpha) - b \cdot \sin(\alpha) - l \cdot \sin(\alpha)\cos(\beta) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The centres of mass position vectors for the above mentioned local coordinate systems are different for each mathematical model simplification. In general, they are listed as:

$$r_1 = {}^2r_2 = [r_{11} \ r_{12} \ r_{13} \ I]^T; \quad r_2 = {}^4r_4 = [r_{21} \ r_{22} \ r_{23} \ I]^T \quad r_3 = {}^6r_6 = [r_{31} \ r_{32} \ r_{33} \ I]^T \quad (4)$$

For the next steps, the real model parameters are:

$$\begin{aligned} a &= 0,171m; & b &= 0,065m; & c &= 0,065m; & d &= 0,031m; & l &= 0,3m \\ m_1 &= 2,938kg; & m_2 &= 1,008kg; & m_3 &= 4,565kg; & g &= 9,81m \cdot s^{-2} \end{aligned} \quad (5)$$

4.2 The first mathematical model - no simplification

The first model (Fig. 4. part 1) has a different approach to the motion equation derivation than the other two models. It uses the matrix form of Lagrange's motion equations of the second kind. This mathematical description is the most accurate of the three listed models because of inertia matrix using with a centre of mass position vectors from the CAD model, which are with transformation matrices all necessary parameters to need to know.

Motion equations matrix form has the general form [2, 16]:

$$D(q) \cdot \ddot{q} + H(q, \dot{q}) + G(q) = T \quad \Rightarrow \quad D(\alpha, \beta, \varphi) \cdot \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \\ \ddot{\varphi} \end{bmatrix} + H(\alpha, \dot{\alpha}, \beta, \dot{\beta}, \varphi, \dot{\varphi}) + G(\alpha, \beta, \varphi) = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \quad (6)$$

in which:

$$\begin{aligned} D_{i,j} &= \sum_{r=\max\{i,j\}}^n tr \left[\frac{\partial^0 T_r}{\partial q_j} \cdot {}^r \bar{I}_r \left(\frac{\partial^0 T_r}{\partial q_i} \right)^T \right] \\ H_i &= \sum_{k=1}^n \sum_{m=1}^n H_{ikm} \dot{q}_k \dot{q}_m \\ H_{ikm} &= \sum_{r=\max\{i,k,m\}}^n tr \left[\frac{\partial^2 T_r}{\partial q_k \partial q_m} \cdot {}^r \bar{I}_r \left(\frac{\partial^0 T_r}{\partial q_i} \right)^T \right] \\ G_i &= - \sum_{r=i}^n m_r \left({}^0 \bar{g} \right)^T \cdot \frac{\partial^0 T_r}{\partial q_i} \cdot {}^r \bar{r}_r \end{aligned} \quad (7)$$

As could be seen, the only two other variables need to be known. These parameters are found in the SolidWorks CAD model.

The centres of mass position vectors are:

$$r_1 = [0,057 \ -0,053 \ 0,058 \ I]^T; \quad r_2 = [-0,15 \ 0 \ -0,004 \ I]^T; \quad r_3 = [0,099 \ 0 \ -0,065 \ I]^T \quad (8)$$

The pseudo-inertia matrices are:

$$\bar{I}_1 = \begin{bmatrix} 0,0120 & -0,0101 & 0,0108 & 0,1675 \\ -0,0101 & 0,0124 & -0,0101 & -0,1557 \\ 0,0108 & -0,0101 & 0,0117 & 0,1704 \\ 0,1675 & -0,1557 & 0,1704 & 2,9380 \end{bmatrix}; \quad \bar{I}_2 = \begin{bmatrix} 0,0352 & 0 & -0,0005 & 0,1512 \\ 0 & 0,0005 & 0 & 0 \\ -0,0005 & 0 & 0,0002 & -0,0040 \\ 0,1512 & 0 & -0,0040 & 1,0080 \end{bmatrix}; \quad \bar{I}_3 = \begin{bmatrix} 0,1069 & 0 & -0,0140 & 0,4519 \\ 0 & 0,0011 & 0 & 0 \\ -0,0140 & 0 & 0,0268 & -0,2967 \\ 0,4519 & 0 & -0,2967 & 4,4650 \end{bmatrix} \quad (9)$$

The specific solution for the system shown in equation (6) is:

$$D\mathbf{1} = \begin{bmatrix} D_{11} & 5,008 \cdot 10^{-5} \sin(\beta + \varphi) - 0,04242 \sin(\beta) & 5,008 \cdot 10^{-5} \sin(\beta + \varphi) \\ 5,008 \cdot 10^{-5} \sin(\beta + \varphi) - 0,04242 \sin(\beta) & 0,2712 \cos(\varphi) + 0,5546 & 0,1356 \cos(\varphi) + 0,108 \\ 5,008 \cdot 10^{-5} \sin(\beta + \varphi) & 0,1356 \cos(\varphi) + 0,108 & 0,108 \end{bmatrix} \quad (10)$$

$$D_{11} = 0,1356 \cos(2\beta + \varphi) + 0,2228 \cos(2\beta) + 0,05875 \cos(2\beta) + 0,1977 \cos(\beta) + 0,1356 \cos(\varphi) + 0,05292 \cos(2\beta + 2\varphi) + 0,3389$$

$$\mathbf{H1} = \begin{bmatrix} \dot{\beta}^2 [5,008 \cdot 10^{-5} \cos(\beta + \varphi) - 0,04242 \cos(\beta)] + 1,002 \cdot 10^{-3} \cos(\beta + \varphi) \dot{\beta} \dot{\varphi} - 2 \dot{\alpha} \dot{\beta} [0,1356 \sin(2\beta + \varphi) + 0,2228 \sin(2\beta) + \\ + 0,02938 \sin(\beta + \varphi) + 0,09885 \sin(\beta) + 0,05292 \sin(2\beta + 2\varphi)] + 5,008 \cdot 10^{-5} \cos(\beta + \varphi) \dot{\varphi} - 2 \dot{\alpha} \dot{\varphi} [0,06779 \sin(2\beta + \varphi) + \\ + 0,02938 \sin(\beta + \varphi) + 0,06779 \sin(\varphi) + 0,05292 \sin(2\beta + 2\varphi)] \\ \alpha^2 [0,05292 \sin(2\beta + 2\varphi) + 0,1356 \sin(2\beta + \varphi) + 0,2228 \sin(2\beta) + 0,02938 \sin(\beta + \varphi) + 0,09885 \sin(\beta)] - \\ - 0,1356 \sin(\varphi) \dot{\varphi}^2 - 0,2712 \sin(\varphi) \dot{\beta} \dot{\varphi} \\ \alpha^2 [0,06779 \sin(2\beta + 2\varphi) + 0,02938 \sin(\beta + \varphi) + 0,06779 \sin(\varphi) + 0,05292 \sin(2\beta + 2\varphi)] + 0,1356 \sin(\varphi) \dot{\beta}^2 \end{bmatrix} \quad (11)$$

$$\mathbf{G1} = \begin{bmatrix} 0 \\ 4,433 \cos(\beta + \varphi) + 14,92 \cos(\beta) \\ 4,433 \cos(\beta + \varphi) \end{bmatrix} \quad (12)$$

4.3 The second mathematical model - shape simplification

The second model (Fig. 4. part 2) uses shape simplification as described in the introduction part. The Lagrange equations of the second kind were used to find the motion equations. In order to use Lagrange equations, it is necessary to define the kinetic and potential energy of the whole system. The transformation matrices and the centre of mass position vectors are used to determine the absolute position and velocity of each simplified part. This mathematical description is not as accurate as the previous model but should be sufficient even for simulation purposes because of the mass distribution cause - it is analyzed in results comparison.

Due to the motion equations derivation complexity (each part has similar but not the same approach to find kinetic and potential energy) will be described only one simplified part (Fig. 1. part 4) kinetic/potential energy determination and then the motion equations will be derived.

The position vector in the global coordinate system:

$$\mathbf{p}_1 = \mathbf{T}_2 \cdot \mathbf{r}_2 = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 1 \end{bmatrix}^T \quad (13)$$

The velocity vector in the global coordinate system:

$$\mathbf{v}_1 = \frac{d\mathbf{p}_1}{dt} = \begin{bmatrix} v_{11} & v_{12} & v_{13} & 1 \end{bmatrix}^T \quad (14)$$

The kinetic energy:

$$dE_{K1} = \frac{1}{2} \cdot dm_1 \cdot (v_{11}^2 + v_{12}^2 + v_{13}^2) \quad (15)$$

$$E_{K1} = \iint_D dE_{K1}$$

The potential energy (gravitation is in the global axis Y direction, therefore the p_{12} position is used):

$$E_{P1} = m_1 \cdot g \cdot p_{12} \quad (16)$$

The kinetic and potential energy of the other six parts (Fig. 1.) is determined in a similar way like for the above-mentioned part. The total kinetic and potential energy of the system is:

$$E_K = \sum_{i=1}^7 E_{Ki} \quad E_P = \sum_{i=1}^7 E_{Pi} \quad (17)$$

The Lagrangian:

$$L = E_K - E_P \quad (18)$$

The final motion equations:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\alpha}} \right) - \frac{\partial L}{\partial \alpha} = T_1; \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\beta}} \right) - \frac{\partial L}{\partial \beta} = T_2; \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\varphi}} \right) - \frac{\partial L}{\partial \varphi} = T_3; \quad (19)$$

Equation (19) can be also rewritten into the motion equations matrix form shown in equation (6). For the parameters in equation (5) and the corresponding centres of mass position vectors, the motion equations for the second mathematical model are:

$$\mathbf{D2} = \begin{bmatrix} D_{11} & -0,04211\sin(\beta) & 0 \\ -0,04211\sin(\beta) & 0,2666\cos(\varphi)+0,5497 & 0,1333\cos(\varphi)+0,1045 \\ 0 & 0,1333\cos(\varphi)+0,1045 & 0,1045 \end{bmatrix} \quad (20)$$

$$D_{11} = 0,1333\cos(2\beta + \varphi) + 0,2226\cos(2\beta) + 0,05776\cos(\beta + \varphi) + 0,1975\cos(\beta) + 0,1333\cos(\varphi) + 0,05227\cos(2\beta + 2\varphi) + 0,3314$$

$$\mathbf{H2} = \begin{bmatrix} -\dot{\alpha}\dot{\varphi}[0,1333\sin(2\beta + \varphi) + 0,05776\sin(\beta + \varphi) + 0,1333\sin(\varphi) + 0,1045\sin(2\beta + 2\varphi)] - 0,04211\cos(\beta)\dot{\beta}^2 - \\ -\dot{\alpha}\dot{\beta}[0,2666\sin(2\beta + \varphi) + 0,4451\sin(2\beta) + 0,05776\sin(\beta + \varphi) + 0,1975\sin(\beta) + 0,1045\sin(2\beta + 2\varphi)] \\ \dot{\alpha}^2[0,05227\sin(2\beta + 2\varphi) + 0,1333\sin(2\beta + \varphi) + 0,2226\sin(2\beta) + 0,02888\sin(\beta + \varphi) + 0,09873\sin(\beta)] - \\ -\dot{\varphi}[0,2666\sin(\varphi)\dot{\beta} + 0,1333\sin(\varphi)\dot{\varphi}] \\ \dot{\alpha}^2[0,05227\sin(2\beta + 2\varphi) + 0,06665\sin(2\beta + \varphi) + 0,02888\sin(\beta + \varphi) + 0,06665\sin(\varphi)] + 0,1333\sin(\varphi)\dot{\beta}^2 \end{bmatrix} \quad (21)$$

$$\mathbf{G2} = \begin{bmatrix} 0 \\ 4,359\cos(\beta + \varphi) + 14,9\cos(\beta) \\ 4,359\cos(\beta + \varphi) \end{bmatrix} \quad (22)$$

This mathematical description is not too complicated compared to the previous equations (10), (11) and (12). Nevertheless, there is one more solution with a simpler mathematical description determination.

4.4 The third mathematical model - mass point simplification

The third model (Fig. 4. part 3) uses mass point simplification which quantity is equal to the DOF number. The Lagrange equations of the second kind were also used to the motion equations determination. There is the same approach as in the previous chapter 4.3, but all the fixed parts are replaced by the mass points located in their centre of mass. Their number is three because the model has three DOF. This mathematical description is not as accurate as previous two models and should not be used for the simulation purposes because all the mass is concentrated in the centre of mass of each fixed body parts only. This means the mass distribution has been disturbed.

However, this model is easiest to derive and should be the most appropriate for the design control law purposes because of its expected simplicity. The differences between dynamic behaviour will be determined in the next chapter. The third model's motion equations matrix form for the parameters in equation (5) and appropriate centres of mass position vectors are:

$$\mathbf{D3} = \begin{bmatrix} D_{11} & -0,01537\sin(\beta + \varphi) - 0,04248\sin(\beta) & -0,01537\sin(\beta + \varphi) \\ -0,01537\sin(\beta + \varphi) - 0,04248\sin(\beta) & 0,2712\cos(\varphi) + 0,4783 & 0,1356\cos(\varphi) + 0,04474 \\ -0,01537\sin(\beta + \varphi) + 0,04474 & 0,1356\cos(\varphi) + 0,04474 & 0,04474 \end{bmatrix} \quad (23)$$

$$D_{11} = 0,1356\cos(2\beta + \varphi) + 0,2168\cos(2\beta) + 0,05875\cos(\beta + \varphi) + 0,1977\cos(\beta) + 0,1356\cos(\varphi) + 0,02237\cos(2\beta + 2\varphi) + 0,2865$$

$$\mathbf{H3} = \begin{bmatrix} -\dot{\alpha}\dot{\varphi}[0,1356\sin(2\beta + \varphi) + 0,05875\sin(\beta + \varphi) + 0,1356\sin(\varphi) + 0,04474\sin(2\beta + 2\varphi)] - 0,04248\cos(\beta)\dot{\beta}^2 - \\ -\dot{\alpha}\dot{\beta}[0,2712\sin(2\beta + \varphi) + 0,4335\sin(2\beta) + 0,05875\sin(\beta + \varphi) + 0,1977\sin(\beta) + 0,04474\sin(2\beta + 2\varphi)] - \\ -0,01537\cos(\beta + \varphi)\dot{\beta}^2 - 0,01537\cos(\beta + \varphi)\dot{\varphi}^2 - 0,03073\cos(\beta + \varphi)\dot{\beta}\dot{\varphi} \\ \dot{\alpha}^2[0,02237\sin(2\beta + 2\varphi) + 0,1356\sin(2\beta + \varphi) + 0,2168\sin(2\beta) + 0,02938\sin(\beta + \varphi) + 0,09885\sin(\beta)] - \\ -\dot{\varphi}[0,2712\sin(\varphi)\dot{\beta} + 0,1356\sin(\varphi)\dot{\varphi}] \\ \dot{\alpha}^2[0,02237\sin(2\beta + 2\varphi) + 0,06779\sin(2\beta + \varphi) + 0,02938\sin(\beta + \varphi) + 0,06779\sin(\varphi)] + 0,1356\sin(\varphi)\dot{\beta}^2 \end{bmatrix} \quad (24)$$

$$\mathbf{G3} = \begin{bmatrix} 0 \\ 4,433\cos(\beta + \varphi) + 14,92\cos(\beta) \\ 4,433\cos(\beta + \varphi) \end{bmatrix} \quad (25)$$

As could be seen, the simplest model with only three mass points does not mean simplest final motion equations. The second mathematical model is more accurate and not too complex, but its determination was hardest.

5. Comparison of results

The resulting motion equations are compared with the physical model torque outputs in Fig. 5. The SimMechanics physical model is considered to be the most accurate dynamic model here and therefore, any deviation indicates an inaccurate approximation.

Fig. 5. The physical and the three different mathematical models comparison

As shown in Fig. 5., all the model's dynamic behaviour is very similar, only in the third DOF third mathematical model output is more obvious deflection. The motion, which the listed torques correspond to, was chosen as the most dynamic technological movement as could be required. For more detailed research, the Fig. 5. is enlarged in Fig. 6.

Fig. 6. The physical and the three different mathematical models comparison - detailed view

The differences between all the mathematical models and the physical model are clear now. The red thick solid line shows the physical model. The black dashed line associated with the first and the most accurate mathematical model almost overlaps the thin blue line related to the second model (shape simplification). All three of the above-mentioned lines are highly similar and the dynamic behaviour for all of them is essentially the same. The last green dot line associated with the third mathematical model (mass point simplification) is also noticeably similar, but this model should not be used for a simulation purpose, thus the real model substitution, because of its dynamic deviation compares to the physical model. The dynamic deflection is not crucial, but the other two mathematical models are better for this purpose for certain.

6. Conclusion

The effort in this work was devoted to the dynamic behaviour comparison of three different mathematical models. As could be found, everything starts with a CAD model. It is an essential part of every accurate dynamic model determination. The CAD model was exported to the SimMechanics physical model which is considered to be the most accurate replacement of a real physics system. All the dynamic behaviours of the mathematical models have been compared with the physical model to see how accurate the mathematical models are.

The first mathematical model derivation uses matrices of inertia from CAD model. That means an accurate description of mass distribution directly in motion equations. This mathematical model is the most accurate, but a physical interpretation is not clear during a derivation. Also, calculation errors are very difficult to find.

The second mathematical model uses an integration approach with every part simplification. This approach is the most complicated in terms of derivation, but it is intuitive and accuracy can be continuously explored. Its accuracy is, surprisingly, almost as precise as the first mathematical model. It means that the shape of any part is not decisive in the case of an appropriate approximation selection.

The third mathematical model derivation is similar to the second type, but all stationary objects are replaced with mass points. The accuracy of a mass distribution description is lowest and the shape of motion equations is not as simple as expected. However, the own motion equations derivation is the simplest.

Ultimate dynamic behaviour is highly similar to the first and second mathematical model and based on their accuracy; both mathematical models can be even used as simulation models. The third mathematical model has also satisfactory accuracy which is more than enough for the control law design but should not be used as a real model substitution for simulation purposes.

All things considered, the shape precision directly in motion equations has not a significant influence in a dynamic behaviour description as could be expected if a centre of mass position is known. Further work could be focused on the use of derived motion equations to the design control law and comparison of on that basis designed regulators accuracy.

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