ON-LINE IDENTIFICATION OF SIMULATION EXAMPLES FOR FORGETTING METHODS TO TRACK TIME VARYING PARAMETERS USING THE ALTERNATIVE COVARIANCE MATRIX IN MATLAB

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ABSTRACT

The paper compares the abilities of forgetting methods to track time varying parameters of two different simulated models with different types of excitation. The observed parameters in the simulations are the integral sum of the Euclidean norm, deviation of the parameter estimates from their true values and a selected band prediction error count. As supplementary information, we observe the eigenvalues of the covariance matrix. In the paper we used a modified method of Regularized Exponential Forgetting with Alternative Covariance Matrix (REFACM) along with Directional Forgetting (DF) and three standard regularized methods.

KEYWORDS: online identification, time varying parameters, covariance matrix, forgetting

INTRODUCTION

This paper is devoted to online identification methods and their possible practical application along with adaptive control; while monitoring long-term, time operation of variant dynamic systems. An emphasis is set on long-term operation and consequently the working mechanism with non-informative data. The process of algorithm realization is also elaborated. Online identification methods are explored, where non-informative data that could possibly destabilize numerical computation of the identified system parameters is weighted by the chosen method to ensure „forgetting“. The contributions in this paper lay within two newly created algorithms and their modifications for online identification; based on the technique for utilizing an alternative covariance matrix. All algorithms are verified by simulations in Matlab Simulink software environment. Finally, the results obtained through the simulation algorithms mentioned in the article are compared to other commonly used algorithms.

PROBLEM STATEMENT

Let us consider a stochastic system in which observations are made at discrete time instants $k = 1, 2, \dots$. A directly manipulated input u_k and an indirectly affected output y_k (both possibly multivariate) can be distinguished in the data pair $d_k = (u_k, y_k)$. The collection of all data observed in the system up to time t is denoted by $D_t = (d_1, d_2, \dots, d_t)$. The dependence of a new pair of data (u_k, y_k) on previous observations D_{k-1} can be described by a conditional probability density function (p.d.f) with the following structure

$$p(y_k, u_k | D_{k-1}, \theta_k) = p(y_k | u_k, D_{k-1}, \theta_k) p(u_k | D_{k-1}) \quad (1)$$

Incomplete knowledge of the system behavior is expressed through a vector of unknown time varying parameters $\theta_k \in \Theta$. Note that the input generator described by the second term does not depend on these parameters directly, it is expected to utilize only prior information and information contained in observed data. These techniques are used in Regularized Exponential Forgetting (REF) and Stabilized Linear Forgetting (SLF).

REF AND SLF TECHNIQUE

Suppose that no explicit model of parameter changes is known. Yet, we can quantify our prior information (and possibly information taken from data already available) by introducing an alternative probability density function (p.d.f.) $p^*(\theta_{k+1}/D_k)$. The problem is then to construct a p.d.f. $p(\theta_{k+1}/D_k)$ based on two hypotheses described by the p.d.f. $p(\theta_k/D_k)$ (the case of no parameter changes) and the alternative p.d.f. $p^*(\theta_{k+1}/D_k)$ (the case of worst expected changes). For simplicity in this section we will use the notation $p_0(\theta)$, $p_1(\theta)$ a $p^*(\theta)$ for the posterior, alternative and resulting p.d.f.'s, respectively. In [3], are formulated the task of choosing p^* given p_0 and p_1 as a Bayesian decision making problem. A short review of their solutions can be expressed: Let,

$$p_{\theta,P} = \frac{1}{\sqrt{2\Pi}} |P|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\theta - \hat{\theta})^T P^{-1}(\theta - \hat{\theta})\right) \quad (2)$$

where $\hat{\theta}$ and P denote the mean and covariance of a particular p.d.f., then the following solutions show:

EF:

$$\hat{\theta}^* = \hat{\theta}_0, P^{*-1} = \lambda P_0^{-1} + (1 - \lambda) P_1^{-1} \quad (3)$$

LF:

$$\hat{\theta}^* = \hat{\theta}_0, P^* = \lambda P_0 + (1 - \lambda) P_1 \quad (4)$$

Let's consider the model of the system with time varying parameters θ_k , see [3]. In order to be able to track parameter variations we complement the standard recursive last square (RLS) algorithm by exponential or linear forgetting according to (3) or (4) respectively. In addition the alternative mean is set equal to posterior mean $\hat{\theta}_{k+1|k}^{\text{alt}} = \hat{\theta}_{k|k}$ and for simplicity the alternative covariance is set equal to the prior covariance $P_{k+1|k}^{\text{alt}} = P_{1,0} = Q$. With this choice we can use a general forgetting algorithm with the following choice of forgetting operator

$$F\{P_{k|k}, Q\} = [\lambda P_{k|k}^{-1} + (1 - \lambda) Q^{-1}]^{-1} \quad (5)$$

which construct the harmonic mean for REF and

$$F\{P_{k|k}, Q\} = \lambda P_{k|k} + (1 - \lambda) Q \quad (6)$$

Constructs the arithmetic mean for SLF. In both cases the prior covariance matrix Q isn't forgotten and is repeatedly taken into account for every step k seen in [1].

AUGMENTING REF WITH ACM

The involved REF augmentation considers addition and keeping the initial information in Alternative Covariance Matrix (ACM) form, see [2]. The augmentation is based on the modified

Dyadic reduction algorithm, where instead of adding a-priori covariance matrix Q , ACM is computed at each step. ACM is stabilizing the evolution of matrix $P(0)$ after the recursive update. This operation is necessary for the REF algorithms to be augmented by the stabilization component in ACM form. The aforementioned stabilization component prevents the destabilization of the original algorithms at long running applications; when slow time changes are to be expected in the observed parameters in relation to the sampling period. The modified algorithm REF augmented with ACM is termed REFACM.

METHODOLOGY FOR THE VERIFICATION OF THE SIMULATIONAL ALGORITHM

Two different models were created in the verification of properties (for the introduced algorithms) in the observation of time variant parameters of dynamic systems.. These two models (model no. 1. and no. 2.) have a different approach to input excitation (input signal generator A and B). Algorithm quality has been compared through the use of DF, which is considered to be the standard in the field. The Praly Forgetting (PF) algorithm featured in [4] has also been considered, using the weighted covariance matrix P . All algorithms were subject to the same test with identical length using the two featured models.

All results were graphically evaluated, and analyzed in a table where algorithm quality has been shown numerically through parameters IS and PE.

Description of model no. 1 and no. 2

In the case of model 1., a second order model is considered with external disturbance $v_{(i)}$ according to:

$$y_k = \sum_{i=1}^2 a_i y_{k-i} + \sum_{i=0}^2 b_i u_{k-i} + \sum_{i=0}^2 d_i v_{k-i} + e_k, e_k \approx N(0, \sigma^2) \quad (7)$$

The values of constant parameters are given by: $a_2 = -0.9$, $b_0 = 0.5$, $b_1 = -0.25$, $b_2 = 0.1$, $d_1 = 0.8$, $d_2 = 0.2$ and $\sigma = 0.1$.

The time variant parameter has been chosen as $a_{(1)} = 0.98$, which has been kept constant for half of the n simulation steps, then at time $t = n / 2$ its value is changed to $a_{(1)} = -0.98$. The outside disturbance has been simulated as a square signal periodically changing its value from $+1$ to -1 every 100 simulation steps. The identification has been made difficult mainly by the rare occurrence of disturbances, which contain minimal information about the parameter $d_{(i)}$.

For the needs of the simulation, two input signal generators have been assumed:

- Input signal generator A: discrete white noise generator
- Input signal generator B: input signal generated using the following equation:

$u_k^* = 0.8u_{k-1}^* + 0.2u_k$, where u_k^* is normally distributed white noise and u_{k-1}^* is the previous input value. For model no. 2. only one change has been realized in comparison to model no. 1. This has been carried out by altering the time variant parameter $a_{(1,k)} = 0.98 \cos(2\pi k/250)$. In this case, two different input generators were considered as well:

- Input signal generator A: discrete white noise generator
- Input signal generator B: input signal generated similarly to model no 1., where $u_{(k)}$ has been chosen only from the interval $u_{(k)} \sim (0.5, 1)$.

VERIFICATION – MATLAB SIMULINK

From the previously mentioned algorithms DF has been chosen along with an additional three normalized methods: REF, SLF and PF. For the simulation verification a set of S-Function libraries has been created along with a common universal user interface. This interface allows the user to select input data, simulated the model and observed algorithm. The output of the discussed simulations is a graphical representation of the observed parameters along with a data file containing the results for the following analysis. Integral sum (IS) of the Euclidian norm of parameter error and prediction error PE has been shown, which is the amount exceeded by the interval $\pm 3\sigma^2$. The simulation experiments will be marked by the character pair XY, where X is the number of the utilized model (no. 1 or no. 2) and Y represents the generator utilized (A or B).

EVALUATION OF SIMULATION RESULTS

This section shows all results in table form. For the detailed description of the algorithm behaviour during the simulations with different lengths (simulations lasting $n = 6000$, $12\ 000$ and $120\ 000$) have been evaluated as shown in Tables (1 to 3). It is clear in Table (2); that using a simulation length of $n = 12\ 000$ steps artefacts of long lasting runs already start appearing. The result is the confirmation of REFACM algorithm quality in comparison to REF, which in case 1A achieved better results than REF. Algorithm PF also achieved excellent results. The data featured in Table (3) fully confirm the previous considerations of the REFACM algorithm quality. It is clear that ACM behaves as if a constraint has been enforced on parameter trending, which also implies the improvement of IS parameters in comparison to the results achieved by REF. The convergence of the REF covariance matrix is faster and finite in contrast to REFACM, where the convergence is slower and also the addition of excited ACM cannot be finite. The achieved simulation results and REFACM algorithm behaviour at $6\ 000$ and $120\ 000$ simulation steps, show that as the running length increases the quality improves in contrast to REF, see Figure 1. In Table (4) we have shown the influence of weighting factor λ to the quality of REFACM algorithms. Best results in simulations were observed with $\lambda=0.8$.

6 000 steps	1A	1B	2A	2B
DF	IS=356,6	IS=957,3	IS=3499,4	IS=4016,0
	PE=130	PE=131	PE=1723	PE=717
SLF	IS=153,4	IS=208,6	IS=928,5	IS=2683,3
	PE=18	PE=12	PE=210	PE=131
REF	IS=118,3	IS=120,3	IS=740,9	IS=1336,7
	PE=18	PE=12	PE=174	PE=103
PF	IS=125,9	IS=114,4	IS=921,9	IS=1807,2
	PE=23	PE=13	PE=197	PE=173
REFACM	IS=127,4	IS=177,8	IS=963,8	IS=1985,1
	PE=27	PE=34	PE=239	PE=166

Tab. 1 - Simulation length 6000 steps

12 000 steps	1A	1B	2A	2B
DF	IS=645,7	IS=1604,6	IS=6701,0	IS=7474,5
	PE=253	PE=214	PE=3892	PE=1488
SLF	IS=219,2	IS=257,5	IS=1533,4	IS=3589,9
	PE=18	PE=13	PE=363	PE=179
REF	IS=168,9	IS=253,9	IS=1457,2	IS=2525,3
	PE=13	PE=14	PE=351	PE=188
PF	IS=103,5	IS=165,4	IS=1760,8	IS=3210,8
	PE=14	PE=15	PE=379	PE=355
REFACM	IS=162,9	IS=296,3	IS=1844,9	IS=4507,3
	PE=21	PE=31	PE=654	PE=439

Tab. 2 - Simulation length 12 000 steps

120 000 steps	REF	REFACM
1A	IS=1035,4	IS=880,8
	PE=17	PE=51
1B	IS=3419,5	IS=2963,7
	PE=12	PE=29

Tab. 3 - Simulation length 120 000 steps only for best performing algorithms REF and REFACM

REFACM	$\lambda = 0.8$	$\lambda = 0.5$	$\lambda = 0.2$
1A	IS=127,4	IS=164,2	IS=182,8
	PE=27	PE=34	PE=48
1B	IS=177,8	IS=298,9	IS=496,9
	PE=34	PE=50	PE=94
2A	IS=963,6	IS=870,1	IS=1031,6
	PE=239	PE=289	PE=327
2B	IS=1985,1	IS=1520,0	IS=1732,8
	PE=166	PE=145	PE=167

Tab. 4 - Influence of weighing factor λ , for REFACM algorithm

Fig. 1 - Simulation results for REF and REFACM algorithms (lower is better)

CONCLUSION

The simulation verification tests featured in the previous section, evaluated in the Matlab Simulink environment confirms that the quality of the tested algorithms is diverse. For long running simulations, the best results are achieved by the REFACM algorithm – which is the focus of this paper. The quality of REFACM in comparison with other algorithms confirms the advantages of using ACM given the specific conditions featured in this work.

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