A METHODOLOGY FOR GEAR FATIGUE TESTS AND THEIR EVALUATION (PART 2)

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SHRNUTÍ

Článek dává návod, jak statisticky zpracovat výsledky experimentálních zkoušek ozubených kol na pulsátoru. Za předpokladu zkoušení materiálu podle metody stair-case jsou uvedeny algoritmy, jak vyhodnotit pravděpodobnostní průběh meze únavy v ohybu. Vyhodnocení je demonstrováno na reálných výsledcích únavových zkoušek.

KLÍČOVÁ SLOVA: METODIKA, PULZÁTOROVÉ ZKOUŠKY, ÚNAVA, STUPŇOVITÁ METODA, PRAVDĚPODOBNOSTNÍ PRŮBĚH MEZE ÚNAVY V OHYBU

ABSTRACT

This article provides guidelines on how to statistically treat the results from single tooth fatigue tests. Assuming the methodology for testing is followed, algorithms are presented here to evaluate the probability plot of the fatigue limit in bending. The evaluation is demonstrated using actual results from fatigue tests.

KEYWORDS: METHODOLOGY, SINGLE-TOOTH FATIGUE, STAIRCASE, PROBABILITY PLOT OF THE FATIGUE LIMIT IN BENDING

1. INTRODUCTION

In the first part we demonstrated mechanisms for the design and performance of fatigue tests. Procedures were also presented for the design of the test specimen, shape of the test jaws and a suitable setting of initial values for testing the bending fatigue properties.

This second part will focus on the staircase methodology for fatigue testing. A specific example will be given of calculating the fatigue limit probability in bending from actual test results. At the conclusion of the article we introduce the complete results of testing performed on two materials, 18CrNiMo7-6 (1.7186) and 21MnCrB5-4 (1.6587). Four specimens were produced from both of these carburizing steels with a rated degree of deformation min. 3:1 and quality MQ according to standard ISO [6]. Half of them were subject to shot peening with shot size of 0.6 mm with intensity ALMA type A at least to the required hardness 59 HRC. This creates a thin surface layer with a significant residual compressive stress, which has a positive effect on increasing the fatigue properties [2]. From the test results it is possible to evaluate the benefit of shot peening and apply it in the future in gear calculations.

The parameters of all the test specimens are shown in Figure 1.

label	z	BRP	mn [mm]	x [-]	da [mm]	dc [mm]	material	note
F1	52	ČSN 01 4607	4,0	0	216	180	21MnCrB5-4	-
F2							21MnCrB5-4	s-p
G1							18CrNiMo7-6	s-p
G2							18CrNiMo7-6	-
H3	70	ČSN 01 4607	4,0	0	288	250	21MnCrB5-4	-
H4							21MnCrB5-4	s-p
J1							18CrNiMo7-6	-
J2							18CrNiMo7-6	s-p

FIGURE 1: The characteristics of test specimens

OBRÁZEK 1: Charakteristika zkušebních vzorků

The tests were performed with a displaced repeating cycle on the Schenk PVT pulsator at the Tatra, a. s. dynamic testing laboratory. During the seven months of intensive testing a total of 111 experiments were performed on the eight specimens. The examination was based on a staircase method.

2. PROCESSING THE EXPERIMENTAL RESULTS

A data file was obtained for statistical processing. Before evaluation, we excluded outlying results. These arose mainly due to incorrect choice of starting values at the start of testing. They can be identified, for example, using the median absolute deviation.

In the first phase of evaluation it is appropriate to convert the load to a stress in the gear tooth root fillet. For this we can use FEM, or conversion constant K_p given by equation (1).

$$K_p = \frac{Y_{Fa} \cdot Y_{Sa}}{b \cdot m_n} = \frac{1,364 \cdot 2,003}{15 \cdot 4} \quad (1)$$

$$K_p = 45.61 \cdot 10^{-3} \text{ mm}^{-2}$$

The calculation algorithm for coefficients Y_{Fa} , Y_{Sa} is dependent on the geometry of the basic rack tooth profile, and the actual addendum modification coefficient x' , diameter d_x , i.e. the point of contact between the jaw and the test sample, see Figure 2. The solution coefficients are set forth in standards DIN [7] and ISO [6]. From the same figure it follows that the diameter d_x is significantly smaller than the tip diameter d_a . Diameter d_x is also smaller than working diameter d_w due to the angle δ .

FIGURE 2: Diagram showing the clamping of the test specimen between the two jaws

OBRÁZEK 2: Schéma upnutí zkušební vzorku mezi dvěma čelistmi

From Figure 2 we can also derive equations for calculating the diameter d_x that must be entered in place of d_a when calculating the coefficients Y_{Sa} and Y_{Fa} .

$$d_x = \frac{d_b}{\cos \alpha_x} = \frac{195.456}{\cos 22,593} \quad (2)$$

$$d_x = 282.327 \text{ mm}$$

$$d_b = z \cdot m_n \cdot \cos \alpha_t = 70 \cdot 4 \cdot \cos 20 \quad (3)$$

$$d_b = 263.114 \text{ mm}$$

$$\alpha_x = \arctan \left(\frac{W}{d_b} - \delta \right) = \arctan \left(\frac{92,485}{263,114} - 0,05236 \right) \quad (4)$$

$$\alpha_x = 21,26^\circ$$

$$W = m_n \cdot \sin \alpha_n \left[(z_m - 0,5 \cdot \operatorname{sgn} z) \cdot \pi + 2 \cdot x \cdot \tan \alpha_n + z \cdot \operatorname{inv} \alpha_n \right] \quad (5)$$

$$W = 4 \cdot \sin 20 \left[(8 - 0,5 \cdot 1) \cdot \pi + 2 \cdot 0 \cdot \tan 20 + 70 \cdot \operatorname{inv} 20 \right]$$

$$W = 92.485 \text{ mm}$$

$$x' = x - \left(\frac{W - W_m}{2 \cdot m_n \cdot \sin \alpha_n} \right) = 0 - \left(\frac{92,485 - 92,53}{2 \cdot 4 \cdot \sin 20} \right) \quad (6)$$

$$x' = 0.01645$$

$$x' = x - \left(\frac{E_w - 2 \cdot T_w}{2 \cdot m_n \cdot \sin \alpha_n} \right) \quad (7)$$

The notation used in equations (2–7) is defined in the list of notations and abbreviations at the end of the article. When calculating actual unit corrections coefficient x' , we can proceed in two ways. We can use equation (6), where we consider the average value measured over the teeth W_m , or we can apply equation (7). In this case, we consider only the upper span over the teeth E_w and tolerance span over the teeth T_w that are commonly given in technical drawings.

3. EVALUATION OF FATIGUE TESTS

An example of fatigue limit evaluation is performed only for test specimen J1. The results for the other specimens are given in master thesis [2]. After the conversion of forces to stress, we create a new table of input values (Table 1). From these values we can produce a staircase chart as shown in Figure 3. During the testing of specimens it less often occurred that there was

no damage to the tooth (empty circle), and therefore in Table 2 this case was evaluated using the staircase method. It is easy to recognize errors in testing procedures. After the fracture of a tooth, the next test will be performed at a lower level, and vice versa.

TABLE 1: Recalculated values of loads for specimen J1

TABULKA 1: Přepočtené hodnoty zatížení vzorku J1

n	specimen	σ_m [MPa]	σ_a [MPa]	number of cycles
1	J1 1-1	684	456	>3 000 000
2	J1 2-2	730	502	171 800
3	J1 3-3	730	502	>3 000 000
4	J1 4-4	775	547	70 400
5	J1 5-5	730	502	>3 000 000
6	J1 6-6	775	547	79 800
7	J1 7-7	775	547	154 200
8	J1 8-8	730	502	273 500
9	J1 9-9	684	456	>3 000 000
10	J1 10-10	730	502	>3 000 000
11	J1 11-11	775	547	51 000
12	J1 12-12	775	547	686 300
13	J1 13-13	730	502	>3 000 000
14	J1 14-14	821	593	51 000
15	J1 15-15	821	593	47 700
16	J1 16-16	775	547	81 500

TABLE 2: Evaluation using the staircase method

TABULKA 2: Vyhodnocení metodou stair-case

FIGURE 3: Chart of the staircase method

OBRAZÉK 3: Graf stupňovité zkoušky

The median value of amplitude obtained using equation (8).

$$\overline{\sigma_{ac}} = \sigma_{ac0} + d \cdot \left(\frac{A}{F} \pm \frac{1}{2} \right) = 456 + 45,61 \cdot \left(\frac{4}{6} + \frac{1}{2} \right) \quad (8)$$

$$\overline{\sigma_{ac}} = 509 \text{ MPa}$$

Here the width of level d corresponds to conversion constant K_p given by equation (1). This fact follows from the test setting where level width was 1 kN. In order to calculate the standard

deviation with sufficient precision, the inequality (9) must be satisfied.

$$\frac{B \cdot F - A^2}{F^2} = \frac{4 \cdot 6 - 4^2}{6^2} = 0,2 \not\geq 0,3 \quad (9)$$

In this case, inequality isn't satisfied. Even so, we attempted to calculate the standard deviation of amplitudes using equation (10).

$$S_{\sigma_{ac}} = 1,62 \cdot d \cdot \left(\frac{B \cdot F - A^2}{F^2} + 0,029 \right) = 1,62 \cdot 45,61 \cdot \left(\frac{4 \cdot 6 - 4^2}{6^2} + 0,029 \right) \quad (10)$$

$$S_{\sigma_{ac}} = 18,6 \text{ MPa}$$

Values of standard deviations thus obtained can be considered as indicative only, and should not be separately used for extrapolation.

The last column in Table 2 indicates the probability of failure on level I , and is given by equation (11).

$$P_i = \frac{\Sigma F}{F + 1} \cdot 100 = \frac{6}{6 + 1} \cdot 100 \quad (11)$$

$$P_3 = 86 \%$$

where ΣF represents the accumulation of damaged or intact samples. Only the case which occurred less often is taken into account. A similar procedure is used when calculating the average amplitude or its standard deviation. Using this we can depict graphically the probability distribution shown in Figure 4 as substitution in equations (8) and (10) for the results obtained with single tooth fatigue test. Ideally, the substitution should correspond to the measured values. The vertical axis is the normal distribution quantile u_p , which has a linear character. From Figure 4 it can be seen that the consensus is, in this case, only symbolic.

FIGURE 4: Plot of the probability distribution σ_{ac} for specimen J1

OBRAZÉK 4: Průběh rozdělení pravděpodobnosti σ_{ac} vzorku J1

Equations (8–10) are based on the staircase method according to the original work of Dixon and Mood [1]. At present this method is embodied in the standard ISO [4], which is valid for general engineering.

When tested with constant lower stress σ_d , we are able from the Haigh diagram [3] to determine the value of the allowable bending fatigue stress σ_{FE} :

$$\sigma_{FE} = 2 \cdot \overline{\sigma_{ac}} + \frac{4}{7} \cdot \sigma_d = 2 \cdot 509 + \frac{4}{7} \cdot 228 \quad (12)$$

$$\sigma_{ac} = 1149 \text{ MPa}$$

The same procedure was used with the other seven specimens. The procedure for calculating the amplitude using the staircase method is, however, quite time consuming and laborious. Therefore, we suggest an alternative method. It consists in principle in evaluating only the intact samples, i.e. those that showed no tooth damage after $3 \cdot 10^6$ cycles (empty circle), see Figure 5. In this case it is possible to merge all the amplitudes from specimens with 52 and 70 teeth from the same material, which is not possible when using the staircase method. This is due to the increased number of levels influenced by the different geometry. Evaluation of average value $\overline{\sigma_{ac}}$ and standard deviation $S_{\sigma_{ac}}$, is realized with a larger number of values, which in terms of statistics is much better [2]. To calculate these two values we can use the internal functions of spreadsheet programs such as Excel.

FIGURE 5: Separation of measurements on samples of material 18CrNiMo7-6

OBRAZEK 5: Separace naměřených hodnot na vzorcích z materiálu 18CrNiMo7-6

An overview of all the allowable stress numbers for bending obtained using the staircase method and alternative method for both tested materials is shown in Table 4.

TABLE 4: Allowable stress number for bending for 50% probability of failure [MPa]

TABULKA 4: Meze únavy v ohybu pro 50% pravděpodobnost vzniku poruchy [MPa]

21MNCRB5		18CRNIMO7		21MNCRB5		18CRNIMO7	
s-p	-	s-p	-	s-p	-	s-p	-
stair-case method				alternative method			
1425	1048	1445	1001	1382	1055	1293	918

These results have probability of damage 50%, similar to values in the standard ČSN [5]. However, comparing the results with equivalent material 14220 (16MnCr5) we find that the values in Table 4 for material not subject to shot-peening are about 30% higher. Based on this finding, use of the ČSN standard for the dimensioning of gears from carburizing steels cannot be recommended. On the other hand, standards DIN [7] and ISO [6] present allowable stress numbers for bending with a one percent probability of fracture damage. For comparison we can use relationships (6–8) from the first part of this article, or experimentally obtained standard deviations of the amplitudes $S_{\sigma_{ac}}$, which can be matched with a standard deviation of fatigue limit in bending by educated estimation. Individual values from Table 4 can be converted using equation (13).

$$\sigma_{FE1\%} = \sigma_{FE50\%} + u_{p1\%} \cdot \overline{S_{\sigma_{ac}}} \quad (13)$$

where $u_{p1\%}$ is normal distribution quantile and $\overline{S_{\sigma_{ac}}}$ is average standard deviation from test results.

This gives the allowable stress number for bending with a similar probability as quoted in standards DIN and ISO, and we can proceed to their comparison with experimental results.

TABLE 5: Allowable stress number for bending for 1% probability of failure [MPa]

TABULKA 5: Meze únavy v ohybu pro 1% pravděpodobnost vzniku poruchy [MPa]

21MNCRB5		18CRNIMO7		21MNCRB5		18CRNIMO7		16MNCR6		CARB. STEEL	
experiment (staircase)				experiment (Gauss)				DIN [7]		ISO [6]	
s-p	-	s-p	-	s-p	-	s-p	-	s-p	-	s-p	-
1291	903	1311	856	1243	916	1154	779	-	920	-	1000

4. CONCLUSION

This article demonstrates the procedure for evaluating the probability course of the fatigue limit in bending from the results of single tooth fatigue tests. Besides the calculation algorithms, an illustrative example of evaluation for two structural materials is also presented. The results are compared with the allowable

value of equivalent materials according to standard ČSN [5], DIN [7] and ISO [6]. With regard to this comparison it can be stated that the experimental results for both materials are similar to those of the fatigue limit in bending in standards ISO and DIN. The still valid ČSN standard significantly underestimates the result. It is also possible to evaluate the benefits of shot-peening with respect to improving the fatigue properties in bending of carburizing steels. This final operation has resulted in an increase in the fatigue limit of steel 21MnCrB5 by almost 40%, and in the case of materials 18CrNiMo7 by 50%! Given the difficulty in performing tests, an increase in the fatigue limit by 30–40% for the strength calculation of gears from these materials can be assumed.

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LIST OF NOTATIONS AND ABBREVIATIONS

A	Constant for calculation amplitude of nominal stress number for bending [MPa]
b	Facewidth [mm]
B	Constant for calculation amplitude of nominal stress number for bending [MPa]
d_a	Tip diameter [mm]
d_b	Base diameter [mm]
d_w	Pitch diameter [mm]
d_x	Contact jaw and specimen diameter [mm]
E_w	Upper deviation of normal tooth thickness [mm]
F	Constant for calculation amplitude of nominal stress number for bending [MPa]
K_p	Conversion constant [mm ⁻²]
m_n	Normal module [mm]
n	Numbers of samples [-]
N	Numbers of cycles [-]
P	Probability of damage [%]
$s-p$	shot peening
$S_{\sigma_{ac}}$	Amplitude of nominal stress number for bending standard deviation [MPa]
T_w	Normal tooth thickness tolerance [mm]
u_p	Normal quantile [-]
W	Span measurement [mm]
W_m	Real span measurement [mm]
x	Addendum modification coefficient [-]
x'	Real addendum modification coefficient [-]
Y_{Fa}	Form factor [-]
Y_{Sa}	Stress correction factor [-]
z	Numbers of teeth [-]
z_m	Number of teeth measured over [-]
α_n	Normal flank angle [°]

α_t	Transverse rack flank angle [°]
α_x	Contact jaw and a specimen angle [°]
δ	Slope support surface of jaw angle [°]
σ_a	Amplitude stress [MPa]
σ_d	Lower stress [MPa]
σ_{ac}	Amplitude of nominal stress number for bending [MPa]
$\frac{\sigma_{ac}}{\sigma_m}$	Amplitude of nominal stress number for bending mean value [MPa]
σ_{FE}	Allowable stress number for bending [MPa]
σ_m	Mean stress [MPa]

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