H_∞ Optimal Control Design of Static Var Compensator Coupling Hybrid Active Power Filter Based on Harmonic State-Space Modeling

Cheng GONG, Wai-Kit SOU, and Chi-Seng LAM

Abstract—Compared with the conventional active power filter (APF), the static var compensator coupling hybrid active power filter (SVC-HAPF) has distinct characteristics of low DC-link operating voltage, which can lower the system and operational costs. It also has a wider operational range than the conventional hybrid active power filter (HAPF). In this paper, a harmonic state-space (HSS) modeling is firstly proposed to address the nonlinearity issue of the SVC-HAPF. The HSS based SVC-HAPF model is linear time-invariant (LTI), which enables the optimal control techniques. Then, the linear matrix inequality (LMI) based H_∞ optimal control design is proposed, which finds the optimal controller parameters via convex optimization algorithm. The proposed controller for the SVC-HAPF achieves high steady-state accuracy, fast transient response, and fixed switching frequency simultaneously. Finally, simulation and experimental results are also provided to verify the effectiveness and performance of the optimal controller for the SVC-HAPF in comparison with the recent developed state-of-the-art multi-quasi-proportional-resonant (MQPR) controller, which shows superior control performances.

Index Terms— H_∞ optimal control, harmonic state-space model, hybrid active power filter, power quality, static var compensator.

I. INTRODUCTION

POWER quality problems, such as harmonic distortion and poor power factor, lead to serious stability problems and additional power losses in emerging power electronic enabled power systems [1], [2]. Conventionally, passive power filters (PPFs) are widely used to compensate the reactive power and eliminate the harmonic current [3]. However, PPFs suffer from fixed compensation capability and resonance problems. Active power filters (APFs) can overcome the problems of PPFs

[4]–[6], but the operational power loss and the initial cost of the APF are much higher because of its high DC-link voltage requirement. The hybrid active power filter (HAPF) could be considered as a cost-effective solution to cancel out harmonic current as well as reactive power [7]–[11]. However, the conventional HAPF has a very limited compensation range. To extend the compensation range while keeping low DC-link voltage characteristics, a static var compensator (SVC) was applied to the HAPF to improve its compensation range.

In 2014, the SVC coupling HAPF (SVC-HAPF) was first proposed [10], [12], in which its SVC part is used to dynamically compensate a wide range of reactive power, thus significantly reduces the current and voltage ratings of the active inverter part. And its active inverter part can also compensate the harmonic current. Due to the advantages of this HAPF topology, additional functions such as unbalance compensation and renewable energy integration are further investigated in the literatures [13]–[16].

The objective of the SVC-HAPF is to achieve power system current quality compensation, thus the current controller design is important for enhancing the compensating performance. However, the utilization of thyristors in the SVC part causes highly nonlinear dynamics, which leads to the controller design a challengeable task. In [13]–[16], a hysteresis current controller was applied for the SVC-HAPF. The hysteresis current controller is a simple and fast control strategy. However, its variable switching frequency leads to large output current ripple and grid stability issues, limit the usage of hysteresis current controller [17]. An alternative solution is applying the PWM-based control strategy which can fix the switching frequency. Recently, a multi-quasi-proportional-resonant (MQPR) controller with gain scheduling for the SVC-HAPF was proposed [18], which achieved a high steady-state accuracy as well as a fixed switching frequency. In the MQPR design, the nonlinear SVC part is modeled as two linear circuits for approximation. However, its gains design is not optimized, and the proportional gain is designed the same for both fundamental and all considered harmonics order [18], this also leads to slow transient response. Due to the nonlinearity caused by the thyristors in the SVC part, the control design with optimizing system performance is still not available for the SVC-HAPF among the existing literatures [10]–[18].

To overcome the nonlinearity and improve the control per-

Manuscript received June 29, 2021; revised August 11, 2021; accepted August 30, 2021. Date of publication September 30, 2021; date of current version September 24, 2021. This work was supported in part by The Science and Technology Development Fund, Macau SAR (FDCT) (File no. 025/2017/A1, 0028/2020/A1, SKL-AMSV(UM)-2020-2022) and in part by the University of Macau (File no. MYRG2017-00090-AMSV, MYRG2018-00039-FST). (Corresponding author: Chi-Seng Lam.)

All authors are with the State Key Laboratory of Analog and Mixed-Signal VLSI, University of Macau, Macao 999078, China, also with the Institute of Microelectronics, University of Macau, Macao 999078, China, and also with the Department of Electrical and Computer Engineering, Faculty of Science and Technology, University of Macau, Macao 999078, China (e-mail: cslam@um.edu.mo; c.s.lam@ieee.org).

Digital Object Identifier 10.24295/CPSSSTPEA.2021.00021

formance for the SVC-HAPF, the harmonic state space (HSS) based H_∞ optimal control design [23], [24] is proposed for the SVC-HAPF in this paper. The HSS model is firstly proposed to overcome the nonlinearity issue of the SVC-HAPF. Based on the HSS modeling technique [19]–[21], the nonlinear SVC-HAPF can be modeled as a linear time-invariant (LTI) system with high accuracy. Then the linear matrix inequality (LMI) based controller design for the LTI systems [22] can be developed for the SVC-HAPF. Moreover, a γ -iteration algorithm which solves the LMI iteratively to find the optimal H_∞ controller is proposed. Comparing to the state-of-the-art MQPR controller, the proposed optimal H_∞ controller shows superior steady-state and transient performances.

The contribution of this paper is summarized as follows:

- 1) Propose a HSS model for the SVC-HAPF, in which the nonlinear SVC-HAPF can be modeled as a linear time-invariant (LTI) system, thus enables its optimal control design methods.
- 2) Propose a LMI based H_∞ optimal control design for the SVC-HAPF with low steady-state error, fast transient response, and fixed switching frequency.
- 3) Propose an overall control block diagram for the SVC-HAPF with H_∞ optimal controller.
- 4) Verify the performance and robustness of the proposed H_∞ optimal controller for the SVC-HAPF by simulation and experimental results in comparison with the state-of-the-art MQPR controller.

In the following, the circuit configuration and system model of the three-phase three-wire SVC-HAPF is provided in Section II. The harmonic state-space model of the SVC-HAPF is derived in Section III. Section IV provides its LMI based H_∞ optimal controller design. The overall control block diagram, simulation and experimental verifications are presented in Sections V and VI. Section VII draws the conclusion of this paper.

II. CIRCUIT CONFIGURATION AND SYSTEM MODELING OF THREE-PHASE SVC-HAPF

A three-phase SVC-HAPF is shown in Fig.1, where the subscript “x” denotes phase a, b, and c. v_{sx} , v_{pccx} , v_{cx} , v_{invx} , and v_{dc} are the system, point of common coupling (PCC), filter capacitor, inverter, and the DC-link voltages for each phase, respectively. i_{sx} , i_{Lx} , i_{Tx} and i_{cx} are the system, load, thyristor-controlled inductor, and compensating currents for each phase, respectively. L_F , C_F and L_T represent the coupling inductor, the filter capacitor, and the thyristor-controlled inductor. $T_x(t)$ is on-off state ($T_x(t) = 1$ or 0) of the phase x anti-parallel thyristors.

The time-domain system model of the SVC-HAPF can be derived as follows [10]:

$$L_F \frac{di_{cx}}{dt} = v_{pccx} - v_{cx} - v_{invx} \quad (1)$$

$$C_F \frac{dv_{cx}}{dt} = i_{cx} - i_{Tx} \quad (2)$$

Fig. 1. Circuit configuration of the three-phase SVC-HAPF.

$$L_T \frac{di_{Tx}}{dt} = T_x(t) v_{cx} \quad (3)$$

Based on the SVC-HAPF system model (1)–(3), the time-domain state-space model of SVC-HAPF is provided as [22]:

$$\frac{dx_x}{dt} = A_x(t)x_x + B_{ux}u_x + B_{wx}w_x \quad (4)$$

$$y_x = C_x x_x + D_x w_x \quad (5)$$

where $x_x = [i_{cx}, v_{cx}, i_{Tx}]^T$ are the vector of state variables, $u_x = v_{invx}$ is considered as the control input, $w_x = [i_{cx}^*, v_{pccx}]^T$ contains the reference compensating currents i_{cx}^* and PCC voltages, y_x is defined as the compensating current error signal $\varepsilon_x = i_{cx}^* - i_{cx}$, and the time-domain state-space matrices A_x , B_{ux} , B_{wx} , C , and D are given in the follows:

$$\begin{cases} A_x(t) = \begin{bmatrix} 0 & -1/L_F & 0 \\ 1/C_F & 0 & -1/C_F \\ 0 & T_x(t)/L_T & 0 \end{bmatrix} \\ B_{ux} = \begin{bmatrix} -1/L_F \\ 0 \\ 0 \end{bmatrix}, B_{wx} = \begin{bmatrix} -1/L_F & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \\ C_x = [1 \ 0 \ 0], D_x = [0 \ -1] \end{cases} \quad (6)$$

III. HARMONIC STATE SPACE MODELING OF SVC-HAPF

As the state-space matrix A_x contains the thyristor on-off state signal T_x , the system described by (4)–(6) is not a LTI system. To address this concern, the HSS model [19]–[21] of the SVC-HAPF is proposed in the following.

The HSS modeling technique is based on the complex Fourier series, where an arbitrary signal $x(t)$ with the fundamental angular frequency ω_1 is given by the series:

Fig. 2. Block diagram of SVC-HAPF HSS model.

$$x(t) = \sum_{h=-\infty}^{+\infty} X_h e^{jh\omega_1 t} \quad (7)$$

where X_h is the h -th order harmonic coefficient of the series, which can be calculated by:

$$X_h = \frac{1}{T} \int_0^{T} x(t) e^{-jh\omega_1 t} dt \quad (8)$$

where T is the fundamental period.

Based on the Fourier series and harmonic balance theory [25], the time-domain state-space SVC-HAPF model (4)–(6) can be transformed into the frequency-domain HSS model, as shown in Fig. 2, that can be derived as [19]–[21]:

$$\frac{dX_x}{dt} = A_x^H X_x + B_{ux}^H U_x + B_{wx}^H W_x \quad (9)$$

$$Y_x = C_x^H X_x + D_x^H W_x \quad (10)$$

where X_x , U_x , and W_x are the vectors of harmonic coefficients of the corresponding time-domain signals x_x , u_x , and w_x , which are given as:

$$\begin{cases} X_x = [\dots & X_{x-2} & X_{x-1} & X_{x0} & X_{x1} & X_{x2} & \dots]^T \\ U_x = [\dots & U_{x-2} & U_{x-1} & U_{x0} & U_{x1} & U_{x2} & \dots]^T \\ W_x = [\dots & W_{x-2} & W_{x-1} & W_{x0} & W_{x1} & W_{x2} & \dots]^T \end{cases} \quad (11)$$

And the HSS matrices are given as follows:

$$\begin{cases} N = \text{diag}[\dots, -j2\omega_1, -j\omega_1, 0, j\omega_1, j2\omega_1, \dots] \\ T_x^H = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \ddots \\ \cdots & T_{x0} & T_{x-1} & T_{x-2} & \cdots \\ \cdots & T_{x1} & T_{x0} & T_{x-1} & \cdots \\ \cdots & T_{x2} & T_{x1} & T_{x0} & \cdots \\ \ddots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \\ A_x^H = \begin{bmatrix} -N & -1/L_F \cdot I & O \\ -1/C_F \cdot I & -N & -1/C_F \cdot I \\ O & T_x^H/L_T & -N \end{bmatrix} \\ B_{ux}^H = \begin{bmatrix} -1/L_F \cdot I \\ O \\ O \end{bmatrix}, B_{wx}^H = \begin{bmatrix} -1/L_F \cdot I & O \\ O & O \\ O & O \end{bmatrix} \\ C_x^H = [I \ O \ O], D_x^H = [O \ -I] \end{cases} \quad (12)$$

where I is the identity matrix, O is the null matrix, T_x^H is the matrix contains the harmonic components of T_x , which are written as T_{xh} for all positive and negative integer h .

Generally, the HSS model is in infinite dimension. However, the high order harmonic components are usually small and can be simplify neglected. Thus, a finite dimension model can be obtained with a truncation of matrix size.

IV. H_∞ OPTIMAL CONTROLLER DESIGN FOR SVC-HAPF BASED ON LINEAR MATRIX INEQUALITY

From (9)–(12), the HSS model of the SVC-HAPF can be built as a LTI system, which enables the optimal control design methods for the SVC-HAPF. In this paper, the H_∞ optimal control design method for the SVC-HAPF is proposed.

The H_∞ controller design is to find an output feedback controller that stabilizes the SVC-HAPF system and meets the H_∞ performance constraint:

$$\|T_{wz}\|_\infty = \|D_x^H + C_x^H(sI - A_x^H)B_w^H\|_\infty < \gamma \quad (13)$$

where T_{wz} is the closed-loop transfer function from W_x to Y_x , γ is a positive constant for the system H_∞ performance evaluation. To obtain the H_∞ controller satisfies (13), the following LMI problem is formulated:

$$\begin{bmatrix} A_{CL}^T P + O A_{CL} & P B_{CL} & C_{CL}^T \\ B_{CL}^T P & -\gamma I & D_{CL}^T \\ C_{CL} & D_{CL} & -\gamma I \end{bmatrix} < 0 \quad (14)$$

$$P > 0 \quad (15)$$

where P is auxiliary variable, A_{CL} , B_{CL} , C_{CL} , and D_{CL} are the closed-loop HSS matrices which are given in the following:

$$A_{CL} = A_x^H + B_{ux}^H K_x^H C_x^H \quad B_{CL} = B_{wx}^H \quad (16)$$

$$C_{CL} = C_x^H + D_x^H K_x^H D_x^{HT} \quad D_{CL} = D_x^H \quad (17)$$

where K_x^H is the control gain matrix. The LMI problem described in (14)–(17) can be efficiently solved by using the well-developed software packages [26].

From (13), the H_∞ performance index is represented by a constant γ , the smaller γ yields better system performance. To find a minimum γ constrained by the system dynamics, a γ -iteration algorithm is proposed, which is shown in Fig. 3. First of all, the data of HSS matrices (12) is collected, and γ is initialized as a small step value $\Delta\gamma$. Then the LMI problem (14)–(17) is solved with the initial value of γ . If the LMI problem is infeasible (that means there is no K_x^H meets the conditions (14)–(17)), the γ is updated by:

$$\gamma = \gamma + \Delta\gamma \quad (18)$$

This algorithm iteratively solves the LMI problem corresponding to the updated γ value, until there is a feasible solution. If the LMI problem has a feasible solution, then return the control gain matrix K_x^H as the optimal control gain matrix K_{opt}^H .

Fig. 3. Flow chart of the proposed γ iteration algorithm.

Therefore, the H_∞ optimal controller is obtained as:

$$U_x = K_{optx}^H Y_x = \begin{bmatrix} \ddots & \vdots & \vdots & \vdots & \ddots \\ \cdots & K_{optx-11} & K_{optx01} & K_{optx11} & \cdots \\ \cdots & K_{optx-12} & K_{optx02} & K_{optx12} & \cdots \\ \cdots & K_{optx-13} & K_{optx03} & K_{optx13} & \cdots \\ \ddots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} Y_{x-1} \\ Y_{x0} \\ Y_{x1} \\ Y_{x2} \\ \vdots \end{bmatrix} \quad (19)$$

The H_∞ optimal controller (19) in the HSS cannot be implemented directly, which should be transformed back into the time domain signals, which can be derived as:

$$u_x(t) = \sum_{h=-n}^{+n} U_{xh} e^{jh\omega t} = \sum_{h=-n}^{+n} \sum_{m=-n}^{+n} K_{optxhm} Y_{xh} e^{jh\omega t} \quad (20)$$

However, the negative harmonic components of Fourier series are proposed for mathematical completeness, which is not convenient for the practical implementation of (20). To address this concern, the following equivalent expression of (20) are used:

$$y_{xh}(t) = Y_{xph} \cos(h\omega t) + Y_{xqh} \sin(h\omega t) = Y_{xh} e^{jh\omega t} + Y_{x-h} e^{-jh\omega t} \quad (21)$$

where $y_{xh}(t)$ is the h -order harmonic component of $y_x(t)$, Y_{xph} and Y_{xqh} are active and reactive amplitudes components of $y_{xh}(t)$, which can be extracted by using band-pass filters (BPFs) [27]. Using the second order BPFs for each harmonic order with the zero-damping ratio, the s -domain transfer function $C_{H_\infty}(s)$ of the H_∞ optimal controller can be derived as follows:

$$\begin{aligned} u_x(s) &= v_{inx}^*(s) = C_{H_\infty}(s) y_x(s) \\ &= \sum_{m=0}^n K_{optx0m} + \sum_{h=1}^n \frac{2 \sum_{m=0}^n K_{optxhm} s}{s + (h\omega_l)^2} (i_{cx}^*(s) - i_{cx}(s)) \end{aligned} \quad (22)$$

Fig. 4. Overall control block diagram of the SVC-HAPF with the H_∞ optimal control.

V. OVERALL CONTROL BLOCK DIAGRAM

According to the previous works [13]–[16], the SVC part and the active inverter part should be controlled coordinately. The overall control strategy of the SVC-HAPF is summarized in Fig. 4.

For the SVC part, the SVC utilizes the instantaneous reactive power theory [28] to calculate the firing angles α_x of the thyristors and compensate the load reactive power. The expression of the required impedance of the SVC part X_{SVC} can be expressed in terms of instantaneous values as:

$$X_{SVC}(\alpha_x) = \frac{\pi X_{L_F} X_{C_F}}{X_{C_F} (2\pi - 2\alpha_x + \sin 2\alpha_x) - \pi X_{L_T}} + X_{L_F} = \frac{\|v_{pcc}\|^2}{\sqrt{3} \cdot \bar{q}_{Lx}} \quad (23)$$

where X_{L_F} , X_{C_F} , and X_{L_T} are impedances of coupling inductor, the filter capacitor, and the thyristor-controlled inductor, $\|v_{pcc}\|$ and $\bar{q}_{Lx}$ are the norm of the three-phase instantaneous grid voltage and average load reactive power, which are obtained with the help of the low-pass filter (LPF) and the following equations:

$$\|v_{pcc}\|^2 = v_{pccf}^2 + v_{pccbf}^2 + v_{pcccf}^2 \quad (24)$$

$$\begin{bmatrix} q_{La} \\ q_{Lb} \\ q_{Lc} \end{bmatrix} = \begin{bmatrix} v_{pccbf} \cdot i_{Lc} - v_{pcccf} \cdot i_{Lb} \\ v_{pcccf} \cdot i_{La} - v_{pccbf} \cdot i_{Lc} \\ v_{pccbf} \cdot i_{Lb} - v_{pccbf} \cdot i_{La} \end{bmatrix} \quad (25)$$

In (24) and (25), v_{pccf} , i_{Lx} and q_{Lx} are the instantaneous grid voltage filtered by bandpass filters (BPFs), load current and instantaneous load reactive power, respectively. A look-up table (LUT) of the corresponding firing angle α_x and the X_{SVC} in (23) is installed inside the controller to directly obtain the α_x .

The active inverter part is to compensate for the harmonic power and reactive power left by the SVC part. Therefore, the reference current value i_{cx}^* can be calculated through the well-known instantaneous p - q theory [28] as:

$$\begin{bmatrix} i_{ca}^* \\ i_{cb}^* \\ i_{cc}^* \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \frac{1}{v_\alpha^2 + v_\beta^2} \begin{bmatrix} 1 & 0 \\ -1/2 & \sqrt{3}/2 \\ -1/2 & -\sqrt{3}/2 \end{bmatrix} \cdot \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \cdot \begin{bmatrix} \tilde{p}_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} \quad (26)$$

where $\tilde{p}_{\alpha\beta}$ is three-phase instantaneous load harmonic active power that is obtained by passing through the instantaneous load active power $p_{\alpha\beta}$ to the high pass filter (HPF) and $q_{\alpha\beta}$ is the three-phase instantaneous load reactive power which includes AC component $\tilde{q}_{\alpha\beta}$ and DC component $\bar{q}_{\alpha\beta}$. The $p_{\alpha\beta}$ and $q_{\alpha\beta}$ can be expressed as:

$$\begin{bmatrix} p_{\alpha\beta} \\ q_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix} \cdot \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (27)$$

In (26) and (27), the voltage (v_α and v_β) and currents (i_α and i_β) in the α - β coordinate are transformed from a - b - c coordinate by using Clarke transformation. After obtaining the reference compensating current and its actual compensating error, the compensating error ($\varepsilon_x = i_{cx}^* - i_{cx}$) can be calculated, then this error will input into the H_∞ optimal controller in (22).

After performing all the calculation, the H_∞ optimal controller will generate a voltage reference signal v_{invx}^* to compare with the fixed frequency triangular carrier wave for generating inverter trigger signals. The switching frequency of the switching pulse is 10 kHz which is same as the triangular carrier wave.

VI. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation Results

In this section, simulation results of applying different controllers for the SVC-HAPF are provided by using MATLAB/Simulink. The system, the SVC-HAPF and the load parameters are provided in Table I, in which the DC-link voltage is modeled by a DC voltage source to simplify the verification of the validity and effectiveness of the H_∞ optimal controller for the SVC-HAPF. The simulation results are presented in Fig. 5 and summarized in Table II.

Figs. 5(a), 5(c) and Table II show the SVC-HAPF compensating performance with the MQPR controller under inductive and capacitive loads compensation. The steady-state compensation performance is satisfactory for both load conditions. In steady-state, the worst-case THD_{isx} can be reduced to 2.03% and 2.13% for inductive and capacitive loads, respectively. However, it takes a long transient time from the SVC-HAPF starting operation to the steady-state (65 ms and 57 ms for inductive and capacitive loads), which is because the gains design is not optimized and the proportional gain is also the same for the fundamental and all considered harmonics order, but our proposed H_∞ optimal controller design provides the

TABLE I
PARAMETERS OF SYSTEM, SVC-HAPF, LOADS FOR SIMULATION AND EXPERIMENT

Parameters		Value
Grid parameters	v_{sd}, f_s	110 V, 50 Hz
Inverter parameters	v_{dc}, f_{sw}	60 V, 10 kHz
SVC parameters	L_F, L_T, C_F	5 mH, 30 mH, 160 μ F
Sampling time	T_s	100 μ s
Non-linear load	R_{NL}, L_{NL}	50 Ω , 30 mH
Inductive load	R_L, L_L	15 Ω , 70 mH
Capacitive load	R_C, C_L	25 Ω , 140 μ F

TABLE II
SIMULATION RESULTS BEFORE AND AFTER SVC-HAPF COMPENSATION

		P_{sx} (W)	Q_{sx} (var)	PF	T_{st} (ms)	Worst THD _{isx} (%)
Inductive Loads	Before Comp.	650	450	0.78	/	18.6
	MQPR controller	660	5	0.99	65	2.03
	H_∞ optimal controller	660	3	0.99	12	1.83
Capacitive Loads	Before Comp.	750	-250	0.93	/	19.5
	MQPR controller	760	-4	0.99	57	2.13
	H_∞ optimal controller	760	-2	0.99	13	1.91

optimized gains for the fundamental and each of the harmonics order.

In Figs. 5(b), 5(d), and Table II, the SVC-HAPF compensating performance with the proposed H_∞ optimal controller is demonstrated. From Figs. 5(b) and 5(d), the steady-state compensation performance is better than the MQPR controller under both load conditions, which reduces the worst-case THD_{isx} to 1.83% and 1.91% for inductive and capacitive loads. Moreover, the transient response is much faster than the MQPR controller, which are 12 ms and 13 ms for inductive load and capacitive load, respectively.

In conclusion, with the help of HSS modeling and optimization-based control design, the proposed H_∞ optimal controller provides superior steady-state and transient performance simultaneously compared with the state-of-the-art MQPR controller [18].

B. Experimental Results

To verify the simulation results, a three-phase three-wire 110 V-5 kVA SVC-HAPF experimental prototype was built in the laboratory. Table I shows the experimental setting of the system and controllers' parameters. The digital control system of the SVC-HAPF consists of two digital signal processors (DSPs) TMS320F2812s for active inverter part and SVC part

Fig. 5. Simulation results of SVC-HAPF compensating performance by using different controllers under inductive and capacitive load conditions: (a) MQPR controller under inductive load; (b) proposed H_∞ optimal controller under inductive load; (c) MQPR controller under capacitive load; (d) proposed H_∞ optimal controller under capacitive load.

control. The switching devices employed in SVC-HAPF are Mitsubishi IGBT intelligent power modules PM300DSA60 for the active inverter part and SanRex PK110FG160 thyristors for the SVC part, respectively. The experimental results are presented in Fig. 6 and summarized in Table III.

Figs. 6(a), 6(c) and Table III show the SVC-HAPF compensating performance with the MQPR controller under inductive and capacitive loads compensation. The steady-state compensation performance is satisfactory for both load conditions. In steady-state, the worst-case THD_{isx} can be reduced to 5.9% and 5.8% for inductive and capacitive loads, respectively. However, it takes a long transient time from the SVC-HAPF starting operation to the steady-state (85 ms and 79 ms for inductive and capacitive loads), which is because the gains design is not optimized, and the proportional gain is also the same for the fundamental and all considered harmonics order.

In Figs. 6(b), 6(d), and Table III, the SVC-HAPF compensating performance with the proposed H_∞ optimal controller is demonstrated. From Figs. 6(b) and 6(d), the steady-state compensation performance is better than the MQPR controller for inductive and capacitive loads, which reduces the worst-case THD_{isx} to 5.4% under both load conditions. Moreover, the transient response is much faster than the MQPR controller,

TABLE III
EXPERIMENTAL RESULTS BEFORE AND AFTER SVC-HAPF COMPENSATION

		P_{sx} (W)	Q_{sx} (var)	PF	T_{st} (ms)	Worst THD _{isx} (%)
Inductive Loads	Before Comp.	610	450	0.78	/	19.4
	MQPR controller	660	10	0.99	85	5.9
	H_∞ optimal controller	660	10	0.99	15	5.4
Capacitive Loads	Before Comp.	740	-250	0.93	/	19.7
	MQPR controller	770	-10	0.99	79	5.8
	H_∞ optimal controller	770	-10	0.99	19	5.4

which are 15 ms and 19 ms for inductive load and capacitive load, respectively.

Therefore, both simulation and experimental results verify that the proposed H_∞ optimal controller based on HSS and LMI provides superior steady-state and transient performance simultaneously compared to the state-of-the-art MQPR controller

Fig. 6. Experimental results of SVC-HAPF compensating performance by using different controller under inductive and capacitive load conditions: (a) MQPR controller under inductive load; (b) proposed H_∞ optimal controller under inductive load; (c) MQPR controller under capacitive load; (d) proposed H_∞ optimal controller under capacitive load.

proposed in [18].

In conclusion, as the gains design of the MQPR controller is not optimized and the proportional gain is designed to be the same for both fundamental and all considered harmonics order, the transient response of the MQPR controller is slow. On the contrary, our proposed H_∞ optimal controller design provides the optimized gains for both fundamental and each of the harmonics order, which results in superior steady-state and transient performances.

VII. CONCLUSION AND FUTURE WORKS

To address the nonlinearity and improve the performance of the SVC-HAPF, a H_∞ optimal control design based on HSS modeling for the SVC-HAPF is proposed in this paper. First, the HSS model in LTI form of the SVC-HAPF is proposed to address the system nonlinearity issue. Then, the H_∞ optimal control design based on the LMI is proposed, which can improve the system performance. Finally, simulation and experimental results are provided to verify the effectiveness and performance of the H_∞ optimal controller for the SVC-HAPF in comparison with the recent developed state-of-the-art MQPR controller, which shows superior steady-state and transient control performances.

In addition, with the help of the proposed HSS model, the

linear control designs are unlocked for the nonlinear SVC-HAPF. Therefore, more recent works for linear optimal control proposed by the control system society could be developed on the SVC-HAPF, which can potentially further improve the system performance and reduce the algorithm complexity.

REFERENCES

- [1] W. E. Reid, "Power quality issues-standards and guidelines," in *IEEE Transactions on Industry Applications*, vol. 32, no. 3, pp. 625–632, May/Jun. 1996.
- [2] B. K. Bose, "Power electronics, smart grid, and renewable energy systems," in *Proceedings of the IEEE*, vol. 105, no. 11, pp. 2011–2018, Nov. 2017.
- [3] C. -J. Wu, J. -C. Chiang, S. -S. Yen, C. -J. Liao, J. -S. Yang, and T. -Y. Guo, "Investigation and mitigation of harmonic amplification problems caused by single-tuned filters," in *IEEE Transactions on Power Delivery*, vol. 13, no. 3, pp. 800–806, Jul. 1998.
- [4] H. Akagi, "Active harmonic filters," in *Proceedings of the IEEE*, vol. 93, no. 12, pp. 2128–2141, Dec. 2005.
- [5] C. Lascu, L. Asiminoaei, I. Boldea, and F. Blaabjerg, "Frequency response analysis of current controllers for selective harmonic compensation in active power filters," in *IEEE Transactions on Industrial Electronics*, vol. 56, no. 2, pp. 337–347, Feb. 2009.
- [6] Q. -N. Trinh and H. -H. Lee, "An advanced current control strategy for three-phase shunt active power filters," in *IEEE Transactions on Industrial Electronics*, vol. 60, no. 12, pp. 5400–5410, Dec. 2013.
- [7] S. Srianthumrong and H. Akagi, "A medium-voltage transformerless AC/DC power conversion system consisting of a diode rectifier and a shunt hybrid filter," in *IEEE Transactions on Industry Applications*, vol. 39, no.

- 3, pp. 874–882, May/Jun. 2003.
- [8] B. Singh, V. Verma, A. Chandra, and K. Al-Haddad, “Hybrid filters for power quality improvement,” in *IEEE Proceedings-Generation, Transmission and Distribution*, vol. 152, no. 3, pp. 365–378, May 2005.
- [9] A. Luo, C. Tang, Z. K. Shuai, W. Zhao, F. Rong, and K. Zhou, “A novel three-phase hybrid active power filter with a series resonance circuit tuned at the fundamental frequency,” in *IEEE Transactions on Industrial Electronics*, vol. 56, no. 7, pp. 2431–2440, Jul. 2009.
- [10] S. Rahmani, A. Hamadi, K. Al-Haddad, and L. A. Dessaint, “A combination of shunt hybrid power filter and thyristor-controlled reactor for power quality,” in *IEEE Transactions on Industrial Electronics*, vol. 61, no. 5, pp. 2152–2164, May 2014.
- [11] L. Wang, C. -S. Lam, and M. -C. Wong, “Hybrid structure of static var compensator and hybrid active power filter (SVC/HAPF) for medium-voltage heavy loads compensation,” in *IEEE Transactions on Industrial Electronics*, vol. 65, no. 6, pp. 4432–4442, Jun. 2018.
- [12] L. Wang, C. -S. Lam, and M. -C. Wong, “A hybrid-STATCOM with wide compensation range and low DC-link voltage,” in *IEEE Transactions on Industrial Electronics*, vol. 63, no. 6, pp. 3333–3343, Jun. 2016.
- [13] L. Wang, C. -S. Lam, and M. -C. Wong, “Unbalanced control strategy for a thyristor-controlled LC-coupling hybrid active power filter in three-phase three-wire systems,” in *IEEE Transactions on Power Electronics*, vol. 32, no. 2, pp. 1056–1069, Feb. 2016.
- [14] C. -S. Lam, L. Wang, S. -I. Ho, and M. -C. Wong, “Adaptive thyristor-controlled LC-hybrid active power filter for reactive power and current harmonics compensation with switching loss reduction,” in *IEEE Transactions on Power Electronics*, vol. 32, no. 10, pp. 7577–7590, Oct. 2016.
- [15] L. Wang, C. -S. Lam, and M. -C. Wong, “Selective compensation of distortion, unbalanced and reactive power of a thyristor-controlled LC-coupling hybrid active power filter (TCLC-HAPF),” in *IEEE Transactions on Power Electronics*, vol. 32, no. 12, pp. 9065–9077, Dec. 2017.
- [16] L. Wang, C. -S. Lam, and M. -C. Wong, “Analysis, control, and design of a hybrid grid-connected inverter for renewable energy generation with power quality conditioning,” in *IEEE Transactions on Power Electronics*, vol. 33, no. 8, pp. 6755–6768, Aug. 2017.
- [17] L. Wang, C. -S. Lam, and M. -C. Wong, “Minimizing inverter capacity design and comparative performance evaluation of SVC-coupling hybrid active power filters,” in *IEEE Transactions on Power Electronics*, vol. 34, no. 2, pp. 1227–1242, Feb. 2018.
- [18] W. -K. Sou, C. -W. Chao, C. Gong, C. -S. Lam, and C. -K. Wong, “Analysis, design and implementation of multi-quasi-proportional-resonant controller for thyristor-controlled LC-coupling hybrid active power filter (TCLC-HAPF),” in *IEEE Transactions on Industrial Electronics*, Jan. 2021, Early Access.
- [19] G. N. Love and A. R. Wood, “Harmonic state space model of power electronics,” in *Proceedings of 2008 13th International Conference on Harmonics and Quality of Power*, Wollongong, NSW, Australia, 2008, pp. 1–6.
- [20] J. R. C. Orillaza and A. R. Wood, “Harmonic state-space model of a controlled TCR,” in *IEEE Transactions on Power Delivery*, vol. 28, no. 1, pp. 197–205, Jan. 2013.
- [21] J. Kwon, X. Wang, F. Blaabjerg, C. L. Bak, V. -S. Sulearea, and C. Busca, “Harmonic interaction analysis in a grid-connected converter using harmonic state-space (HSS) modeling,” in *IEEE Transactions on Power Electronics*, vol. 32, no. 9, pp. 6823–6835, Sept. 2017.
- [22] S. Skogestad and I. Postlethwaite, in *Multivariable Feedback Control: Analysis and Design*, vol. 2, John Wiley & Sons, New York, USA, 2007.
- [23] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, in *Linear Matrix Inequalities in System and Control Theory*. SIAM, USA, 1994.
- [24] P. Gahinet and P. Apkarian, “A linear matrix inequality approach to H_∞ control,” in *International Journal of Robust and Nonlinear Control*, vol. 4, no. 4, pp. 421–448, 1994.
- [25] N. M. Wereley and S. R. Hall, “Linear time periodic systems: transfer function, poles, transmission zeroes and directional properties,” in *Proceedings of 1991 American Control Conference*, Boston, MA, USA, 1991, pp. 1179–1184.
- [26] D. -W. Gu, P. H. Petkov, and M. M. Konstantinov, in *Robust Control Design With MATLAB*, Springer, New York, NY, USA, 2005.
- [27] J. C. Willems, S. Hara, Y. Ohta, and H. Fujioka, in *Perspectives in Mathematical System Theory, Control, and Signal Processing*, vol. 398, Springer, Berlin, Heidelberg, Germany, 2010.
- [28] H. Akagi, E. H. Watanabe, and M. Aredes, in *Instantaneous power theory and applications to power conditioning*. John Wiley & Sons, Hoboken, New Jersey, USA, 2017.

Cheng Gong received the M.Sc. degree in electrical engineering from the Guangxi University, Nanning, China, in 2017. He is currently pursuing the Ph.D. degree in electrical and computer engineering in University of Macau (UM), Macau, China.

His research interests include power quality compensator, converter controller design, and power electronic based power systems.

Wai-Kit Sou received the B.Sc. degree in electrical and computer engineering from the University of Macau (UM), Macao, China, in 2019. He is currently working toward the Ph.D. degree in electrical and electronics engineering, UM.

In 2017 and 2018, he was the Vice President and President of UM Student Branch under IEEE Macau Section and he is also a member in the board of supervisors of PES/PELS Joint Chapter in IEEE

Macau Section since 2018. From 2020, he is the secretary of IES Chapter under IEEE Macau Section and the member of Power Quality Subcommittee of the IEEE IES PETC.

His current research interests include power electronics, energy saving and power quality compensation.

Chi-Seng Lam received the Ph.D. degree in electrical and electronics engineering from the University of Macau (UM), Macau, China, in 2012. He completed the Clare Hall Study Program at the University of Cambridge, Cambridge, U.K., in 2019.

From 2006 to 2009, he was an Electrical and Mechanical Engineer with UM. From 2009 to 2012, he simultaneously worked as a Laboratory Technician at UM, and pursued his Ph.D. degree in part-time. In 2013, he was a Postdoctoral Fellow with The Hong Kong Polytechnic University, Hong Kong, China. He is currently an Associate Professor with the State Key Laboratory of Analog and Mixed-Signal VLSI and the Institute of Microelectronics, UM, and also with the Department of Electrical and Computer Engineering, Faculty of Science and Technology, UM. He has co-authored or co-edited four books and more than 130 technical journals and conference papers. He holds four U.S. and three Chinese patents. His research interests include power quality compensators, renewable energy generation, integrated power electronics controllers, power management integrated circuits, and wireless power transfer.

Dr. Lam was the recipient or co-recipient of the IEEE PES Chapter Outstanding Engineer Award in 2016, the Best Track Paper Award of APPEEC 2019, the Best Paper Award of ICTA 2019, the Merit Paper Award of the 3rd RIUPEEEEC Conference in 2005, the Macao Science and Technology Invention Award (Second Class and Third Class) and the Research and Development Award for Postgraduates (Ph.D.) in 2018, 2014, and 2012, respectively. He was the Vice-Chair of the IEEE Macau Section from 2016 to 2020 and the Chair of the IEEE Macau CAS Chapter from 2017 to 2018. He is currently the Chair of the IEEE Macau IES Chapter, the Vice-Chair of the IEEE Macau PES/PELS Joint Chapter and the Power Quality Subcommittee Chair of the IEEE IES Technical Committee on Power Electronics. He currently serves as an Associate Editor for the *IEEE Transactions on Industrial Electronics*, the *IEEE Open Journal of the Industrial Electronics Society* and the *IEEE Access*.