A METHODOLOGY FOR GEAR FATIGUE TESTS AND THEIR EVALUATION (PART 1)

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SHRNUTÍ

Článek popisuje mechanismy pro realizaci únavových zkoušek ozubených kol na pulsátoru, které vedou k získání pravděpodobnostního průběhu meze únavy v ohybu. Na základě realizovaných experimentů je navržena metodika pro návrh a realizaci tohoto typu zkoušek. Pozornost je rovněž věnována konstrukci zkušebních vzorků a tvaru zkušebních čelistí.

KLÍČOVÁ SLOVA: **METODIKA, PULZÁTOROVÉ ÚNAVOVÉ ZKOUŠKY, STUPŇOVITÁ METODA, PRAVDĚPODOBNOSTNÍ PRŮBĚH MEZE ÚNAVY V OHYBU**

ABSTRACT

This article describes the mechanisms for performing fatigue tests on gears using a single tooth bending test, which enables determination of the probability of fatigue limit in bending. The testing methodology is proposed based on experiments carried out. Attention is also paid to the design of the test specimen and shape of the jaws.

KEYWORDS: **METHODOLOGY, SINGLE TOOTH FATIGUE TEST, STAIRCASE METHOD, PROBABILITY PLOT OF FATIGUE LIMIT IN BENDING**

1. INTRODUCTION

Besides the standard strength calculations, one of the most widely used tools for the design of machine parts is the calculation of fatigue. In many parts, due to the nature of their service load, the calculation of fatigue life is the only way to arrive at the correct design solutions. The load method determines whether it is necessary to dimension the components in terms of static, time or permanent (fatigue) strength. Gears and, for example, shafts are cyclically loaded even when subject to contact strain. In this case, calculations should be directed at the fatigue strength or high-cyclic fatigue.

Acquisition of a probability plot for the fatigue limit in bending for the chosen material is an expensive and time consuming process. Against this background, it is advisable to proceed very cautiously in order to properly interpret the test results. Therefore, this article provides a general methodology for the testing gears using a single tooth bending test including specimen selection, the design and shape of the jaws and the setting of the starting values.

2. PRINCIPLES OF TESTING FATIGUE PROPERTIES IN GEARS

Generally we can test the fatigue properties of gears using running tests where real gear mechanisms are tested under conditions that correspond to actual operational loading. These tests are very time consuming. Usually you can identify in these types of test contact damage of teeth by pitting. Only with difficulty can damage due to tooth bending fracture be evaluated. Therefore for determining allowable bending values it is customary to use the single tooth bending test (see Figure 1). This type of testing is suitable only for hardened gears. In the case of a "soft" tooth surface, the specimen material at the point of contact with the test jaws can be damaged earlier than would have happened due to a bending fracture [3]. In contrast to running tests, the single tooth bending test is in principle based on a certain degree of idealization. Differences can be identified in load cycle characteristics, the distribution of dynamic forces, slip conditions, and the shape of the test specimen. Despite these differences, the determined values of fatigue limit in bending for the same materials are more or less identical. The conclusions of the study [2] reported a maximum difference of 5 % in the case of carburizing steels.

FIGURE 1: Test machine with clamped specimen
OBRÁZEK 1: Zkušební stroj s upnutým vzorkem

Other factors that argue in favor of the single tooth bending test when compared to running testing are its simplicity, speed and in particular variability. Thanks to these advantages this form of testing is commonly used in industrial practice, as opposed to running tests, which are usually only used for specialized operations at research and academic institutions.

3. METHODOLOGY OF SINGLE TOOTH BENDING

One of the sufficiently accurate methods allowing evaluation of a relatively small number of test samples is the staircase method. This method was developed by two American statisticians, W.J. Dixon and A.M. Mood, during the Second World War [1]. Since its inception, it has undergone many modifications, although the fundamental principles remain essentially unchanged. The test should start at the level where mean values can be expected. If a failure occurs, the next sample is tested at a level below. In an event that the sample is not damaged, the next test will take place at a higher level. The test continues until the available samples are exhausted with the level of each subsequent test being raised or lowered depending on the previous result. The intervals between the levels should be approximately equal to the standard deviation, although this might not be exact. However, the intervals should not be greater than twice the standard deviation [1].

The resulting chart of the staircase method, shown in Figure 2, should be centered on a mean value of at least three levels. This method is more efficient than a group method [7], which for determining the mean and standard deviation requires a relatively large number of samples $n > 50$. The number of samples is reduced by about a half.

FIGURE 2: Illustration chart of the staircase method
OBRÁZEK 2: Ukázka grafu stupňovité metody

The mean value $\overline{\sigma_{ac}}$ and standard deviation $s_{\sigma_{ac}}$ are calculated using the damaged or undamaged samples, depending on which occurred least frequently during testing. The levels of stress σ_{aci} , which is evenly distributed over the interval d , are indexed descending so that $i = 0$ is the lowest level of stress σ_{ac0} and $i = k$ is for the highest stress level [8]. Evaluation using this method is relatively laborious. An example of such evaluation will be presented in the second part of this article.

4. SIZE OF TEST SPECIMENS

For the design of the test specimen it is necessary to realize that an unsuitable shape can have a significant influence on the test results of fatigue properties. Although we cannot determine the ideal shape of the specimen, we can at least try to outline some basic principles that should be followed in a design.

For correct evaluation of the results using the staircase method it is necessary to perform at least 15–20 tests [7], preferably on the same specimen. In view of the fact that at the start of the test it is possible to have several failed attempts at finding the mean value, it is advisable to perform a higher number of tests. The number of possible tests on one specimen n is directly proportional to the number of teeth z which corresponds to the prescribed number of teeth z_{zk} , through which the test sample is clamped in the test jaws. Usually for testing we use a real gear wheel with number of teeth z , for example $z = 57$ which corresponds to $z_{zk} = 8$ [14]. By putting these values into equations (1) to (3) we determine that the proposed sample will be fully utilized.

$$n = (z_{zk} - 1) \cdot \text{floor}\left(\frac{z}{2(z_{zk} - 1)}\right) + \text{floor}\left(\frac{z - 2(z_{zk} - 1) \cdot \text{floor}\left(\frac{z}{2(z_{zk} - 1)}\right)}{z_{zk}}\right), \quad (1)$$

where function floor() returns the next lowest integer value.

$$n = (8 - 1) \cdot \text{floor}\left(\frac{57}{2(8 - 1)}\right) + \text{floor}\left(\frac{57 - 2(8 - 1) \cdot \text{floor}\left(\frac{57}{2(8 - 1)}\right)}{8}\right) = 28$$

Of course, a question may arise as to whether the layout of this test may lead to a weakening of the tooth area z_x , thereby affecting the outcome of these tests. Analysis using FEM showed that due to the absence of a rim, a significant stress during loading can be observed on previous tooth $z_x - 1$ only. Therefore, one tooth is omitted from testing. A similar test procedure arrangement is also selected in standard SAE [13].

With regards to the requirements for assessment of the fatigue tests results, we could consider using larger specimens. Whilst in such case we could perform more tests on one sample, there is the disadvantage of increased sample weight. This brings with it an increase in roll moment due to the weight of the specimen, which may in turn have a negative effect on the rigidity of the clamping, but especially on the test results. Increasing the number of performable tests on a specimen cannot even be achieved with a reduction of module m_n . The standard ISO [10] states that the value of allowable bending stress is obtained on a specimen with module $m_n = (3-5)$ mm and width $b = (10-50)$ mm. When designing the specimen we must also take into account the ratio b/m_n . To avoid uneven loading of the tooth it should not be too large.

5. SHAPE OF TEST JAWS

Success of the fatigue test is dependent upon on the shape of the test jaws. On the basis of performed tests it is possible to summarize a few basic steps that should lead to the optimum shape and design of jaw.

The test jaw can be divided into two parts: functional and clamping, which serves only for attachment to the test machine, so it will not be addressed further. Much more important is the functional part. Its shape should take the form of a wedge with apex angle $2\alpha_n$, which corresponds to a hypothetical coverage of the test specimen with a rack. It is also necessary to ensure easy set up of correct contact between the test jaw and the surface of specimen. This can be achieved by suitable design geometry of test jaw so that the bottom is in contact with tip of tooth $z_x - 1$, as shown in Figure 3. After loading under force F_z there is a deformation of tooth f , whilst between the jaws and the specimen a space Δ is created and the load will act only on the test tooth z_x .

For smooth running of the test it is necessary to bear in mind the tilting contact surfaces of the test jaw. This stabilizes clamping and also enables smooth operation of the hydraulic or resonant

FIGURE 3: The design of the test jaws
OBRAZEK 3: Konstrukce zkušební čelisti

FIGURE 4: The computational model
OBRAZEK 4: Výpočtový model

pulsator, as well as configuration of the direction of the applied loading force F_z . A sufficient angle is $d = (3-5)^\circ$. Assuming that the test is performed with a maximum number of teeth z_{zk} , we achieve almost identical orientation of loading α_F , which is considered in standard DIN [11]. The diagram of tooth loading using a test jaw with diameter d_x is shown in Figure 4.

6. SETUP AND PERFORMANCE OF TESTS

Without prejudice we can state that the start of fatigue testing properties (the initial conditions) on a pulsator can have a decisive influence on the quality of the results obtained. It is certainly appropriate to propose a procedure to determine the starting values for performing this type of testing. Only a correct estimate of the mean amplitude of allowable bending fatigue stress σ_{ac} will save both machine working time and a considerable number of samples.

In the first phase it is necessary to determine the relationship between the load F_z and local bending stress at tooth root σ_F with the aid of equation (2) or FEM.

$$\sigma_F = \frac{F_z \cdot K_F}{b \cdot m_n} \cdot Y_{Fa} \cdot Y_{Sa} \cdot Y_\epsilon \cdot Y_\beta \quad (2)$$

In the single tooth bending tests we can consider coefficients K_F , Y_β , Y_ϵ as a unit. Then we obtain the relationship:

$$\sigma_F = F_z \cdot K_p \rightarrow K_p = \frac{Y_{Fa} \cdot Y_{Sa}}{b \cdot m_n} \quad (3)$$

The method for calculating algorithm coefficients Y_{Fa} , Y_{Sa} in the case of a single tooth bending test case will be explained in the second part of this article.

Furthermore, we find an approximate value of the fatigue limit in bending σ_{FE} of the test material for 50% damage probability. Here we can proceed on the basis of published works, standards or experience with the fatigue properties of equivalent materials. In the case of carburizing steels we can make use of the relationships (4) and (5). This approach gives a similar fatigue limit probability as in the case of determining initial amplitude of allowable bending fatigue stress σ_{ac} .

$$\sigma_{FE50\%} = \sigma_{FE1\%} - u_{p1\%} \cdot S_{\sigma_{FE}}^{DIN} \quad (4)$$

$$S_{\sigma_{FE}}^{DIN} = \frac{\sigma_{FE10\%} - \sigma_{FE1\%}}{u_{p10\%} - u_{p1\%}} \quad (5)$$

If we know only the value of allowable bending fatigue stress for 1% probability of damage, such as from the standards ISO [10]

or DIN [12], we can, assuming a normal Gaussian distribution, estimate the 50% probability of damage using equation (6).

$$\sigma_{FE50\%} = \frac{\sigma_{FE1\%}}{f_{xF}} \quad (6)$$

where a value f_{xF} is the mean coefficient for recalculation. Its value for common structural steel is $f_{xF} = 0.8-0.9$ [6].

Although it might seem much easier to choose the fatigue limit value according to standard ČSN [9], which gives 50% probability of damage, we do not recommend this procedure. This is due to significantly lower values compared to the equivalent materials according to standards ISO [10] or DIN [12].

If we know the value $\sigma_{FE50\%}$, we can proceed to design a test cycle where the asymmetry coefficient R should have a value close to zero. Lower stress σ_d in this case allows the safe running of tests based on the principle of resonance, while respecting the assumption of asymmetry coefficient $R = 0$, where $\sigma_d = 0$. We can determine the lower stress using equation:

$$\sigma_d \approx 0,2 \cdot \sigma_{FE50\%} \quad (7)$$

It is also necessary to calculate the initial value of the amplitude load of the test specimen:

$$\sigma_{ac} \approx 0,45 \cdot \sigma_{FE50\%} \quad (8)$$

Equations (7) and (8) were derived from thesis [4] and correspond with the results of tests [5]. For correct evaluation of the staircase method, the width of level d is also important. According to the method recommended by the authors [1], it should correspond in value to approximately the standard deviation $s_{\sigma_{ac}}$, and should not exceed double this value. A relationship for its determination is, however, not given. To estimate the value of width d , we can use equation (9).

$$d \approx S_{\sigma_{FE}}^{DIN} \approx 0,1 \cdot \sigma_{ac} \quad (9)$$

The results of equations (7), (8) and (9) are now converted from stress [MPa] to force [N] using constant K_p given by equation (3), and suitably adjusted to take into account the capabilities of the testing machine.

When performing the tests it is recommended to maintain constant lower stress σ_d as shown in Figure 5b. This method has, as opposed to testing at a constant preload σ_m see Figure 5a, similar loading characteristic to driving or driven gears. This assumption corresponds to the definition of the fatigue limit in bending σ_{FE} , which is given by the upper stress of the repeated cycle.

FIGURE 5: Test load at constant mid-stress a) and constant lower stress b)
OBRAZEK 5: Zkušební zatížení při konstantním středním a) a dolním napětí b)

9. CONCLUSION

This article provides general guidance on performing single tooth bending testing from initial design methodology to the beginning of testing. Assuming a staircase testing method, tools are defined for the design of the test specimen with an emphasis on its maximum utilization. Attention is also paid to the optimal design of test jaws for smooth tests. This article also presents empirical relationships for setting the initial test values. Their correct estimation can save machine time and reduce sample wastage by streamlining the entire test procedure.

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