DIRECT DISPLACEMENT PROBLEM OF THE 6-DOF GOUGH-STEWART PLATFORM

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This Publication has to be referred as: Gebel, E[lena] (2016). Direct Displacement Problem of the 6-DOFGough-Stewart Platform, Proceedings of the 27th DAAAM International Symposium, pp.0452-0456, B. Katalinic (Ed.), Published by DAAAM International, ISBN 978-3-902734-08-2, ISSN 1726-9679, Vienna, Austria
DOI: 10.2507/27th.daaam.proceedings.067

Abstract

In order to obtain direct displacement solutions of parallel manipulator without divergence in real time, a Newton-Raphson algorithm was proposed for direct displacement analysis of six-degree-of-freedom (DOF) Gough-Stewart platform. Based on geometrical frame of parallel mechanism, the highly nonlinear equations of kinematics were arranged into the homogeneous transformation matrix. The procedure of Newton-Raphson algorithm was programmed in Matlab/Simulink. The performance of the proposed algorithm for the 6-PSS parallel mechanism was analysed and confirmed. Applying Newton-Raphson algorithm, the real generalized pose of movable base is solved by using the given position of actuators.

Keywords: parallel mechanism; direct displacement problem; Newton-Raphson algorithm.

1. Introduction

With development of the manufacturing technology, robots are used in industrial applications where high accuracy, repeatability and stability of operations are required. Various parallel robots with six degree of freedom have been widely used in many fields including flight simulators, rehabilitation robotics and coordinate measuring devices [1]. Compared to other kinds of robots, the parallel robots have special kinematic characteristics because of their limited actuator displacements. As we usually assume that the number of bars of the parallel robot is equal to its degree of freedom, and only one actuator is needed for each bar [2]. Then based on the Chebyshev criterion, each bar of the 6-DOF parallel robot should be a 6-DOF serial kinematic chain. Once one prismatic joint and two spherical joints are applied in one bar, the only chain structure for 6-DOF is P-S-S (Fig. 1).

The workspace of 6-DOF parallel robot is hard to compute and express [3 – 7], because it is a complex six dimensional volume with non-linear boundaries. Direct kinematics is referred to the calculation technique of the movable base's position from the preset values of actuator joints. For the most 6- DOF parallel robot, the nonlinear direct kinematic equations can be solved by the algorithms using Newton method [4, 11], homotopy method [5], neural network [6] and algebraic elimination [7]. The singularity of 6-DOF parallel robot is a special mechanism pose where resultant displacement from actuators acted upon the movable base generates into a dimension less than six. The singularities usually lead the direct kinematic algorithm of 6-DOF parallel robot to converge into wrong solutions if improper initial value is chosen.

Fig. 1. Kinematic scheme of the 6-DOF parallel mechanism

This paper aims to obtain the common algorithm for the direct displacement analysis based on the Newton method and then the common kinematic constraints of 6-PSS parallel robot for a proper motion domain determination. Compared to the complex six-dimensional workspace, a proper motion range for the actuators is more convenient for application purpose.

2. Direct displacement problem

There are two kinds of the situation: firstly, at the beginning of work (calibration of the robot) and secondly, the real-time control of the parallel mechanism, when it is needed to solve the direct displacement problem. Mentioned above the task for a parallel mechanism is more complex than the inverse kinematics because the corresponding mathematical model can have a lot of possible solutions.

The direct displacement problem is referred to as the expression of the functional dependence $P = f(L)$ describing the ratio between the upper base position and given values of actuator joints. In common way the mathematical model of the direct displacement problem of 6-DOF Gough-Stewart platform can contain six (three angles and the coordinates of the end-effector) of nine (three coordinates of three points on the movable base) unknown variables.

Based on the principals of analytical algebra the equations of the holonomic stationary ideal link between the spherical joints of the upper and lower bases are expressed in the following way:

$$\begin{cases} l_4^2 = (x_{A1} - x_{A2})^2 + (y_{A1} - y_{A2})^2 + (z_{A1} - z_{A2})^2; \\ l_5^2 = (x_{A1} - x_{B2})^2 + (y_{A1} - y_{B2})^2 + (z_{A1} - z_{B2})^2; \\ l_6^2 = (x_{B1} - x_{B2})^2 + (y_{B1} - y_{B2})^2 + (z_{B1} - z_{B2})^2; \\ l_1^2 = (x_{B1} - x_{C2})^2 + (y_{B1} - y_{C2})^2 + (z_{B1} - z_{C2})^2; \\ l_2^2 = (x_{C1} - x_{C2})^2 + (y_{C1} - y_{C2})^2 + (z_{C1} - z_{C2})^2; \\ l_3^2 = (x_{C1} - x_{A2})^2 + (y_{C1} - y_{A2})^2 + (z_{C1} - z_{A2})^2. \end{cases} \quad (1)$$

Where distance l_i is equal to the magnitude of the co-directional vector B_iC_i with the same bar of the parallel mechanism so $l_2 = |C_1A_2|$, $l_3 = |C_1A_2|$, $l_4 = |A_1A_2|$, $l_5 = |A_1B_2|$, $l_6 = |B_1B_2|$.

Failing dependences for the calculation the movable base's position are found out by using the apparatus of analytical geometry in space:

$$\begin{cases} (x_{A2} - x_{B2})^2 + (y_{A2} - y_{B2})^2 + (z_{A2} - z_{B2})^2 = 3R_2^2; \\ (x_{A2} - x_{C2})^2 + (y_{A2} - y_{C2})^2 + (z_{A2} - z_{C2})^2 = 3R_2^2; \\ (x_{C2} - x_{B2})^2 + (y_{C2} - y_{B2})^2 + (z_{C2} - z_{B2})^2 = 3R_2^2. \end{cases} \quad (2)$$

The bases of the parallel mechanism are equilateral triangles $A_1B_1C_1$ and $A_2B_2C_2$ rotated to each other at the angle 60° . The vertices of the upper and lower basis are specified using by the magnitudes of the circle radii R_1 and R_2 which has been set before.

The resultant mathematical model is combined the formulae (1) and (2) and present the non-linear equation system which can be solved by well-known numerical methods.

For the more convenient mode of control of the studied mechanism, the results of the direct positional problem should be rearranged to the homogeneous matrix that is offered by J. Denavit and R. Hartenberg. The equation of the plane passing through three points namely vertexes of movable base $A_2B_2C_2$ are derived by:

$$\begin{vmatrix} x - x_{A2} & x_{B2} - x_{A2} & x_{C2} - x_{A2} \\ y - y_{A2} & y_{B2} - y_{A2} & y_{C2} - y_{A2} \\ z - z_{A2} & z_{B2} - z_{A2} & z_{C2} - z_{A2} \end{vmatrix} = 0 \quad (3)$$

Rearrange the expression (3) to the equation of the straight line in a space which is passed through the given point:

$$(x - x_{A2}) \cdot d_1 - (y - y_{A2}) \cdot d_2 + (z - z_{A2}) \cdot d_3 = 0 \quad (4)$$

Where variables d_i ($i=1, \dots, 3$) are the coordinates of the normal vector and equal to the determinants of square matrices:

$$\begin{aligned} d_1 &= (y_{B2} - y_{A2}) \cdot (z_{C2} - z_{A2}) - (y_{C2} - y_{A2}) \cdot (z_{B2} - z_{A2}); \\ d_2 &= (x_{B2} - x_{A2}) \cdot (z_{C2} - z_{A2}) - (x_{C2} - x_{A2}) \cdot (z_{B2} - z_{A2}); \\ d_3 &= (x_{B2} - x_{A2}) \cdot (y_{C2} - y_{A2}) - (x_{C2} - x_{A2}) \cdot (y_{B2} - y_{A2}). \end{aligned} \quad (5)$$

Transfer vector $(x_{O2} \ y_{O2} \ z_{O2})^T$ in the matrix of the homogeneous transformation is found out by using the following formulae:

$$(x_{O2} \ y_{O2} \ z_{O2})^T = \left(\frac{x_{A2} + x_{B2} + x_{C2}}{3}, \frac{y_{A2} + y_{B2} + y_{C2}}{3}, \frac{z_{A2} + z_{B2} + z_{C2}}{3} \right)^T \quad (6)$$

The directional cosines of the normal vector $n = (d_1; -d_2; d_3)$ to the plane that is described through (4) is calculated as follows:

$$\begin{aligned} a_x &= \cos \alpha_{O2z} = \frac{d_1}{\sqrt{d_1^2 + d_2^2 + d_3^2}}; \\ a_y &= \cos \beta_{O2z} = \frac{-d_2}{\sqrt{d_1^2 + d_2^2 + d_3^2}}; \\ a_z &= \cos \gamma_{O2z} = \frac{d_3}{\sqrt{d_1^2 + d_2^2 + d_3^2}}. \end{aligned} \quad (7)$$

Elements s_x, s_y, s_z of the transformation matrix which are characterized the rotation around the axis O_2y are equal to the angle between the unit vector of O_2z axes and vector $\overrightarrow{O_2C_2}$

$$\begin{aligned} s_x &= \cos \alpha_{O2y} = \frac{x_{C2} - x_{O2}}{\sqrt{(x_{C2} - x_{O2})^2 + (y_{C2} - y_{O2})^2 + (z_{C2} - z_{O2})^2}}; \\ s_y &= \cos \beta_{O2y} = \frac{y_{C2} - y_{O2}}{\sqrt{(x_{C2} - x_{O2})^2 + (y_{C2} - y_{O2})^2 + (z_{C2} - z_{O2})^2}}; \\ s_z &= \cos \gamma_{O2y} = \frac{z_{C2} - z_{O2}}{\sqrt{(x_{C2} - x_{O2})^2 + (y_{C2} - y_{O2})^2 + (z_{C2} - z_{O2})^2}}. \end{aligned} \quad (8)$$

Constituents of the rotation angle around the O_2x axes are computed through the vector product of the unit – vectors of the O_2x - and O_2z - axes:

$$\begin{aligned} n_x &= \cos \alpha_{O2x} = s_y \cdot a_z - s_z \cdot a_y; \\ n_y &= \cos \beta_{O2x} = s_z \cdot a_x - s_x \cdot a_z; \\ n_z &= \cos \gamma_{O2x} = s_x \cdot a_y - s_y \cdot a_x; \end{aligned} \quad (9)$$

Combining (6) – (9), the matrix of homogeneous transformation is formed to rearrange the position of the end-effector from relative coordinate system $O_2x_2y_2z_2$ to absolute fixed system $O_1x_1y_1z_1$:

$$T = \begin{bmatrix} n_x & s_x & a_x & x_{O2} \\ n_y & s_y & a_y & y_{O2} \\ n_z & s_z & a_z & z_{O2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

Thus, the mathematical model of the direct displacement problem is presented in form of homogeneous matrix (10). The solution of this task is carried out by using the different methods, for examples: the exceptions [7], the polynomial continuation [8], Grobner bases [9], interval analysis [10] etc. The first mention above the method gives no sustainable solution in numerical form since there are solution omissions and side roots. The second method is more stable than first. Using the last two methods excludes the problems mentioned above and a permit providing the given accuracy, though the time taken for its usage is more than it is needed for real-time control of a mechanism.

If priory information (initial approximation) for Gough-Stewart platform is known, the iterative Newton-Raphson or Gauss-Newton methods [4, 11] are more appropriate. For the studied mechanism the direct displacement algorithm is given as follows:

1. Input the desire actuators displacement l_i ($i = 1 \div 6$), and calculate the corresponding T_i for the Gough-Stewart platform, choose the initial position:

$$L_0 = f(X_0). \quad (11)$$

2. Calculate a position correction through the above-mention nonlinear direct displacement algorithm:

$$X_1 = X_0 + A(L_1 - f(X_0)). \quad (12)$$

Where matrix A is obtained as $A = \left(\frac{\partial f}{\partial l_i} \right)^{-1} \Big|_{X_0}$.

3. If $\|f(p, \Delta p)\| \leq \varepsilon_p$, then $p = p + \Delta p$; else back to step 2) for new Δp . Where $\| \cdot \|$ denotes the norms in corresponding parameters space.
4. If $\|f(p, \Delta p)\| \leq \varepsilon_c$, then output the present p as the result; else back to step 2) for a new iteration.

3. Simulation of Gough -Stewart platform

Fig. 1 shows the 6-DOF parallel mechanism we used in this research. Based on the mechanism structure parameters and values of radii $R_1 = 100\text{mm}$ and $R_2 = 25\text{mm}$, the terminal coordinates in the corresponding coordinates system are presented in Table. 1.

	G₁ – Homogeneous coordinates of joints on the lower base in $O_1x_1y_1z_1$ frame, mm	G₂ – Homogeneous coordinates of joints on the upper base in $O_2x_2y_2z_2$ frame / mm
Chain 1. B ₁ C ₂	(100, 0, 0, 0)	(-12.5, -21.651, 0, 0)
Chain 2. C ₁ C ₂	(-50, -86.603, 0, 0)	(-12.5, -21.651, 0, 0)
Chain 3. C ₁ A ₂	(-50, -86.603, 0, 0)	(-12.5, 21.651, 0, 0)
Chain 4. A ₁ A ₂	(-50, 86.603, 0, 0)	(-12.5, 21.651, 0, 0)
Chain 5. A ₁ B ₂	(-50, 86.603, 0, 0)	(25, 0, 0, 0)
Chain 6. B ₁ B ₂	(100, 0, 0, 0)	(25, 0, 0, 0)

Table 1. Terminal coordinates for the 6-DOF parallel mechanism

The uniqueness problem for direct kinematic calculation is more complex, which can be transformed into an objective relation proof. In this research, we proposed a robust direct displacement algorithm to determine the uniqueness of direct kinematic solution by numerical verification.

Normally, the numerical direct displacement analysis methods like Newton-Raphson method [4, 11], Jacobi method [12] and Powell method [13] can achieve high accuracy in the parallel robot analysis when the initial pose is not far from the final result. All of these methods may fail to refresh the value of p when $\det(A) = 0$. Then the iteration can be trapped by a local minimum solution. Even the methods without Jacobian matrix (like Powell method) may be affected by the bifurcation or singularities.

Here presented Newton-Raphson method to obtain the direct displacement solution of the 6-PSS parallel mechanism. For researched Gough-Stewart platform, once the actuators output L is known, the whole kinematic information of G_1 is determined. Newton-Raphson method searches the solution from actuator workspace directly, which means all the temporary status appearing during the iteration can be realized by real 6-PSS parallel mechanism.

Fig .2. Procedure of direct displacement method

Fig. 2 shows the procedures of the direct kinematic method. The blue sketches are the initial status of the PSS parallel mechanism, the black sketches are the final status for the direct kinematic solutions. In Fig. 2, the numerical iteration steps are limited by penalty coefficient, and there is a maximum step limit for the modification of l_i

5. Conclusion

In this paper, a through direct displacement analysis of 6-PSS parallel mechanism has been experienced. The method for direct displacement problem for the 6-DOF Gough-Stewart platform is proposed respectively. The simulation results validate the effectiveness of the proposed method.

In the future work, the proposed method will be applied for available workspace determination, singularity analysis and bifurcation avoidance.

6. References

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