ROBUST STABILIZATION OF A CONTINUOUS STIRRED TANK REACTOR MODEL

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Abstract

The main goal of this contribution is to outline an approach to robust stabilization of a continuous stirred tank reactor (CSTR) by means of PI controller. The linearized mathematical model of the CSTR with an exothermic reaction is supposed in the form of the fourth order interval system and its robust stabilization is based on combination of a method for calculating/plotting the stability boundary, the sixteen plant theorem and an algebraic approach to control design. The validity of the technique is verified on a simulative example of CSTR control.

Keywords: Robust stability; continuous stirred tank reactor; CSTR; interval system; PI controller; stability boundary; stability region.

1. Introduction

Chemical reactors belong among the most critical and interesting processes in all chemical engineering [1], [2]. They are frequently affected by complex behaviour and bounded with potential safety problems. From the viewpoint of control engineering, they represent the systems suitable for application of robust control [3], [4]. A common type of reactor is known as a continuous stirred tank reactor (CSTR) [1], [2]. The mathematical model of CSTR, robustly stabilized in this contribution, was constructed in [5]. Furthermore, the same work [5] presented stabilization of the CSTR by using the technique from [6], [7] and the pole-placement method. Besides, robust static output feedback control was applied to this CSTR in [8]. The robust stabilization in the paper [9] (which published the same results as can be found here) was based on the similar idea as in the contribution [5], but it used the alternative methods, namely the combination of the Kronecker summation method [10] and the algebraic approach to control design under the ring of proper and stable rational functions (R_P s) [11], [12], [13], [14], [15]. The same combination and its application to a nonlinear electronic model described by the interval system have been already studied in [16], [17].

This contribution is focused on brief description of design of robustly stabilizing PI controllers for a CSTR in which an exothermic reaction occurs. The controlled CSTR is modelled as a fourth order interval system. The PI controller is designed by means of combination of a method for calculating/plotting the stability boundary, sixteen plant theorem and an algebraic approach. The robust stabilization is verified on a simulative example. The same results as in this work have been already published in the journal paper [9].

2. Key results

The controlled process is represented by a CSTR for hydrolysis of propylene oxide to propylene glycol. The detailed description of this CSTR can be found e.g. in [5], [8], [9]. Some physical parameters (reaction enthalpy, pre-exponential factor, and overall heat transfer coefficient) of the CSTR are not known exactly but they are assumed to vary within the intervals. This contribution adopts the mathematical model of the CSTR introduced in [5], where it was obtained in the linearized form of the fourth order interval transfer function:

$$G(s) = \frac{[-0.0291, -0.0245]s^2 + [-0.0199, -0.0127]s + [-0.000574, -0.0003549]}{s^4 + [0.5801, 0.903]s^3 + [0.1002, 0.2299]s^2 + [0.0062, 0.0142]s + [0.0001094, 0.0002412]} \quad (1)$$

where the input is the volumetric flow rate of the coolant, and the output is the temperature of the reacting mixture.

In the first step, the aim is to determine all possible robustly stabilizing PI controllers by means of plotting the stability boundary. It can be done e.g. via the Tan's (et al.) method [6], [7] or through the Kronecker summation method [10] and their combination with the sixteen plant theorem [3], [18]. A brief overview of these techniques can be found also in [19], [20]. The Fig. 1 shows the open areas of stabilizing or non-stabilizing pairs of proportional (k_p) and integral (k_I) gains of PI controller for the first of the sixteen Kharitonov plants [3], [18]:

$$G_{1,1}(s) = \frac{-0.0245s^2 - 0.0199s - 0.000574}{s^4 + 0.903s^3 + 0.2299s^2 + 0.0062s + 0.0001094} \quad (2)$$

Fig. 1. Areas of stability/instability for the first Kharitonov plant (2)

The stability areas for all sixteen Kharitonov plants [3], [18] are plotted in Fig. 2. The intersection of these stability areas defines the region of stabilizing (k_p, k_I) pairs for all sixteen Kharitonov plants together, i.e. it gives the robust stability region for the original interval plant (1). This region is zoomed and highlighted in Fig. 3.

The next step is to select a convenient PI controller from the obtained robust stability region. This contribution uses an algebraic approach based on general solutions of Diophantine equations in R_{PS} , Youla-Kučera parameterization of controllers, conditions of divisibility in R_{PS} , and additional single parameter tuning. The method is grounded in the ideas from [11], [12] and its details can be found e.g. in [13], [14], [15]. The PI controller designed for a nominal system obtained from the interval plant (1) is:

$$C(s) = \frac{k_P s + k_I}{s} = \frac{-1.238s - 0.029706}{s} \quad (3)$$

Obviously, the controller (3) lies inside the robust stability region from Fig. 3. The full design is described in [9].

Finally, the robust stability is confirmed and the control performance demonstrated in Fig. 4 which shows the set of output signals of the feedback control loop with designed PI controller (3) and 128 selected members of the interval family (1) with all variations of minimum or maximum values of each interval parameter. The level of reference signal was chosen just for the simulation purposes and it does not correspond to any real life values. As can be seen, the only one off-line tuned feedback PI controller with fixed coefficients can be successfully applied to robustly stabilize the fourth order interval model (1) of the CSTR.

Fig. 2. Areas of stability for sixteen Kharitonov plants

Fig. 3. Robust stability region for the interval plant (1)

Fig. 4. Output (controlled) signals of 128 selected members of the interval family (1)

3. Conclusion

This contribution was focused on brief description of an approach to robust stabilization of CSTR by means of PI controller. The CSTR with an exothermic reaction was considered as the fourth order interval plant and its robust stabilization was based on combination of a technique for calculating/plotting the stability boundary, the sixteen plant theorem and an algebraic approach to PI controller design. The validity of the method was verified through a simulative example with a set of CSTR control results.

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5. References

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