## A DESIGN METHOD OF THE CONTROL SYSTEM WITH LOW SENSITIVITY AND ROBUST STABILITY USING DOUBLE FEEDBACK CONTROL FOR MINIMUM PHASE SYSTEMS HAVING UNCERTAIN RELATIVE DEGREES

## DAISUKE KOYAMA<sup>1</sup>, KOTARO HASHIKURA<sup>2</sup>, MD ABDUS SAMAD KAMAL<sup>2</sup> AND KOU YAMADA<sup>2</sup>

<sup>1</sup>Graduate School of Science and Technology <sup>2</sup>Division of Mechanical Science and Technology Gunma University 1-5-1 Tenjincho, Kiryu 376-8515, Japan { t202b004; k-hashikura; maskamal; yamada }@gunma-u.ac.jp

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ABSTRACT. In this paper, we examine a design method of a control system using double feedback control for the Single-Input/Single-Output minimum phase systems with varying number of unstable poles and having an uncertain relative degree. Several researchers studied a robust stabilization problem. According to several studies, it is a difficult problem that for the large uncertainty, the control system has a robust stability and low sensitivity. However, Yamada clarifies a necessary and sufficient condition of the control system that low-sensitivity control makes the system robustly stable for the certain class of uncertainty. Yu et al. expand the result of Yamada and propose a design method of a control system using double feedback control. According to Yu et al., since the double feedback control system has a sensitivity characteristics and robust stability, it is suitable to design the control system with low sensitivity and robust stability. The purpose of this paper is to expand the result of Yu et al., and propose a design method of the double feedback control system for a class of uncertainties not considered by Yu et al., which is the set of systems that the number of relative degrees of the nominal plant is not equal to that of the real plant.

**Keywords:** Single-Input/Single-Output minimum phase systems, The systems with varying number of unstable, Robust stabilization problem, Low sensitivity control, Robust control, Uncertain relative degree

1. Introduction. In this paper, we examine a design method of a control system using double feedback control for the Single-Input/Single-Output (SISO) minimum phase systems with varying number of unstable poles and having an uncertain relative degree. Several researchers study about a robust stabilization problem [1-7]. Doyle and Stein build the basis for this problem, and show the conditions for the multiplicative uncertainty and additive uncertainty [1]. Chen and Desoer gave the complete proof of the result of [2]. Kimura considers the robust stabilizability problem for Single-Input/Single-Output systems [8]. Vidyasagar and Kimura expand the findings of Kimura [8] for multiple-input and multiple-output systems [9].

According to [1-3], in order to keep the stability for the large uncertainty, a complementary sensitivity function of the control system must be small value. This means to bring the control systems low parformances in terms of the low sensitivity characteristics such as disturbance attenuation. On the other hand, to produce the control system with disturbance attenuation, we must make a sensitivity function of the control system small. Since the sum of the complementary sensitivity function and the sensitivity function is

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equal to 1, it is well-known to be difficult that on the design of control system, both low sensitivity characteristics and robust stability are obtained.

However, the control system with low sensitivity characteristics is not always made to be unstable. Maeda and Vidyasagar consider this problem as an infinite gain margin one [10, 11]. Nogami et al. clarify the condition that hi-gain controller does not make the system unstable, and proposed a design method [12]. Doyle et al. consider this lowsensitivity control problem from another viewpoint; there exists a class of uncertainty that has the property to make the control system with low sensitivity characteristics robustly stabile, and provide a necessary and sufficient condition to make the control system robustly stable for its class of uncertainty [13]. Thus, for the uncertainty described in [13], we can construct the control system with low sensitivity and robust stability. In this meaning, the uncertainty described in [13] is suitable for designing the highperformance robust control system. The uncertainty described in [13] cannot be applied to a system with varying number of poles in an open right half plane. There exist applications of a system with varying number of poles in the open right half plane such that the number of poles in the open right half plane changes. For example, the number of poles in the open right half plane of a large flexible spacecraft changes when the configuration of the spacecraft is changed [9]. The problem to obtain the robust stability condition for the system with varying number of poles in the closed right half plane is difficult because the problem does not reduce to the small gain theorem. Yamada considers this problem, and obtains the robust stability condition for the system with varying number of poles in the open right half plane [14].

To give a low-sensitivity characteristic to the control system, it is important to consider a control structure of the control system. Morari and Zafiriou propose the Internal Model Control (IMC) structure [15]. The IMC structure is a structure that the controller has a model of the plant, which is called the nominal plant inside. According to [15-17], the IMC structure can give the control system a low sensitivity characteristics by using the feedback of an error between the output of a model of the plant, which is called the nominal plant, and that of the plant. However, the IMC structure has a problem that for a plant with an unstable system, the IMC structure does not provide a high performance characteristic such as disturbance attenuation [15-19]. This problem is caused by the reason that if the plant is unstable, any reference input will make the output grow without bound, since the inverse system has an unstable zero [16]. To overcome this problem, several researchers propose several design methods of IMC structure [18-21]. Kaya proposes a two-degree-of-freedom IMC structures by using the two-degree-of-freedom control [19]. The two-degree-of-freedom IMC structure is provided to eliminate the aforementioned shortcomings of the original IMC structure, since single controller is split into two controllers for set-point tracking and disturbance attenuation. Zhou and Ren overcome the problem of IMC structure and propose a new structure named Generalized Internal Model Control (GIMC) structure [21]. There exist several applications of the GIMC structure [22-27]: suspension systems [23, 24, 27]; active actuators [25]; automotive electric power steering system [26, 27] and so on. Okajima et al. propose a model error compensator control structure [28-30]. Model error compensator control structure is a control structure that the output trajectory of the nominal plant can be made close to that of the model. The model error compensator control structure is applied to several control such as an indoor platoon driving system of welfare personal vehicles [31]. On the other hand, Yu et al. expand the result of [14] and propose a design method of a control system using double feedback control [32]. The control system using double feedback structure, which is called the double feedback control system, has a structure that the two-degree-of-freedom control system is included in the two-degree-of-freedom control system. According to [32], the double feedback control system can have the robust stability and the low sensitivity characteristic in the meaning of reducing influence of uncertainty to the output by using the result of [14]. The low sensitivity characteristic of the double feedback control system is better than that of a original two-degree-of-freedom control system. This design method proposed by Yu et al. is suitable to design a high performance robust control system in the meaning of the disturbance attenuation and reducing the influence of uncertainty to the output for the system with varying number of unstable poles. However, the design method of [32] cannot be applied to the system having an uncertain relative degree. According to [33, 34], several plants having an uncertain relative degree exist. Thus, in order to design a control system with robust stability and low sensitivity for the plant having an uncertainty relative degree, it is important to clarify a condition to make the double feedback control system robustly stable for its plant, and propose a design method of its control system.

According to [35], it is clarified a robust stability condition that for the plant with varying number of poles in the closed right half plane having uncertain relative degree, the low sensitivity control guarantees robust stability. By expanding the result of [35], we can obtain a design method of a control system by using double feedback control for systems with varying number of poles in the closed right half plane having uncertain relative degree.

In this paper, we examine a design method of a control system using double feedback control for the SISO minimum phase systems with varing number of unstable poles and having an uncertain relative degree. The purpose of this paper is summarized as follows: 1) we clarify a condition that the control system by the double feedback is robustly stable for the plant with varing number of unstable poles and having an uncertain relative degree; 2) we propose a design method of a control system by using double feedback. This paper is organized as follows. In Section 2, we describe the preliminary results and problem in this paper. That is, we explain the double feedback control system by comparing the twodegree-of-freedom control system. In Section 3, the robust stability condition of the double feedback control system is clarified. In Section 4, we examine a sensitivity characteristic of double feedback control system. In Section 5, we present a design method for double feedback control system with low-sensitivity and robust stability. Section 6 gives some concluding remarks.

Notations.

R the set of real numbers.

 $R_{+e} \quad R \cup \{\infty\}.$ 

 $|\cdot|$  absolute value of  $\cdot$ .

R(s) the set of real relational functions with s.

 $RH_{\infty}$  the set of stable proper real relational functions.

 $\|\cdot\|_{\infty}$   $H_{\infty}$  norm of  $\cdot$ .

2. **Preliminary Result and Problem Formulation.** In this section, we explain the preliminary results of the two-degree-of-freedom control system and the problem considered in this paper.

Consider the two-degree-of-freedom control system shown in Figure 1. Here,  $G(s) \in R(s)$  is strictly proper and a SISO minimum phase plant,  $C_1(s) \in R(s)$  and  $F_1(s) \in RH_{\infty}$  are controllers,  $r(s) \in R(s)$  is an input and  $y(s) \in R(s)$  is an output.

The nominal plant of G(s) denotes  $F_0(s) \in R(s)$ , in which  $F_0(s)$  is proper and of minimum-phase. The relationship between G(s) and  $F_0(s)$  is written by the form in

$$G(s) = F_0(s)(1 + \Delta(s)), \tag{1}$$

where  $\Delta(s)$  is the uncertainty, and assume that the relative degree of  $F_0(s)$  is not always equal to that of G(s).



FIGURE 1. The two-degree-of-freedom control system

The transfer function of the two-degree-of-freedom control system in Figure 1 from r(s) to y(s) is written as

$$y(s) = F_1(s) \left(1 + H_1(s)\right) r(s), \tag{2}$$

where  $H_1(s) \in R(s)$  is written by

$$H_1(s) = \frac{\frac{1}{1+C_1(s)F_0(s)} \frac{\Delta(s)}{1+\Delta(s)}}{1 - \frac{1}{1+C_1(s)F_0(s)} \frac{\Delta(s)}{1+\Delta(s)}}.$$
(3)

The two-degree-of-freedom control system in Figure 1 reduces an influence of  $\Delta(s)$  to y(s) if the gain of  $H_1(s)$  is made to be small. From (2) and (3), in order to have  $H_1(s)$  small value, it is needed to make a function  $S_1(s)$  denoted by

$$S_1(s) = \frac{1}{1 + C_1(s)F_0(s)} \tag{4}$$

small value.

To design the control system with low sensitivity and robust stability for the uncertainty with varying number of poles in the closed right half plane having uncertain relative degree, we adopt the class of uncertainty  $\Delta(s)$  given by the following definition.

**Definition 2.1.** [35] G(s) is called the elementary of the set  $\Omega$  if following expressions hold.

• The number of zeros of G(s) in the closed right half plane is equal to that of  $F_0(s)$ .

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$$0 \le \rho(G(s)) - \rho(F_0(s)) \le 2$$
(5)

holds true, where  $\rho(G(s))$  is the relative degrees of G(s) and  $\rho(F_0(s))$  is that of  $F_0(s)$ . •  $\Delta(s)$  satisfies

$$\left|\frac{\Delta(j\omega)}{1+\Delta(j\omega)}\right| \le |W(j\omega)| \quad (\forall \omega \in R_{+e}),\tag{6}$$

where  $W(s) \in R(s)$  is an upper bound of  $\Delta(s)$  satisfying

$$\lim_{\omega \to \infty} W(j\omega) = 1.$$
<sup>(7)</sup>

• G(s) and  $F_0(s)$  are of minimum phase.

When G(s) is the element of the set  $\Omega$ , we denote  $\Delta(s) \in \Omega$ .

According to [35], for  $\Delta(s) \in \Omega$ , we can construct the control system in Figure 1 with robust stability and low sensitivity. A necessary and sufficient condition of the control system in Figure 1 with robust stability and low sensitivity characteristics is summarized as follows.

**Theorem 2.1.** [35] Assume that  $C_1(s)$  stabilizes the nominal plant  $F_0(s)$ ,  $0 \le \rho(G(s)) - \rho(F_0(s)) < 2$  and  $F_1(s)/F_0(s) \in RH_{\infty}$ . The two-degree-of-freedom control system in Figure 1 is robustly stable for  $\Delta(s) \in \Omega$  if and only if

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$$\left\|S_1(s)W(s)\right\|_{\infty} < 1\tag{8}$$

holds.

To prove Theorem 2.1, necessary lemmas are shown.

**Lemma 2.1.** [35] It is assumed that  $G_m(s)$  has a q-th number of zero in the closed right half plane and a  $p_m$ -th number of pole in the closed right half plane, and G(s) has a q-th number of zero in the closed right half plane and p-th number of pole in the closed right half plane, the relative degree of G(s) is equal to that of  $G_m(s)$ . The Nyquist plots of  $1 + \Delta(j\omega) = G(j\omega)/G_m(j\omega)$  for  $-\infty \leq \omega \leq \infty$  encircle the origin (0,0)  $p - p_m$  times in the counter-clockwise direction.

**Proof:** The proof is obvious from Lemma 1 in [35].

**Lemma 2.2.** [35] It is assumed that W(s) satisfies (7) and (5),  $G_m(s)$  has q-th number of zero in the closed right half plane and  $p_m$ -th number of pole in the closed right half plant, and G(s) has q-th number of zero in the closed right half plane and p-th number of pole in the closed right half plane. The Nyquist plot of  $1 + \Delta(j\omega)$  for  $-\infty \leq \omega \infty$  encircles the origin  $(0,0) p - p_m$  times in the counter-clockwise direction.

 Proof: The proof is obvious from Lemma 2 in [35].
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 The proof of Theorem 2.1 is shown by using above lemmas.
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 Proof: The proof is obvious from Theorem 2 in [35].
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 Theorem 2.1 is summarized as follows.
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• In order to design the control system with robust stability and low sensitivity, the controller  $C_1(s)$  needs to minimize  $||S_1(s)W(s)||_{\infty}$ , at worst  $C_1(s)$  must satisfy

$$||S_1(s)W(s)||_{\infty} < 1.$$

• The control system in Figure 1 to satisfy Theorem 2.1 is robustly stable for the plant included in the set Ω.

In order to make the control system have lower sensitivity characteristic than twodegree-of-freedom control system, Yu et al. propose the double feedback control system shown in Figure 2 [32]. Here,  $F_2(s) \in RH_{\infty}$  and  $C_2(s) \in R(s)$  are controller, and  $C_2(s)$ stabilizes  $F_1(s)$ . According to [15], since  $C_2(s)$  stabilizes  $F_1(s) \in RH_{\infty}$ , the parameterization of all stabilizing controller  $C_2(s)$  is written by

$$C_2(s) = \frac{Q(s)}{1 - Q(s)F_1(s)},\tag{9}$$

where  $Q(s) \in RH_{\infty}$  is any function. The double feedback control structure is a structure that the two-degree-of-freedom control system surrounded by dotted line in Figure 2 is included in the two-degree-of-freedom control.

According to [32], the double feedback control system in Figure 2 can attenuate the influence of  $\Delta(s)$  to y(s) less than that of the two-degree-of-freedom control system in Figure 1. The transfer function from r(s) to y(s) in Figure 2 is given by

$$y(s) = F_2(s) \left(1 + H_2(s)\right) r(s), \tag{10}$$

where  $H_2(s) \in R(s)$  is written by

$$H_2(s) = \frac{\frac{1}{1+C_2(s)F_1(s)} \frac{1}{1+C_1(s)F_0(s)} \frac{\Delta(s)}{1+\Delta(s)}}{1-\frac{1}{1+C_2(s)F_1(s)} \frac{1}{1+C_1(s)F_0(s)} \frac{\Delta(s)}{1+\Delta(s)}} = \frac{S_1(s)S_2(s) \frac{\Delta(s)}{1+\Delta(s)}}{1-S_1(s)S_2(s) \frac{\Delta(s)}{1+\Delta(s)}}$$
(11)

and

$$S_2(s) = \frac{1}{1 + C_2(s)F_1(s)}.$$
(12)

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The double feedback control system in Figure 2 can attenuate the influence of  $\Delta(s)$  to y(s) by making gain of  $H_2(s)$  small value. In order to have the gain of  $H_2(s)$  small value, it is needed to make a function  $S_1(s)S_2(s)$  be small.



FIGURE 2. The double feedback control system

In this paper, we consider two problems as follows.

- 1) The first problem is to clarify a robust stability condition of the double feedback control system for  $\Delta(s) \in \Omega$  that low sensitivity control in the meaning of attenuating influence of  $\Delta(s)$  to y(s), which makes  $H_2(s)$  small value, guarantees the robust stability.
- 2) The second problem is to examine a design method for the low sensitivity control of the double feedback control system in Figure 2.

3. The Robust Stability Condition of the Double Feedback Control System. In this section, we clarify the robust stability condition of double feedback control system in Figure 2.

The robust stability condition is summarized in Theorem 3.1.

**Theorem 3.1.** Assume that  $C_i(s)$  (i = 1, 2) stabilizes  $F_{i-1}(s)$ ,  $0 \le \alpha = \rho(G(s)) - \rho(F_0(s)) < 2$  and  $F_i(s)/F_{i-1}(s) \in RH_{\infty}$ . The double feedback control system in Figure 2 is robustly stable for  $\Delta(s) \in \Omega$  if and only if

$$\|S_1(s)S_2(s)W(s)\|_{\infty} < 1 \tag{13}$$

holds true.

The proof of Theorem 3.1 is shown by using Lemma 2.1 and Lemma 2.2.

**Proof:** Let a characteristics polynomial of the double feedback control system in Figure 2 be

$$1 + \left\{ \frac{F_1(s)}{F_0(s)} C_2(s) \left( 1 + C_1(s) F_0(s) \right) + C_1(s) \right\} G(s).$$
(14)

Proof is immediately obtained by applying Theorem 2 in [35] to

$$1 + \left\{ \frac{F_1(s)}{F_0(s)} C_2(s) \left( 1 + C_1(s) F_0(s) \right) + C_1(s) \right\} G(s)$$
  
=  $(1 + C_1(s) F_0(s)) (1 + C_2(s) F_1(s)) \left\{ 1 - S_1(s) S_2(s) \right\}.$  (15)

We have the complete proof of this theorem.

Theorem 3.1 is summarized as follows.

• In order to design the control system with robust stability and low sensitivity, the controllers  $C_1(s)$ ,  $F_1(s)$  and  $C_2(s)$  need to minimize  $||S_1(s)S_2(s)W(s)||_{\infty}$ , at worst  $C_1(s)$  must satisfy

$$||S_1(s)S_2(s)W(s)||_{\infty} < 1.$$

• It is not only related with  $C_1(s)$  but also  $C_2(s)$  to minimize  $||S_1(s)S_2(s)W(s)||$ .

• The control system in Figure 2 to satisfy Theorem 3.1 is robustly stable for the plant included in the set  $\Omega$ .

4. Sensitivity Characteristic of Double Feedback Control System. In this section, we examine a sensitivity characteristic of double feedback control system in Figure 2.

According to [32], by comparing between the gain of  $H_2(s)$  and that of  $H_1(s)$ , we can know whether or not the low sensitivity characteristics of the double feedback control system outperform that of the two-degree-of-freedom control system in Figure 1. Since  $H_1(s)$  and  $H_2(s)$  depend on  $F_1(s)$  and  $F_2(s)$ , in order to compare characteristics of  $H_1(s)$ and  $H_2(s)$ , we settle  $F_1(s) = F_2(s)$ . If  $|H_2(j\omega)/H_1(j\omega)| \leq 1$ , then the double feedback control system in Figure 2 has low sensitivity characteristics more than the two-degreeof-freedom control system in Figure 1, in the meaning of reducing the influence of  $\Delta(s)$ to y(s). From (3) and (11),  $H_2(s)/H_1(s)$  is written as

$$\frac{H_2(s)}{H_1(s)} = 1 - K(s), \tag{16}$$

where

$$K(s) = \frac{1 - S_2(s)}{1 - S_1(s)S_2(s)\frac{\Delta(s)}{1 + \Delta(s)}}.$$
(17)

From (17), if

$$1 - S_2(j\omega) \simeq 0,\tag{18}$$

 $H_2(j\omega)/H_1(j\omega)$  is close to 0. In addition, if

$$1 - S_2(j\omega_1) \simeq 1,\tag{19}$$

 $H_2(j\omega)/H_1(j\omega)$  is close to 1. From above discussion, on the frequency range making  $S_2(s)$  small value, the double feedback control system in Figure 2 can achieve low sensitivity characteristics more than the two-degree-of-freedom control system in Figure 1.

5. A Design Method for Double Feedback Control System. In this section, we present a design method for double feedback control system with low-sensitivity and robust stability.

It is assumed that  $C_1(s)$  is settled using appropriate methods. From (9), since  $F_1(s)$  is of minimum-phase and stable, Q(s) is settled as

$$Q(s) = \frac{1}{F_1(s)}\hat{Q}(s),$$
(20)

where  $\hat{Q}(s) \in RH_{\infty}$  is

$$\hat{Q}(s) = \frac{1}{(1 + \tau_q s)^{\alpha_q}},$$
(21)

where  $\tau_q \in R$  is an arbitrary positive real number, which satisfies  $\hat{Q}(j\omega) \simeq 0$  in the wide frequency range, and  $\alpha_q$  is an arbitrary positive integer which makes  $\hat{Q}(s)$  proper.

From (12), (9), and (21), since  $S_2(s)$  in (12) can be rewritten by

$$S_2(s) = 1 - Q(s), (22)$$

 $S_2(s)$  satisfies (18) in the wide frequency range.

6. **Conclusion.** In this paper, we have examined the design method for double feedback control system for the SISO minimum phase systems with varying number of unstable poles and uncertain relative degrees. We have shown the robust stability condition of

the double feedback control system for the SISO minimum phase systems with varying number of unstable poles and uncertain relative degrees. In addition, we have presented a design method of the double feedback control system with robust stability and low sensitivity. However, we do not consider a design method of a control system by using double feedback control with robust stability and low sensitivity for SISO non-minimum phase system with varying number of unstable poles having uncertain relative degree. It is well-known that it is difficult to design a control system with low sensitivity for a non-minimum phase system. In addition, we do not consider an application by using a result of this paper. These will be considered in another papers as future work.

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