FORMULATION OF THE MATHEMATICAL MODEL TO MAXIMIZE THE TOTAL ANNUAL COST SAVINGS OF A POWER TRANSMISSION OR DISTRIBUTION SYSTEM

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ABSTRACT

In this article the authors propose a mixed integer non-linear programming algorithm (MINLP) to calculate the annual total cost savings (TCS) by optimizing the location and size, of Static Synchronous Compensators (STATCOMs) in transmission and distribution systems of electrical energy.

The MINLP algorithm is focused on minimizing the active power losses present in the system lines in order to maximize its annual TCS. The algorithm was implemented in the General Algebraic Modeling System (GAMS) software for the IEEE-33 radial distribution system, the results of which were compared with those presented by other authors.

KEYWORDS: Optimization, Power Losses, STATCOM & Total Cost Savings

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1. INTRODUCTION

One of the methods that has emerged for the optimization of electrical power systems at the transmission and distribution levels is the inclusion of Flexible AC Transmission Systems (FACTS) in specific nodes of the network, which allow the reduction of losses caused by the Joule effect and improve the voltage profiles of the nodes near its connection.

The Static Synchronous Compensator (STATCOM) is a voltage source converter (VSC), which is part of the FACTS, connected in derivation with the transmission and distribution networks capable of regulating the voltage, compensating the reactive power and eliminating current harmonics. The STATCOM allows effective control of power exchanges with the system through the absorption or generation of controllable active and reactive power. [1]

The STATCOM is capable of injecting lagging or leading compensation current associated with a specific load, in order to comply with the technical specifications of the network. The optimal allocation of STATCOMs maximizes the load capacity, minimizes power losses, improves stability, compensates the reactive power and improves power quality. [2]

The authors propose a mathematical model to determine the economic benefit obtained from optimizing the inclusion of STATCOMs in electrical energy systems. The model offers a joint technical-economic solution that maximizes the annual TCS of the network through the minimization of active power losses.

2. LITERATURE REVIEW

In [3] the effect of the STATCOM to mitigate power losses and improve the voltage stability of a transmission system is investigated. They adopt the Newton Raphson algorithm to find the load flow before and after including the STATCOM within the system considering four different scenarios.

In [4] the problem of optimization in size and location for distributed generators (DG) in radial distribution networks is approached by presenting a MINLP.

In [5] the application of the Global Harmony Search Algorithm (GHS) for the optimal location and size of STATCOM in power systems is shown. They use the Newton Raphson method to calculate the load flow while the GHS is used to find the most suitable nodes for the STATCOM installation and their nominal power.

In [6] the optimal location and size of the Distribution Static Compensator (DSTATCOM) in order to reduce power losses and improve current and voltage profiles in distribution networks is exposed. For which an immunological algorithm to determine the best location and size of DSTATCOM is developed taking as parameters the voltage and current profiles, the minimum installation cost of DSTATCOM and the maximum reduction of power loss.

In [7] a new approach to find the optimal location and size of the Distribution Static Compensator (DSTATCOM) using the minimization of total network power losses as an objective function is presented. The GHS algorithm is used to find the optimal location and size of DSTATCOM.

3. MATHEMATICAL MODEL

In this section, the authors present the formulation of the mathematical model to determine the annual TCS in a transmission or distribution system that includes a STATCOM. The model has the following 4 stages:

- Determine the free power flow (FPF).
- Determine the optimal power flow (OPF).
- Locate and size the STATCOMs to include.
- Calculate the annual TCS for the system.

3.1. FPF

FPF is defined as the power flow presented by the system without the operational restrictions in voltage stability, and it is formulated in equations 1 (objective function) [4], 2 (active power balance at each node) and 3 (reactive power balance at each node) [8].

$$\text{Min } Z = \sum_{i=1}^n V_i \sum_{j=1}^n V_j * Y_{ij} * \cos(\delta_i - \delta_j - \theta_{ij}) \quad (1)$$

Subject to:

$$P_{g_i} - P_{d_i} = V_i \sum_{j=1}^n V_j * Y_{ij} * \cos(\delta_i - \delta_j - \theta_{ij}) \quad (2)$$

$$Q_{g_i} - Q_{d_i} = V_i \sum_{j=1}^n V_j * Y_{ij} * \sin(\delta_i - \delta_j - \theta_{ij}) \quad (3)$$

3.2. OPF

OPF is a nonlinear optimization problem that represents the steady state operation of the electrical power system and allows determining the optimal value of the control variables considering the technical operating restrictions [8]. The objective of the OPF within the proposed model is to limit the dispersion in the magnitudes of the nodal voltages that the system presents.

To determine the OPF, a restriction must be added to the model proposed for the FPF as presented in equation 4 [4].

$$V_i^{min} \leq V_i \leq V_i^{max} \quad (4)$$

3.3. STATCOMs

In order to find the optimal location and dimensioning of the STATCOMs, the number of devices to be included in the system, a binary decision variable that will take the value of 1 if the STATCOM is located in the node and 0 if otherwise and the maximum value in kVAr that each device can take must be defined. The above should be included as restrictions to the model as presented in equations 5, 6 and 7.

$$\#STATCOMs \leq \#STATCOMs^{max} \quad (5)$$

$$X_i \leq \#STATCOMs^{max} \quad (6)$$

$$0 \leq Q_{gS} \leq X_i * Q_{gS}^{max} \quad (7)$$

3.4. TCS

The quantification of the annual TCS that is achieved from the incorporation of STATCOMs in electrical energy transmission and distribution systems is obtained by using equations 8 (annual TCS), 9 (annual cost of STATCOM) [6] and 10 (STATCOM cost per dimensioning) [7].

$$TCS = K_p(T * P_{Losses}^{Without STATCOM}) - K_p(T * P_{Losses}^{With STATCOM}) - K_c * STATCOM_{Yearly Cost} \quad (8)$$

$$STATCOM_{Yearly Cost} = STATCOM_{Total Cost} * \frac{(1+B)^{N_{STATCOM}} * B}{(1+B)^{N_{STATCOM}} - 1} \quad (9)$$

$$STATCOM_{Total Cost} = STATCOM_{Cost} * Q_{gS} \quad (10)$$

4. SYSTEM DESCRIPTION

The IEEE-33 radial distribution system (Figure 1) is proposed as a case study, which has a base voltage of 12.66 kV and power demand of 3715 kW and 2300 kVAr. The data associated with the transmission lines and loads discriminated by nodes are presented in Table 1.

Figure 1: IEEE-33 radial distribution system. [9]

Table 1: IEEE-33 radial distribution system data. [10]

Node i	Node j	R_{ij} (Ω)	X_{ij} (Ω)	P_j (kW)	Q_j (kVAr)
1	2	0,0922	0,0477	100	60
2	3	0,4930	0,2511	90	40
3	4	0,3660	0,1864	120	80
4	5	0,3811	0,1941	60	30
5	6	0,8190	0,7070	60	20
6	7	0,1872	0,6188	200	100
7	8	1,7114	1,2351	200	100

8	9	1,0300	0,7400	60	20
9	10	1,0400	0,7400	60	20
10	11	0,1966	0,0650	45	30
11	12	0,3744	0,1238	60	35
12	13	1,4680	1,1550	60	35
13	14	0,5416	0,7129	120	80
14	15	0,5910	0,5260	60	10
15	16	0,7463	0,5450	60	20
16	17	1,2890	1,7210	60	20
17	18	0,7320	0,5740	90	40
2	19	0,1640	0,1565	90	40
19	20	1,5042	1,3554	90	40
20	21	0,4095	0,4784	90	40
21	22	0,7089	0,9373	90	40
3	23	0,4512	0,3083	90	50
23	24	0,8980	0,7091	420	200
24	25	0,8960	0,7011	420	200
6	26	0,2030	0,1034	60	25
26	27	0,2842	0,1447	60	25
27	28	1,0590	0,9337	60	20
28	29	0,8042	0,7006	120	70
29	30	0,5075	0,2585	200	600
30	31	0,9744	0,9630	150	70
31	32	0,3105	0,3619	210	100
32	33	0,3410	0,5302	60	40

5. RESULTS

The proposed mathematical model was implemented in GAMS, through the KNITRO solver, applied to the IEEE-33 radial distribution system.

For the implemented system, the dispersion limits of the nodal voltages in pu, the maximum number of

STATCOMs, their maximum size in kVAr and the parameters K_P , T , K_C , $STATCOM_{Cost}$, B , and $N^{STATCOM}$ were defined with the same values used in [6] and [7] and are presented in Tables 2 and 3.

Table 2: Restrictions for the OPF and STATCOMs

Voltage limits	Number of decives	Maximum size of devices
$0.90 \leq V_i \leq 1.10$	$\#STATCOMs^{\max} = 1$	$Q_{GS}^{\max} = 10000 \text{ kVAr}$

Table 3: Parameter values to calculate the annual TCS

K_P (\$/kWh)	T	K_C	$STATCOM_{Cost}$ (\$/kVAr)	B	$N^{STATCOM}$ (Years)
0,06	8760	1	50	0,1	30

A STATCOM (Table 4) was located and sized, in such a way that the annual TCS of the network was maximized from the minimization of active power losses. The results obtained were compared with those presented in [6] and [7].

Table 4: Optimal location and sizing of the STATCOM

STATCOM		
Node	Location	Size (kVAr)
30	1	1258

Table 5 presents the power losses before and after the inclusion of the STATCOM, achieving a reduction in active and reactive power losses of 28.26% and 27.41% respectively.

Table 5: Losses with and without STATCOM

	Without STATCOM		With STATCOM	
	P (kW)	Q (kVAr)	P (kW)	Q (kVAr)
Losses	210,98	143,12	151,36	103,89
Reduction	NA	NA	28.26%	27.41%

Table 6 compares the active and reactive power losses, the optimal location and sizing of the STACOM, as well as the annual TCS for the methods presented in [6] and [7] and the model proposed by the authors.

Table 6: Comparison of results

	Base Case		[6]	[7]	Proposed Model
Size (kVAr)	NA	NA	962.49	1150	1258
Location (Node)	NA	NA	12	30	30
P _{losses} (kW)	202.67	210.98	171.79	143.97	151.36
Q _{losses} (kVAr)	135.24	143.12	115.26	96.47	103.89
V _{min} (pu)	0.9131	0.9038	0.9258	0.9236	0.9165
Annual TCS (\$)	NA	NA	11130	24264	24664

In [6] and [7] power losses of 202.67 kW and 135.24 kVAr are presented, which differ from those actually obtained for the base case of the IEEE-33 network (210.98 kW and 143.12 kVAr). Therefore, Table 6 shows two columns for the Base Case, the first shows the values used in [6] and [7] and the second the values used by the model proposed by the authors.

When the STATCOM is included in node 30 of the IEEE-33 radial distribution system, a correction is generated in the magnitude of the system voltages, since shunt compensation produces an injection of current towards the node supplying the demand current of the load, reducing the voltage drop and therefore improving the voltage profile (Figure 2).

Figure 2: System voltage profile with and without the inclusion of the STATCOM.

In Figure 3 an overlap of the graphs corresponding to the voltage profile obtained with the model shown and the algorithms proposed in [6] and [7] is presented.

The inclusion of the STATCOM in node 12 [6] allows to achieve an average correction of the voltage profile,

between nodes 8-18, which is 1.25% and 2% higher than that obtained in [7] and with MINLP respectively. On the other hand, the inclusion of the STATCOM in node 30 (MINLP) allows to achieve an average correction of the voltage profile, between nodes 26-33, which is 1.58% and 0.2% higher than that obtained in [6] and [7] respectively.

Figure 3: Overlap of voltage profiles.

6. CONCLUSIONS

The proposed mixed-integer nonlinear programming algorithm (MINLP) offers an alternative technical-economic solution for the optimal inclusion of STATCOMs in electrical power transmission and distribution systems, maximizing the annual TCS of the network through the minimization of active power losses.

The proposed MINLP algorithm allows a cost reduction of 121.6% and 1.65% in relation to that achieved with the algorithms proposed in [6] and [7] respectively.

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ABBREVIATIONS

Z	Active power that is lost through the lines.
P_{gi}	Active power generated at node i.
P_d	Active power demanded at node i.
Q_{gi}	Reactive power generated at node i.
Q_{di}	Reactive power demanded at node i.
V_i	Voltage magnitude of node i.
V_j	Voltage magnitude of node j.
δ_i	Voltage angle of node i.
δ_j	Voltage angle of node j.
Y_{ij}	Magnitude of admittance between nodes i-j.
θ_{ij}	Admittance angle between nodes i-j.
V_i^{min}	Lower limit of the nodal voltages.
V_i^{max}	Upper limit of nodal voltages.
$\#STATCOMs^{max}$	Limit of devices to locate.
X_i	Binary decision variable.
Q_{gS}^{max}	Maximum reactive power with which the STATCOM can be dimensioned.
Q_{gS}	Optimum reactive power of the STATCOM.
K_P	Cost of energy losses.
T	Total hours per year.
$P_{Losses}^{Without STATCOM}$	Active power lost before STATCOM installation.

$P_{Losses}^{With\ STATCOM}$	Power lost after STATCOM installation.
K_C	Time duration ratio.
$STATCOM_{Yearly\ Cost}$	Annual cost of running the device.
$STATCOM_{Total\ Cost}$	Device sizing cost.
B	Asset rate of return.
$N^{STATCOM}$	Longevity of STATCOM.