CALCULUS OF THE COEFFICIENT OF REGENERATIVE LOSSES IN STIRLING MACHINES

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Abstract: The coefficient of regenerative losses, X , includes all the losses produced by the heat transfer in the regenerator. This parameter depends on a large number of variables. Among these are piston speed, cylinder dimensions, regenerator dimensions, materials, gas properties, and the range of operating conditions. These variables are used in a new technique for calculating the parameter X .

The computed values of X were compared with experimental data available in the literature. The technique for calculating the coefficient of regenerative losses X proves to be accurate. This predictive capability is a powerful tool in the design of effective Stirling machines.

Key words: X coefficient, efficiency, power, Stirling engine

1. INTRODUCTION

This paper presents a new technique for calculating the efficiency and power of actual operating Stirling machines. This technique is based on the first law of thermodynamics for processes with finite speed. It is used in conjunction with a new PV/Px diagram and a new method for determining the imperfect regeneration coefficient.

One of the objectives of this paper is to develop a method for determining the imperfect regeneration coefficient X , and to use it for calculating the efficiency and the power output of the Stirling engine.

The thermal efficiency is expressed as a product of the Carnot cycle efficiency and second law efficiency (Petrescu & Harman 1996).

Initially, the thermal efficiency is written as a function of three basis parameters.

$$\eta_{SE} = \eta_{SE} \cdot \eta_{II,irrev} = \eta_{CC} \cdot \eta_{II,irrev,\Delta T} \cdot \eta_{II,irrev,X} \cdot \sum \Delta P_i \quad (1)$$

where

$$\eta_{CC} = 1 - T_0/T_{H,S} \quad (2)$$

is the efficiency of a Carnot cycle operating between the same temperature limits as the Stirling engine.

The second law efficiency

$$\eta_{II,irrev,\Delta T} = 1/(1 + \sqrt{T_0/T_{H,S}}) \quad (3)$$

is determined by the irreversibility caused by the temperature difference between the heat source and the gas in the engine.

The second law efficiency

$$\eta_{II,irrev,X} = \{1 + X(1 - \sqrt{T_0/T_{H,S}})/[(\gamma - 1) \ln \varepsilon]\}^{-1} \quad (4)$$

depends of loses in the regenerator caused by the incomplete regeneration, using of the coefficient of losses, X , (Florea et al., 2009).

The second law efficiency

$$\eta_{II,irrev,X} \cdot \sum \Delta P_i = [1 - (\sum \Delta P_i \cdot \Delta V_i)/(\eta' P_1 V_1 \sqrt{T_{H,S}/T_0} \cdot \ln \varepsilon)]^{-1} \quad (5)$$

with

$$\eta' = (1 - \sqrt{T_0/T_{H,S}}) \cdot \eta_{II,irrev,X} \quad (6)$$

considers the irreversibility losses produced by the pressure drop caused by the finite piston speed.

The optimum point of operation of the Stirling engine, from the point of view of economy, results as a trade off between efficiency and power.

The power output of the engine is:

$$Power_{SE} = \eta_{SE} \cdot mRT_{H,g} \cdot \ln \varepsilon \cdot (w/2S) \quad (7)$$

where s is the compression ratio, w is piston speed, S is the stroke of the piston, and γ is the specific heat ratio.

2. DETERMINATION OF LOSSES, EFFICIENCY AND POWER

Computation of pressure losses, work losses, efficiency and power of processes revealed by the new PV/Px diagrams are made using the first law of thermodynamics for processes with finite speed. The form of first law which includes these conditions is

$$dU = \delta Q - P_{m,i} \left(1 \pm \frac{aw}{c} \pm \frac{b \cdot \Delta P_{thrott}}{2P_{m,i}} \pm f \cdot \Delta P_f \cdot P_{m,i} \right) dV \quad (8)$$

When is applied to processes with finite speed the irreversible work then is

$$\delta W_{irrev} = P_{m,i} \left(1 \pm \frac{aw}{c} \pm \frac{b \cdot \Delta P_{thrott}}{2P_{m,i}} \pm f \cdot \Delta P_f \cdot P_{m,i} \right) dV \quad (9)$$

The work expression for the finite speed isothermal irreversible compression process 1-2 (Fig. 1) can be integrated using the Direct Method (Petrescu et al., 2002).

The result is

$$W_{1,2,irrev} = \int_1^2 P_{m,i} dV + \int_1^2 \left[\frac{aw}{c} + \frac{b \cdot \Delta P_{thrott}}{(2P_{m,cpr,i})} + \frac{\Delta P_f}{P_{m,cpr,i}} \right] P_{m,cpr,i} dV \quad (10)$$

The work losses can be calculated for the compression process 1-2 by using eq. (10),

$$W_{12,losses} = W_{12,irrev} - W_{12,rev} = \left[\left(\frac{aw}{c} \right) P_{m,cpr,i} + \frac{b \cdot \Delta P_{thrott}}{2 \Delta P_f} \right] (V_2 - V_1) \quad (11)$$

Fig. 1. The new PV/Px diagram of the ideal Stirling cycle

Computing and adding the losses produced by finite speed of the pistons, throttling of the gas through the regenerator, the mechanical friction of the entire Stirling engine cycle and introducing them into eq. (5), it becomes (Florea, 1999), (Petrescu et al., 2000):

$$\eta_{II,irrev,\Sigma\Delta P_i} = 1 - \frac{\left(\frac{w}{w_{S,L}}\right) \gamma (1 + \sqrt{\tau}) \ln \varepsilon + 5N \left(\frac{w}{w_{S,L}}\right)^2 + \frac{3(0.94 + 0.045w)}{4\rho}}{(\tau \eta \ln \varepsilon)} \quad (12)$$

The heat input during the expansion process is also irreversible because of the finite speed. In order to consider this influence, an adjusting parameter z is introduced

$$Q_{34} = z \cdot mRT_{H,g} \ln \varepsilon \quad (13)$$

Finally, the real power output of the engine, eq. (7) becomes.

$$Power_{SE,irrev} = \eta_{SE} \cdot z mRT_{H,g} \cdot \ln \varepsilon \cdot (w/2S) \quad (14)$$

3. A METHOD FOR X COEFFICIENT CALCULUS

The efficiency of real Stirling machines is always less than that of the idealized Stirling cycle operating between the same temperature limits. This happens because of the heat losses that occurs in the regenerative processes. The coefficient of regenerative losses, X , includes all of the losses due to incomplete heat transfer in the regenerator. The analysis resulted in differential equations that were integrated. This integration is based on an analysis which gives pessimistic results, X_1 or on a linear distribution of the temperature in the regenerator matrix and gas – fig. 2.

The last mentioned one offers optimistic results, X_2 . The resulting expressions for are:

Fig. 2. Gas and matrix temperature distribution in regenerator

$$X_1 = \frac{1+2M+e^{-B}}{2(1+M)} ; \quad X_2 = \frac{M+e^{-B}}{(1+M)} \quad (15)$$

where

$$M = \frac{m_g c_{v,g}}{m_R c_{v,R}} ; \quad B = (1+M) \frac{h A_R}{m_g c_{v,g}} \cdot \frac{S}{w} \quad (16)$$

$$h = \frac{0.395(4P_m/RT_L)w_g^{0.424} \cdot c_p(T_m) \cdot v(T_m)^{0.576}}{(1-\tau) \left[1 - \frac{\pi}{4((b/d)-1)}\right] D_R^{0.576} \cdot P_r^{2/3}} \quad (17)$$

The sensitivity of X_1 and X_2 related to operating variables, such as piston speed, was determined.

The results based on theory are correlated with experimental values using the following equation

$$X = yX_1 + (1-y)X_2 \quad (18)$$

where y is an adjusting parameter – 0,72.

The loss produced by incomplete regeneration determined using eq. (18) is the final loss to be considered in the analysis.

The second law efficiency caused by irreversibilities from incomplete regeneration is

$$\eta_{II,irrev,X} = \left[1 + (0,72X_1 + 0,28X_2) \cdot \left(\frac{1 - \sqrt{\frac{T_R}{T_{H,S}}}}{\frac{R}{c_p(T) \cdot \ln \varepsilon}} \right)^{-1} \right] \quad (19)$$

The variation of the coefficient of regenerative losses with the piston speed can be revealed for several values of the analysis parameters (d , S , porosity).

4. CONCLUSION

The strong correlation between the analytical results and actual engine performance data also indicates that the Direct Method using the first law for processes with finite speed is a valid method of analysis for irreversible cycles.

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