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DETERMINATION OF STRESS STATE FOR A LAYER WITH A LONGITUDINAL CYLINDRICAL THICK-WALLED TUBE UNDER GIVEN MIXED CONDITIONS ON BOUNDARY SURFACES

A substantially spatial problem of elasticity theory for a layer with a longitudinal circular cylindrical tube solved in it is solved. The layer and tube are rigidly fixed together. It is necessary to study the stress-strain state of the elastic bodies of both the layer and tube.

On the lower boundary of the layer, displacements are given; on the upper boundary of the layer and the inner surface of the tube, stresses; on the boundary of the layer and tube, conjugation conditions. The solution to the spatial problem of the theory of elasticity is obtained using the generalized Fourier method in relation to the system of Lamé's equations in the cylindrical coordinates associated with the tube, and the Cartesian coordinates associated with the boundaries of the layer. By satisfying both the boundary and conjugation conditions, we obtain infinite systems of linear algebraic equations that are solved by the truncation method. As a result, we obtain displacements and stresses at different points of both the elastic layer and elastic tube.

A numerical analysis of the stress-strain state of the elastic body of the layer and tube is carried out. Graphs of the normal stresses on the inner and outer surfaces of the tube are presented.

***Keywords**: thick-walled tube in the layer, Lame's equation, generalized Fourier method.*

Introduction

When designing tunnels, underground facilities and protective screens, there is a need to determine the stress-strain state in such structures. To achieve this, it is necessary to have a calculation method that matches the calculation scheme and allows getting the result with the required accuracy.

There are papers for the layer with a transverse circular cavity or inclusion [1 - 3]. However, the methods used in them can not be applied to the layer with a longitudinal cavity or inclusion.

Papers [4-5] consider the stationary problems of wave diffraction for the layer with a longitudinal cylindrical cavity or inclusion, with the problems based both on the Fourier decomposition method and the image method.

This paper uses an analytical-numerical approach, and is based on the generalized Fourier method [6]. On the basis of this method, also solved are the problems for a half-space with a cylindrical cavity or inclusion [9-11], as well as the one for a cylinder with cylindrical inclusions [12].

Formulation of the Problem

In an elastic homogeneous layer, parallel to its boundaries, an infinite circular cylindrical thick-walled tube with an external radius R_1 and an interior one R_2 is located.

The tube will be considered in the cylindrical coordinate system (ρ, φ, z), and the layer, in the Cartesian coordinate system (x, y, z), which is equally oriented and associated with the system of coordinates of the tube. The upper boundary of the layer is located at the distance $y = h$, the lower one, at the distance $y=-\tilde{h}$. It is necessary to find a solution to the Lamé equation

$$\Delta\vec{U}_j + \left(1-2\sigma_j\right)^{-1}\nabla div\vec{U}_j = 0$$

where σ_j is Poisson's coefficient for the layer ($j = 1$) or the tube ($j = 2$).

On the lower boundary of the layer, the displacements $\vec{U}_1(x,z)\big|_{y=-\tilde{h}} = \vec{U}^{0}_{\tilde{h}}(x,z)$ are given; on the upper boundary of the layer, the stresses $F\vec{U}_1(x,z)\big|_{y=h} = F\vec{U}^{0}_{h}(x,z)$; on the inner surface of the tube, the stresses $F\vec{U}_2(\varphi,z)\big|_{\rho=R_2} = \vec{F}_R^0(\varphi,z)$; on the boundary of the tube and layer, the conjugation conditions

$$\vec{U}_1(\varphi,z)\big|_{\rho=R_1} = \vec{U}_2(\varphi,z)\big|_{\rho=R_1}, \tag{1}$$

$$F\vec{U}_1(\varphi,z)\big|_{\rho=R_1} = F\vec{U}_2(\varphi,z)\big|_{\rho=R_1}, \tag{2}$$

where $F\vec{U}_j = 2\cdot G_j \cdot \left[\frac{\sigma_j}{1-2\cdot\sigma_j}\vec{n}\cdot \mathrm{div}\vec{U}_j + \frac{\partial}{\partial n}\vec{U}_j + \frac{1}{2}\left(\vec{n}\times \mathrm{rot}\vec{U}_j\right)\right]$ is the stress operator; σ_j, G_j, $\vec{U}_j$ are the elastic constants and displacements of the layer (*j = 1*) or the tube (*j = 2*);

$$\begin{aligned}
&\vec{U}_h^0(x,z) = \tau_{yx}^{(h)}\vec{e}_1^{(1)} + \sigma_y^{(h)}\vec{e}_2^{(1)} + \tau_{yz}^{(h)}\vec{e}_3^{(1)},\\
&\vec{U}_{\tilde{h}}^0(x,z) = U_x^{(\tilde{h})}\vec{e}_1^{(1)} + U_y^{(\tilde{h})}\vec{e}_2^{(1)} + U_z^{(\tilde{h})}\vec{e}_3^{(1)},\\
&\vec{F}_R^0(\varphi,z) = \sigma_\rho^{(p)}\vec{e}_1^{(2)} + \tau_{\rho\varphi}^{(p)}\vec{e}_2^{(2)} + \tau_{\rho z}^{(p)}\vec{e}_3^{(2)}
\end{aligned} \tag{3}$$

are known functions; $\vec{e}_j^{(k)}$, $(j = 1,2,3)$ are the unit vectors of the Cartesian (*k = 1*) and cylindrical (*k = 2*) coordinate systems.

All known vectors and functions will be considered as fast falling to zero at great distances from the origin of the coordinate *z* for the tube and the coordinates *x* and *z* for the boundaries of the layer.

Solving the Problem

Choose the basic solutions to the Lamé equation for the specified coordinate systems in form [6]

$$\begin{aligned}
&\vec{u}_k^{\pm}(x,y,z;\lambda,\mu) = N_k^{(d)} e^{i(\lambda z+\mu x)\pm\gamma y};\\
&\vec{R}_{k,m}(\rho,\varphi,z;\lambda) = N_k^{(p)} I_m(\lambda\rho) e^{i(\lambda z+m\varphi)};\\
&\vec{S}_{k,m}(\rho,\varphi,z;\lambda) = N_k^{(p)}\left[(\mathrm{sign}\,\lambda)^m K_m(|\lambda|\rho)\cdot e^{i(\lambda z+m\varphi)}\right]; k=1,2,3;
\end{aligned} \tag{4}$$

$$N_1^{(d)} = \frac{1}{\lambda}\nabla;\ N_2^{(d)} = \frac{4}{\lambda}(\sigma-1)\vec{e}_2^{(1)} + \frac{1}{\lambda}\nabla(y\,\cdot);\ N_3^{(d)} = \frac{i}{\lambda}\mathrm{rot}\left(\vec{e}_3^{(1)}\,\cdot\right);\ N_1^{(p)} = \frac{1}{\lambda}\nabla;$$

$$N_2^{(p)} = \frac{1}{\lambda}\left[\nabla\left(\rho\frac{\partial}{\partial\rho}\right) + 4(\sigma-1)\left(\nabla - \vec{e}_3^{(2)}\frac{\partial}{\partial z}\right)\right];\ N_3^{(p)} = \frac{i}{\lambda}\mathrm{rot}\left(\vec{e}_3^{(2)}\,\cdot\right);$$

$$\gamma = \sqrt{\lambda^2+\mu^2},\ -\infty < \lambda,\mu < \infty,$$

where $I_m(x)$, $K_m(x)$ are the modified Bessel functions; $\vec{R}_{k,m}$, $\vec{S}_{k,m}$, *k*=1, 2, 3 are, respectively, the internal and external solutions to the Lamé equation for the cylinder; $\vec{u}_k^{(-)}$, $\vec{u}_k^{(+)}$ are the solutions to the Lamé equation for the layer.

The solution to the problem will be presented in the form

$$\vec{U}_1 = \sum_{k=1}^{3}\int_{-\infty}^{\infty}\sum_{m=-\infty}^{\infty} B_{k,m}(\lambda)\cdot \vec{S}_{k,m}(\rho,\varphi,z;\lambda)d\lambda + \\ + \sum_{k=1}^{3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(H_k(\lambda,\mu)\cdot \vec{u}_k^{(+)}(x,y,z;\lambda,\mu) + \tilde{H}_k(\lambda,\mu)\cdot \vec{u}_k^{(-)}(x,y,z;\lambda,\mu)\right)d\mu d\lambda, \quad (5)$$

$$\vec{U}_2 = \sum_{k=1}^{3}\int_{-\infty}^{\infty}\sum_{m=-\infty}^{\infty} A_{k,m}(\lambda)\cdot \vec{R}_{k,m}(\rho,\varphi,z;\lambda) + \tilde{A}_{k,m}(\lambda)\cdot \vec{S}_{k,m}(\rho,\varphi,z;\lambda)d\lambda, \quad (6)$$

where, $\vec{S}_{k,m}(\rho,\varphi,z;\lambda)$, $\vec{R}_{k,m}(\rho,\varphi,z;\lambda)$ $\vec{u}_k^{(+)}(x,y,z;\lambda,\mu)$ and $\vec{u}_k^{(-)}(x,y,z;\lambda,\mu)$ are the basic solutions given by formulas (4), and the unknown functions $H_k(\lambda,\mu)$, $\tilde{H}_k(\lambda,\mu)$, $B_{k,m}(\lambda)$, $A_{k,m}(\lambda)$ and $\tilde{A}_{k,m}(\lambda)$ must be found from boundary conditions (3) and conjugation conditions (1) and (2).

For the transition in basic solutions between coordinate systems, we use the formulas [14].

To fulfill the boundary conditions at the boundaries of the layer, we rewrite the basic solutions $\vec{S}_{k,m}$ in (5), using transition [14, formulas 7], in the Cartesian coordinate system through the basic solutions $\vec{u}_k^{(-)}$ (for $y = h$) and $\vec{u}_k^{(+)}$ (for $y=-\tilde{h}$). We equate the resulting vectors, for $y=-\tilde{h}$, to the given $\vec{U}_{\tilde{h}}^{0}(x,z)$, and for $y=-\tilde{h}$, we find the stresses and equate them to $\vec{F}_h^0(x,z)$. We give the vectors $\vec{U}_{\tilde{h}}^{0}(x,z)$ and $\vec{F}_h^0(x,z)$ in advance through the double Fourier integrals.

The resulting system of 6 equations has a determinant

$$\frac{32\cdot G^3\cdot\gamma^5\cdot \mathrm{ch}\,\bar{x}\cdot\left[\bar{x}^2+(3-4\sigma)\cdot\mathrm{ch}^2\,\bar{x}+(1-2\sigma)^2\right]}{\lambda^4}$$

where $\bar{x} = \gamma\left(h+\tilde{h}\right)$, G is the shear modulus. The square brackets of this determinant coincide with known results [15].

From the obtained equations, we find the functions $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$ through $B_{k,m}(\lambda)$.

To take into account conjugation conditions (1), we decompose, in (5), the basic solutions $\vec{u}_k^{(\pm)}$ by means [14, formulas 8], turning them into the solutions $\vec{R}_{k,m}$. We then equate $\rho=R_1$ therein. This will fulfill condition (1).

To take into account conjugation conditions (2), we find the vectors $F\vec{U}_1$ and $F\vec{U}_2$ from solutions (5) and (6), decompose the basic solutions $\vec{u}_k^{(\pm)}$ therein by means of [14, formulas 8], turning them into the solutions $\vec{R}_{k,m}$, and equate ρ = R1. This will fulfill condition (2).

These two conditions give 6 equations, conjugating all the unknowns in equations (5) and (6).

To take into account the boundary conditions on the inner surface of the tube, we apply the stress operator to the right-hand side of (6), and equate (for $\rho=R_2$) to the specified $\vec{F}_R^0(\varphi,z)$ given by the integral and Fourier series.

From the resulting system of equations, we exclude the previously found functions $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$ through $B_{k,m}(\lambda)$. Having gotten rid of the series m and integrals λ, we obtain a collection of nine infinite systems of linear algebraic equations for identifying the unknowns $A_{k,m}(\lambda)$, $\tilde{A}_{k,m}(\lambda)$ and $B_{k,m}(\lambda)$.

For the obtained infinite systems of equations, we will apply the truncation method. The numerical studies show that the determinant of the truncated system does not turn into zero for any m, for $0\le m\le 10$, and, consequently, this system of equations has a unique solution.

Having solved this system of equations, we will find the unknowns, $A_{k,m}(\lambda)$, $\tilde{A}_{k,m}(\lambda)$ and $B_{k,m}(\lambda)$.

We substitute the functions $B_{k,m}(\lambda)$ obtained from the infinite system of equations into the expressions for $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$. This will determine all unknown problems.

Numerical Studies of the Stressed State

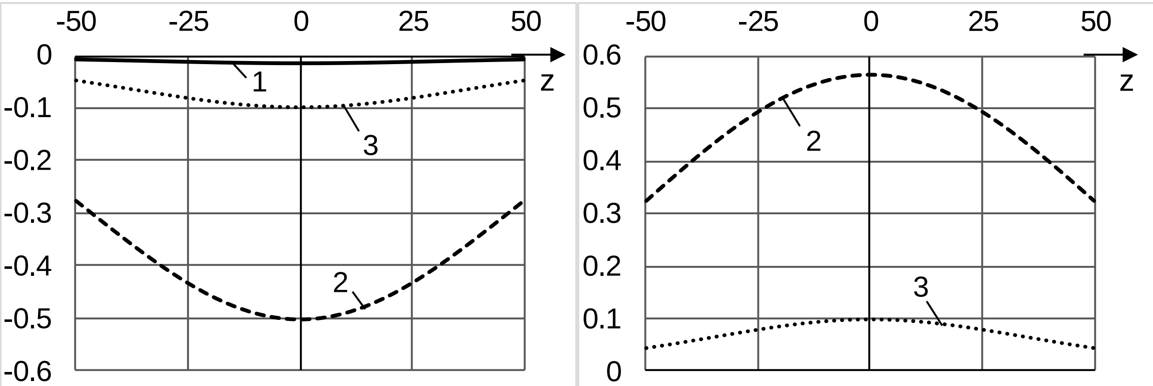
A B30 grade concrete tube is located in a homogeneous isotropic clay layer in parallel with its surfaces. Layer: Poisson's coefficient $\sigma_1 = 0.3$, the elastic modulus E_1=10 kN / cm^2. Tube: Poisson's coefficient σ_2 = 0.16, the elastic modulus E_2= 3250 kN / cm^2. The outer tube radius R_1 = 70 cm, the internal one R_2 = 60 cm. The layer thickness $h+\tilde{h}$ = 340 cm. The distance from the upper boundary of the layer to the tube center h = 170 cm.

With the weight of the processing equipment taken into account, on the upper boundary of the layer, the stresses $\sigma_y^{(h)}(x,z) = -10^8 \cdot \left(z^2+10^2\right)^{-2} \cdot \left(x^2+10^2\right)^{-2}$, $\tau_{yx}^{(h)} = \tau_{yz}^{(h)} = 0$ are given; on the lower boundary of the layer, the displacements $U_x^{(\tilde{h})} = U_y^{(\tilde{h})} = U_z^{(\tilde{h})} = 0$ are given. On the inner surface of the tube, there are no stresses $\sigma_\rho^{(p)} = \tau_{\rho\varphi}^{(p)} = \tau_{\rho z}^{(p)} = 0$.

A finite system of equations of order m = 8 was solved. The accuracy of the fulfillment of the boundary conditions for the indicated values of geometric parameters was equal to 10^{-4}.

Fig. 1 shows the normal stresses along the z axis at the upper point of the tube on the outer and inner surfaces.

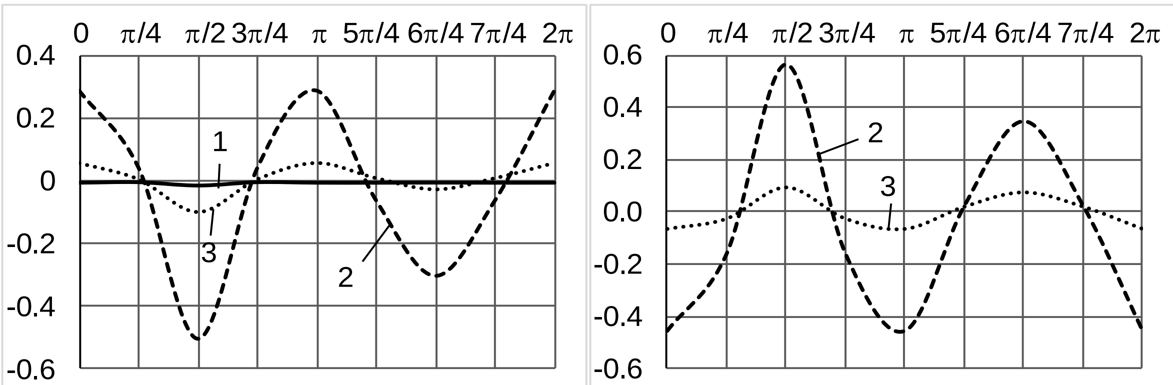
The greatest stresses are σ_φ (Fig. 1, line 2), which reach the maximum values at z = 0: on the outer surface of the tube, the compression σ_φ = -0.5 kN / cm^2; on the inner surface of the tube, the tension σ_φ = +0.564 kN / cm^2. It should be noted that the stresses on the tube surface, along the z axis, fall very slowly (compared to the specified function at the boundary of the layer).



a *b*

Fig. 1. Stresses on the surfaces of the tube along the z axis, at x = 0 (in kN / cm^2): a − on the outer surface (y = + R_1); b − on the inner surface (y = + R_2); 1 − σ_ρ ; 2 − σ_φ ; 3 − σ_z

Fig. 2 shows the stresses on the tube surface (along the radii R_1 and R_2) in the plane z = 0.



a *b*

Fig. 2. Stresses on the surface of the tube along the radii R_1 and R_2, at z = 0 (in kN / cm$_2$): a − on the outer surface; B − on the inner surface; 1 − σ_ρ ; 2 − σ_φ ; 3 − σ_z

Along the radii, the stresses vary from compression to tension and vice versa. Thus, on the outer surface of the tube (Fig. 2a) at the upper and lower points, there is compression, to the right and left, tension. At the inner surface of the tube (Fig. 2b), in the upper and lower points, there is tension, on the left and right, compression. In addition, the stresses in absolute value on the inner surface of the tube are higher than on the outer one.

Along the radius R_1 in the elastic body of the layer, the stresses are very small (in comparison with those in the elastic body of the tube), which is the result of the difference in the layer and tube materials.

Conclusions

On the basis of the generalized Fourier method, the problem for the layer with a longitudinal cylindrical thick-walled tube and different boundary conditions at the boundaries of the layer and tube is calculated.

The proposed analytical-numerical calculation method allows us, with the given accuracy, to determine the stress-strain state of the elastic body, taking into account its infinite boundaries and conjugation conditions for the layer and tube.

The numerical study of the stress-strain state of the concrete tube, which is in a layer of clay under the action of loading on the surface of the layer, proves that the greatest stresses arise on its inner surface. In addition, in comparison with the given function, there is a very slow decrease in the stresses along the *z* axis.

The numerical studies of the algebraic system make it possible to state that its solution can be found with any degree of accuracy by the method of reduction. This is confirmed by the high accuracy of the implementation of boundary conditions.

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