GNSS DATA PROCESSING THROUGH THE WAVELET TRANSFORM

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ABSTRACT

The present work shows an approach to signal processing through the wavelet transform received by positioning equipment, which are transmitted by GNSS constellation satellites. In these techniques of GNSS geopositioning, there are no applications of the wavelet transform, due to the absence of experts in the field or in its absence by complication, for this reason we undertake the task of carrying out this processing with this transform on gps satellite reception data. Situation that allows to establish an approximation to the calculation of speed of displacement in a particular zone. Having as main input a series of data very robust in time over 24 months.

KEYWORDS: Wavelets, Geodynamics, Geodesy, Time Series, Estimation

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1. INTRODUCTION

A branch of study of geosciences, research in geodynamics is the measurement, quantification and monitoring of the movements of the Earth's cortical zone in time. For the accomplishment of this monitoring it is necessary a series of measurements of position obtained in continuous stations GPS that interact and establish geodesic coordinates of the surface on which they are with respect to a terrestrial fixed system.

With the analysis of the received information, it is possible to evidence the movements of the crust in magnitude and direction in a determined interval of time, whose results of the analysis allow to carry out specific studies like:

- Stress analysis (Tension-Compression) that could suffer the electrical, hydraulic, transport, and telecommunication service networks.
- Determination of the direction and magnitude of movement in processes of expansion of volcanoes
- Land stability analysis for the construction of residential and industrial settlements

The execution of these studies leads to better development and planning of urban and rural areas.

Currently, for the Americas region, a model has been proposed to analyze and relate the captured GNSS information to the continental drift. The continental model called VEMOS2009 (Velocity Model SIRGAS-2009) is the result of the application of least squares and statistical methods.

The waveform method has been developed to analyze the variation of signals and data over time. Its main application in geosciences is in the field of geophysics, specifically in the area of seismic, whose main function is to

serve as a tool for the analysis and interpretation of seismic signals received by a series of geophones. However, this tool has been used in other fields of science in which electrical signals, images, economic data, and audio-video signals are analyzed, treated, and interpreted.

2. GNSS GLOBAL POSITIONING SYSTEM

The positioning on the earth's surface through GNSS is mainly a navigation system. The foundation of this navigation is the measurement of pseudoranges called between the user and four satellites. From the known coordinates of the satellites and with a suitable reference frame, the user's antenna coordinates can be determined. From a geometric point of view three gamma measurements are sufficient. A fourth observation is necessary because the receiver's clock is not synchronized with the satellite's clock. This synchronization error is the reason for the expression pseudodistances

Figure 1: Basic principles of positioning with GNSS

The three-dimensional position of the receiver is the point where pseudo distances from a group of satellites intersect.

The functional principle of GNSS is based on measuring the time taken for the signal transmitted by the satellites to reach the user's receiver, and this time interval is multiplied by the speed of light to obtain the receiving satellite distance.

$$\rho_r^s = \rho_r^s + C (\delta t^s - \delta t_r) + I_r^s + T_r^s + M_r^s + \epsilon_p \quad (1)$$

$$\phi_r^s = \rho_r^s + C (\delta t^s - \delta t_r) + \lambda N_r^s - I_r^s + T_r^s + M_r^s + \epsilon_p \quad (2)$$

$$\dot{\phi}_r^s = \dot{\rho}_r^s + C (\delta \dot{t}^s - \delta \dot{t}_r) - \dot{I}_r^s + \dot{T}_r^s + \dot{M}_r^s + \dot{\epsilon}_p \quad (3)$$

Where:

ρ : Pseudo distances between satellite and receiver

ϕ : Phase

C : Speed of light

I : Ionospheric component

T : Component per troposphere

M : Tidal component

N : Ambiguity

ε : Associated error

$\delta t^s - \delta t_r$: Clock difference

3. SIRGAS - GEOCENTRIC REFERENCE SYSTEM FOR THE AMERICAS

In a conventional reference system, the models, numerical constants and algorithms are explicitly described. A frame of reference has the materialization of a reference system through observations. This is established from a set of identification of reliability points in the sky or on the earth's surface, such as the fundamental stations). It is described by a catalog of the positions and precise movements at a given time.

The geocentric reference system for the Americas known as SIRGAS. It has correspondence with the International Terrestrial Reference System and its materialization is the regional densification of the international terrestrial reference framework known as ITRF. Together with the geometric reference system, SIRGAS deals with the realization and definition of a vertical reference system established for ellipsoidal heights as a geometric component and geopotential numbers C , referred to a potential value of gravity reference W_0 as a physical element.

This system is the basis for the development of projects committed to the generation and use of georeferenced information in the region, both nationally and internationally. In addition to providing the reference coordinates for practical applications such as digital geographic information management, spatial data infrastructures, engineering projects, SIRGAS is the basis for a wide range of applications, both scientific and for the observation of cortical land deformation, sea level variation, atmospheric studies and vertical movements.

3.1. SIRGAS AND ITS PARTICULARITIES

SIRGAS establishes coordinates that are associated with a given reference period and their variation in time is taken into account for the calculation of individual velocities of each of the continuous positioning stations or by implementing a velocity model with continental coverage. The densities of the SIRGAS stations associated to different periods and related to dissimilar ITRF solutions, and materialize it from the reference coordinate system and their respective positions, referred to the same time and the same ITRF reference frame, established for a millimetric order.

In addition to the geocentric reference system, SIRGAS establishes the definition and realization of the vertical reference system established in ellipsoidal heights as a geometric component and in geopotential numbers referring to a conventional gravity power value as a physical component.

This system is the platform for the development of projects related to the generation and use of georeferenced information in the region, both locally and regionally.

Also to provide the reference coordinates for practical studies as well as engineering projects, spatial data infrastructures, digital management of geographic information, SIRGAS becomes the platform for a wide variety of scientific applications such as vertical movements, observation of crustal deformations, sea level variation, and atmospheric studies.

3.2. ABOUT THE SIRGAS COORDINATES

The first realization of SIRGAS (SIRGAS95) corresponds to the ITRF94, period 1995.4 and is given by a high precision GPS network with 58 stations 36 distributed over South America. This network was reoccupied in the year 2000, extending to the countries of the Caribbean and Central and North America. For this reason, the original meaning of the acronym SIRGAS (Geocentric Reference System for South America) changed to Geocentric Reference System for the Americas. The second realization of SIRGAS (SIRGAS2000) includes 184 stations and corresponds to the ITRF2000, epoch 2000.4 the accuracy of the coordinates of these two realizations is between $\pm 3 \dots \pm 6$ mm.

The third realization of SIRGAS is the SIRGAS network of Continuous Operation (SIRGAS-CON). It is currently composed of more than 200 GNSS stations in permanent operation, of which 48 belong to the global network of the IGS (International GNSS Service). SIRGAS-CON is calculated weekly by the SIRGAS processing and combination centers. The coordinates and final speeds of the SIRGAS-CON stations are made available to users by the IGS-RNAAC-SIR (IGS Regional Network Associate Analysis Centre for SIRGAS), which operates at the Deutsches Geodätisches Forschungsinstitut, Munich, Germany. The fixed weekly solutions for the SIRGAS-CON network refer to the observation period and the current ITRF, currently the IGS05, an implementation of the ITRF2005. The coordinates of the multiannual solutions refer to a specific time, for example, the solution SIR09P01 corresponds to the IGS05, time 2005.0.

3.3. PRODUCTS OFFERED BY SIRGAS-CON

After combining the continental SIRGAS-CON-C network with the SIRGAS-CON-D densification sub-networks, the following products are generated:

- Semi-free (loosely constrained) weekly coordinates in SINEX format for further calculations, e.g. the global polyhedron of the ISM, multi-year solutions, etc.
- Fixed weekly coordinates adjusted to the IGS05 network (realization of the ITRF2005) for practical applications that require reference coordinates in Latin America.
- Multi-annual solutions (coordinates + velocities) for practical and scientific applications that require the variation of reference coordinates over time.

The final SIRGAS-CON products are provided by the IGS Regional Network Associate Analysis Centre for SIRGAS (IGS-RNAAC-SIR), which currently operates at the Deutsches Geodätisches Forschungsinstitut.

4. FILTERS

A mathematical filter is a linear operator invariant in time. It acts on the input vectors X. The output vector Y is the

convolution operation of X with a fixed vector h . This vector h contains the filter coefficients $h(0), h(1), h(2) \dots$ these filters are digital, so the coefficients $h(n)$ arrive at the instant $t = nT$ at discrete times. The sampling period T is supposed to be one direction. The inputs $X(n)$ and outputs $Y(n)$ come at all times $t = 0, 1, 2, \dots$

The convolution operation of $h * x$ in the time domain:

$$Y(n) = \sum_k h(k) * x(n - k) \quad (4)$$

The convolution operation with the h -vector becomes the multiplication by a function of H . The action of a filter on time and frequency is the concept on which the signal processing is established.

Thus you have a set of filters, which often has two filters, one filter called low pass and one with high pass characteristics. This is done by separating the input signal into frequency bands.

The signal components can be compressed much more efficiently than the original input signal. Allowing them to be sent or stored. In these terms is describing the coding band and its applications. At any time, the signals can be recombined by the synthesis set. For this it is not necessary to preserve the complete results of the analysis filters. They are usually decreased. And only the components, including the low-pass and high-pass components of the filter outputs are kept.

5. WAVELET THEORY

Wavelets are small waves that allow to treat a series of data, to obtain information of interest in another domain and vector space, or also allow to modify the content of the original information. The term wavelet was assigned in the geophysical literature by Morlet (1982), as it was implemented as a technique for processing seismic signals. However, the evolution of wavelets has been carried out on several disciplines such as economics, medicine, engineering, statistics, design, geodesy, and geodynamics.

The wavelets are functions that allow making transformations and analysis of a series of data in the space-time to the spectral domain (frequency-energy) or in the same temporal, likewise, they are used as tools of treatment of signals, filtering, prediction, estimation and compression of data. Although it is considered that wavelets apply the same Fourier principle where we seek to represent an $X(t)$ function in terms of space-time, as an $X(w)$ function whose characteristics are of the spectral domain, ie are given by the amplitude and wavelength through operations such as convolution, the characteristics of wavelets offer certain advantages for the treatment of information, below we will list some of them:

- The Wavelets must have oscillations, which allow to identify changes at different scales. They must have a compact support, that is to say, the amount of information or input energy must be the same as that transformed without modifying any of its components.
- The wavelets allow the transformation of non-stationary information.
- Wavelets allow the transformation of non-stationary information.
- Data processing at different scales is much more efficient than the fast Fourier transform which must be applied each time the width of the transformation window is adjusted.

- The wavelets allow the reconstruction of the transformed information without modifying its content.
- It is possible to obtain reduced versions by levels of the transformed information.
- With wavelets, changes in the information can be identified, giving the possibility of locating them in the time domain.
- Wavelets allow information smoothing processes to be carried out at various scales, without causing loss of the initial information.
- The calculation algorithm is equal or more efficient than the Fast Fourier Transform.

The smoothed version will be obtained by the technique known in Fourier theory as filtering, wherefrom the convolution operation, it is possible to obtain a noise-free or smoothed function. Following the same procedure, it should be possible to obtain noise or difference function. This has been approached from the technique known as filtering, where it has been possible to obtain two types of filters: low-pass (Smoothing), and high-pass (Differences). In the development of this technique, it was determined roughly, that the high pass filter is a modification of the coefficients of the low pass filter (Smoothing), ie by varying the sign or value of some of the coefficients of the low pass filter, it was possible to obtain the function of the differences.

5.1. RELATIONSHIP BETWEEN FILTERS AND WAVELETS

Let $\theta(x)$ be a waveform function such that it serves as the basis for generating the high and low pass filter from the modification of the coefficients that define it.

$$\phi(x) = H\theta(x) \quad \phi(x): (5)$$

$$\varphi(x) = G\theta(x) \quad \varphi(x): (6)$$

Where H is low pass filter and G is high pass filter.

5.2. DIRECT WAVELET TRANSFORMATION (WAVELET ANALYSIS)

It allows obtaining additional information from the decomposition of the signal in terms of coefficients representing smoothed versions and differences from the original signal, at various scales. The direct wavelet transformation allows us to carry out processes of multi-temporal information analysis, noise detection, and filtering.

Figure 2: Wavelet analysis scheme.

6. METHODOLOGICAL PROCEDURE

For the application of wavelets as a technique for the analysis of GNSS positioning time series, it is necessary that the information is regular, in order to preserve the time scale and in this way, the obtaining of coefficients at various levels are not affected by significant differences containing too wide time intervals that over dimension the differences.

To obtain the wavelet coefficients it is necessary that the series is expressed as a dyadic interval that is that its length is equivalent to a power of two, that is:

$$2n = \text{Length of Dyadic Interval} \quad (7)$$

Once the dyadic interval has been selected, the direct wavelet transformation (NDWT) is performed, using a wavelet that meets the conditions of orthonormality, symmetry, must have a large number of fading moments and the most important condition that meets the principle of entropy.

Figure 3: Wavelet Coefficients

From the coefficients obtained, we proceed to determine the wavelet spectrum which allows us to accentuate the most representative coefficients.

7. RESULTS ANALYSIS

Once the wavelet coefficients are obtained, the displacements in the dyadic interval of the received point are determined, rescaling the differences obtained. Employing this procedure some displacement velocities were obtained which were compared with a GNSS velocity model (VEMOS09) developed for the region, as shown in figure 2

Table 1: Displacements BOGA Station Week 1048-1303

Determination method	$V_x \text{ ms}^{-1}$	$V_y \text{ ms}^{-1}$	$z \text{ ms}^{-1}$	$V_N \text{ ms}^{-1}$	$V_E \text{ ms}^{-1}$	$V_h \text{ ms}^{-1}$
VEMOS 09	-0.0149	0.0409	0.0174			

Wavelet	- 0.019294	0.04430	0.018457	0.048050 3	-.007049	-0.01773
Δ	<u>0.004394</u>	<u>0.0034</u>	<u>0.001057</u>			

From the amplitude of the wavelet spectrum, it was possible to detect temporary occurrences of the strongest station displacements and thus carry out an association with data of another nature and among data from nearby stations (approximately 1 km).

GPS WEEK Corresponding date	
1064	31/05/00
1072	26/07/00
1112	02/05/01
1120	27/06/01
1126	08/08/01
1145	19/12/01
1161	10/04/02
1169	05/06/02
1173	03/07/02
1180	21/08/02
1189	23/10/02
1206	19/02/03
1219	21/05/03
1226	09/07/03
1250	23/12/03
1269	05/05/04

GPS WEEK Corresponding date	
1174	2002-07-10
1179	2002-08-14
1200	2003-01-08
1205	2003-02-12
1218	2003-05-11
1221	2003-06-04
1226	2003-07-09
1258	2004-02-15
1275	2004-06-16
1302	2004-12-21
1317	2005-04-06
1327	2005-06-15
1329	2005-06-29
1332	2005-07-18
1334	2005-08-03

Figure 4: Partnership between GNSS stations. BOGA and BOGT, GNSS Interval 1048 – 1430

8. CONCLUSIONS

The displacement value obtained by the wavelet transformation depends on the characteristics of the wavelet used as fading moments and degree of asymmetry; in turn, it also depends on the length of the dyadic interval. For the application of the wavelet method, the Daubechies wavelet with 10 fading moments was used.

The wavelet transformation allows us to obtain displacements scaled to different temporal resolutions, which in our case are weekly, fortnightly, monthly, bimonthly, four-monthly, and so on, according to the length of the diametric interval used for the wavelet transformation. For our case it should be a dyadic interval whose proportion of estimated/measured values does not exceed 13%, and that these belong to a time window of minimum 104 (GNSS Weeks) allowing to classify the displacements in levels where the less detailed level would contain the value of the scaled wavelet displacement of the dyadic interval, while the more detailed level would contain the weekly displacements of the station

The wavelet spectrum allows highlighting the most relevant differences in the time series over thinner or other levels.

The width of the wavelet spectrum allows identifying moments or time correspondences when the

displacement is more pronounced at a given level.

The application of wavelets to GNSS positioning time series allows obtaining products related to station displacement, and its correspondence in a time interval. These obtained products, associated with other types of temporal measurements can be useful to other disciplines related to geospatial and environmental issues, such as geodynamics, geophysics, meteorology, etc.

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