ANALYSIS OF MHD EFFECT ON RAYLEIGH STEP SLIDER BEARING LUBRICATED WITH COUPLE-STRESS FLUIDS

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ABSTRACT: *The magneto-hydrodynamic Rayleigh step slider bearing lubricated with couple-stress fluids is analyzed under transverse magnetic field. The expression for non-dimensional pressure, non-dimensional load carrying capacity, frictional force and the co-efficient of friction is derived and results are discussed for various non-dimensional parameters. It is observed that the effect of Hartmann number and couple-stress parameter enhances the load carrying capacity, frictional force and the co-efficient of friction.*

KEYWORDS: *Couple Stress, Magneto Hydrodynamics & Rayleigh Step Bearing*

Received: Jun 06, 2020; **Accepted:** Jun 26, 2020; **Published:** Aug 24, 2020; **Paper Id.:** IJMPERDJUN2020860

INTRODUCTION

The analysis of interface of conducting fluids with electromagnetic occurrences is called Magnetohydrodynamic (MHD). MHD lubrication is significantly than hydrodynamic lubrication, because most MHD bearing models have been developed from the perspective of mathematical analysis simplicity rather than practicality. The unexpected variation in the viscosity of lubricants with temperature, made the use of an electrically conducting fluid more popular. The first author to represent a study of MHD infinite inclined plane slider bearing in the existence of uniform transverse magnetic field was Snyder [1] and found that a significant increase in load capacity was possible with liquid metal lubricants in the presence of magnetic field. Shukla [2] examined optimum load of MHD slider bearing using film thickness and conductivity functions as bounded control variables and concluded that if the conductivity of bearing surface is of step-type in nature, it is more desirable for a uniformly distributed magnetic field. Hughes [3] analysed the MHD finite step slider bearing in the presence of magnetic field applied both tangentially and transversely to the fluid film. For the transverse field it is found that only a slight increase in pressurization can be affected on open circuit conditions and that short circuit conditions are unfavorable. Anwar [4] analysed the MHD characteristic of inclined slider bearing with arbitrary magnetic field. It is found that a non-uniform applied magnetic field gives higher load capacity as compared with uniform magnetic fields.

In 1918, Lord Rayleigh discovered a film geometry where a step is split the lubricant film into two zones viz; h_1 at the entry zone and h_2 at the exit zone and result holds good when the viscosity of lubricant is considered as function of pressure. Many researchers have been studied the characteristics of the Rayleigh step bearing lubricated with non-Newtonian fluids to name few are Jianming and Gaobing [5], and Naduvnamani and Siddangouda [6] have studied the effect of couple stress on the characteristics of Rayleigh step bearings.

The slider bearings have practical applications in machine design and in other kinds of machine elements in

which rectilinear sliding motions occur. The thrust area of the step bearings is in the clutch plate, gear box, thrust bearings and journal bearings etc. Awareness of the features of bearing is important when taking into consideration specific operational circumstances. Several analyses on lubrication performance of slider bearings have been concluded in the last few years. Later to upgrade the lubricating performance the increased utilization of Newtonian lubricant which has been blended with long chain polymers has been observed. Using additives stabilizes the flow properties and minimizes lubricant sensitivity to change shear rate. A variety of micro-continuum theories have been carried out, but Stokes [7] theory is the simplest micro-continuum theory which allows the existence of Couple stresses and body couples. The load carrying capability, force of friction increases, and frictional coefficient decreases for a slider bearing greased with couple-stress fluid was concluded by Ramanaiah and Sarkar [8]. A relative study is made between parabolic and inclined slider lubricated with couplestress fluid by Moobalaji and John [9] and found that parabolic slider has more superior load carrying capability than inclined slider. The rough slider bearing with couple stress was analyzed by Naduvianamani et al [10]. Buzurke et al [11] examined the porous step slider with couple stress fluids and many authors [12-14] discussed the effect of couple stress lubricants on the characteristics of bearing systems. Combined effect of Magnetic field and couple stress lubricant was discussed by several researchers such as Kashinath and Hanumagowda [15] presented the composite slider bearing with MHD couple stress, and noticed due to magnetic field there is a increase in pressure, load carrying, frictional force and coefficient of friction. Hanumagowda et al [16-21] studied Land Tapered slider bearing and plane slider bearing, and Naduvianamani et al [22] discussed the effect of MHD couple stress on exponential slider bearings and noted that the increase in load carrying capacity. The aim of the current paper is to conduct a study on MHD Rayleigh step slider bearing lubricated with Couple-stress Fluids.

THEORETICAL FORMULATION

The physical configuration of the problem is shown in Figure1 in which lower surface have pure tangential sliding motion relative to the other surface and separate by the lengths L_1 and L_2 . The film thickness in the entry region is h_1 and in exit region is h_2 . The uniform magnetic field B_0 is applied perpendicular to the two plates. The lubricant in the film region is incompressible. Stokes[7] couplestress fluid, body forces and body couples are assumed to be absent. The basic equations governing the hydro-magnetic flow of the lubricant in the fluid region are

$$\mu \frac{\partial^2 u}{\partial y^2} - \eta \frac{\partial^4 u}{\partial y^4} - \sigma B_0^2 u = \frac{\partial p}{\partial x} + \sigma E_z B_0 \quad (1)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$\int_{y=0}^h (E_z + B_0 u) dy = 0 \quad (4)$$

From figure1 the boundary of the upper surface is given by

$$h = \begin{cases} h_1 & \text{for } 0 \leq x \leq L_1 \\ h_2 & \text{for } L_1 \leq x \leq L \end{cases} \quad \text{where } L_1 + L_2 = L$$

Figure1 Rayleigh step slider bearing

Boundary conditions are

At the upper surface $y = h$

$$u=0 \frac{\partial^2 u}{\partial y^2} = 0 \quad v=0 \quad (5)$$

At the lower surface $y = 0$

$$u=U \frac{\partial^2 u}{\partial y^2} = 0 \quad v=0 \quad (6)$$

Solution of equation (1) using condition (4) and the boundary conditions (5) and (6) is

$$u = -\frac{h_2^2 h}{2l\mu M_0^2} \frac{\partial p}{\partial x} \left\{ \frac{B^2 \xi_1 - A^2 \xi_2}{\frac{B^2}{A} \tanh\left(\frac{Ah}{2l}\right) - \frac{A^2}{B} \tanh\left(\frac{Bh}{2l}\right)} \right\} - \frac{U}{2(A^2 - B^2)} \{B^2 \xi_3 - A^2 \xi_4\} \quad (7)$$

where

$$\xi_1 = \frac{\sinh \frac{Ah}{l} - \sinh \frac{Ay}{l} + \sinh \frac{A(h-y)}{l}}{\sinh \frac{Ah}{l}}, \quad \xi_2 = \frac{\sinh \frac{Bh}{l} - \sinh \frac{By}{l} + \sinh \frac{B(h-y)}{l}}{\sinh \frac{Bh}{l}},$$

$$\xi_3 = \frac{\sinh\left(\frac{Ah}{l}\right) - \sinh\left(\frac{Ay}{l}\right) - \sinh \frac{A(h-y)}{l}}{\sinh\left(\frac{Ah}{l}\right)}, \quad \xi_4 = \frac{\sinh\left(\frac{Bh}{l}\right) - \sinh\left(\frac{By}{l}\right) - \sinh \frac{B(h-y)}{l}}{\sinh\left(\frac{Bh}{l}\right)}.$$

Integration of the continuity equation (3) over the film thickness with boundary conditions (5) and (6) results the modified Reynolds equation in the form

$$\frac{\partial}{\partial x} \left[\frac{6h_2^2 h^2}{\mu l M_0^2} \left\{ \frac{A^2 - B^2}{\frac{A^2}{B} \tanh\left(\frac{Bh}{2l}\right) - \frac{B^2}{A} \tanh\left(\frac{Ah}{2l}\right)} - \frac{2l}{h} \right\} \frac{\partial p}{\partial x} \right] = 6U \frac{dh}{dx}$$

Using the non-dimensional quantities, the non-dimensional MHD Reynolds-type equation is

$$\frac{\partial}{\partial x^*} \left\{ f(H, l^*, M_0) \frac{\partial P}{\partial x^*} \right\} = \frac{dH}{dx^*} \quad (8)$$

$$\text{where } f(H, l^*, M_0) = \frac{12H^2}{l^* M_0^2} \left\{ \frac{(A^{*2} - B^{*2})}{\frac{A^{*2}}{B^*} \tanh\left(\frac{B^* H}{l^*}\right) - \frac{B^{*2}}{A^*} \tanh\left(\frac{A^* H}{l^*}\right)} - \frac{l^*}{H} \right\}$$

$$x^* = \frac{x}{L}, \quad P = \frac{ph_2^2}{6\mu UL}, \quad l^* = \frac{2l}{h_2}, \quad H = \frac{h}{h_2}, \quad L_1^* = \frac{L_1}{L}, \quad L_2^* = \frac{L_2}{L}$$

$$A^* = \left[\frac{1 + \left\{ 1 - (l^{*2} M_0^2 / \mu) \right\}^{1/2}}{2} \right]^{1/2}, \quad B^* = \left[\frac{1 - \left\{ 1 - (l^{*2} M_0^2 / \mu) \right\}^{1/2}}{2} \right]^{1/2}$$

The relevant boundary conditions for pressure are

$$P = 0 \text{ at } x^* = 0, 1 \quad (9)$$

Integrating both sides of equation (8) twice w.r.t. x^* and using the boundary conditions (9) we get

$$P_1^* = \frac{(h_1^* - 1)L_2^* x^*}{L_1^* f^*(1, l^*, M_0) + L_2^* f^*(h_1^*, l^*, M_0)} \quad 0 \leq x^* \leq L_1^* \quad (10a)$$

$$P_2^* = \frac{(h_1^* - 1)L_1^* (1 - x^*)}{L_1^* f^*(1, l^*, M_0) + L_2^* f^*(h_1^*, l^*, M_0)} \quad L_1^* \leq x^* \leq 1 \quad (10b)$$

The non-dimensional load carrying capacity is

$$W^* = \frac{Wh_2^2}{6\mu UL^2} = \int_0^{L_1^*} P_1^* dx^* + \int_{L_1^*}^1 P_2^* dx^*$$

Using (10a) and (10b) the non dimensional load carrying capacity is obtained in the form

$$W^* = \frac{(h_1^* - 1)L_1^* L_2^*}{2 \{ L_2^* f^*(h_1^*, l^*, M_0) + L_1^* f^*(1, l^*, M_0) \}} \quad (11)$$

The components of stress tensor required for calculating frictional force is

$$\tau_{yx} \Big|_{y=0} = \mu \frac{\partial u}{\partial y} - \eta \frac{\partial^3 u}{\partial y^3}$$

The frictional force on the bearing surface is

$$F = \int_0^L \left[-\frac{\mu U l M_0^2}{2h_1^2(A^2 - B^2)} \left\{ \frac{A^2}{B} \coth\left(\frac{Bh}{2l}\right) - \frac{B^2}{A} \coth\left(\frac{Ah}{2l}\right) \right\} - \frac{h}{2} \frac{\partial p}{\partial x} \right] dx$$

The dimension less frictional force is

$$\begin{aligned} F^* &= -\frac{F h_1}{6\mu U L} \\ &= G(h_1^*, l^*, M_0) L_1^* + G(1, l^*, M_0) L_2^* \\ &\quad + \frac{(h_1^* - 1)^2 L_1^* L_2^*}{2\{f(1, l^*, M_0) L_1^* + f(h_1^*, l^*, M_0) L_2^*\}} \end{aligned} \quad (12)$$

$$\text{where } G(h_1^*, l^*, M_0) = \frac{l^* M_0^2}{24(A^{*2} - B^{*2})} \left\{ \frac{A^{*2}}{B^*} \coth\left(\frac{B^* h_1^*}{l^*}\right) - \frac{B^{*2}}{A^*} \coth\left(\frac{A^* h_1^*}{l^*}\right) \right\}$$

$$G(1, l^*, M_0) = \frac{l^* M_0^2}{24(A^{*2} - B^{*2})} \left\{ \frac{A^{*2}}{B^*} \coth\left(\frac{B^*}{l^*}\right) - \frac{B^{*2}}{A^*} \coth\left(\frac{A^*}{l^*}\right) \right\}$$

The co-efficient of friction is given by

$$C = \frac{F^*}{W^*} \quad (13)$$

Limiting case of the Present Study

As $M_0 \rightarrow 0$ equations (11), (12) and (13) reduces to the case studied by Ramanaiah and Sarkar[8](case2.1). The comparison of present analysis with Ramanaiah and Sarkar[8] is given in table1.

RESULTS AND DISCUSSIONS

The effect of MHD on Rayleigh step slider bearing lubricated with couple stress fluids is studied. The results are discussed for various non-dimensional parameters namely Hartmann number M_0 , couple-stress parameter l^* , bearing length L_1^* as follows.

Pressure

Figure 2 shows the variation of non-dimensional pressure P^* with x^* for distinct values of M_0 and l^* with fixed values $L_1^* = 0.7$, $h_1^* = 2$. It is observed that the pressure increases with increasing values of Hartmann number M_0 and couple stress parameter l^* as compared with Non-magnetic case and Newtonian case. The deviation of non-dimensional pressure P_{max}^* with L_1^* as function of M_0 and l^* with fixed values $h_1^* = 2$ is depicted in Figure 3. It is found that pressure is maximum for L_1^* lies between 0.7 and 0.8.

Figure 3 Variation of non-dimensional pressure P_{max}^* with L_1^* for different values of M_0 and l^* with $h_1^* = 2$. Figure 2 Variation of non-dimensional pressure P^* with x^* for different values of M_0 and l^* with $L_1^* = 0.7, h_1^* = 2$.

Load Carrying Capacity

Figure 4 shows the variation of non-dimensional load carrying capacity W^* with step height h_1^* for various values of M_0 and l^* with fixed values $L_1^* = 0.7$. It is observed that the effect of Hartmann number M_0 and couple stress parameter l^* , increases load carrying capacity. The non-dimensional load increases with h_1^* until a maximum is obtained, and there after decreases with h_1^* . In Figure 5 the graph of non-dimensional load carrying capacity W^* with L_1^* for different values of M_0 and l^* with fixed values $h_1^* = 2$ is plotted and observed that maximum load carrying capacity is for L_1^* lies in the interval 0.7 and 0.8.

Table 1: Present analysis is compared with work done by Ramanaiah and sarkar [8] by varying MHD and couple stress parameter and values are tabulated for W^*, F^*, C and with $L_1^* = 0.5$.

Ramanaiah and Sarkar[8]				Present analysis				
analysis				$M_0 = 0$	$M_0 = 0$	$M_0 = 2$	$M_0 = 4$	$M_0 = 6$
l^*	$h_1^* = 2$	$h_1^* = 3$	$h_1^* = 2$	$h_1^* = 3$	$h_1^* = 2$	$h_1^* = 2$	$h_1^* = 2$	$h_1^* = 2$
W^*	0	0.02778	0.01786	0.02778	0.01786	0.03384	0.04708	0.06242
	0.01	0.02778	0.01786	0.02778	0.01786	0.03385	0.04709	0.06246
	0.1	0.02804	0.01793	0.02804	0.01793	0.03424	0.04808	0.06462
	0.5	0.03395	0.01943	0.03339	0.01943	0.04076	0.05942	0.08411
	1.0	0.04829	0.02351	0.04829	0.02351	0.05656	0.07931	0.11227
	10	1.92456	0.53089	1.92390	0.53089	1.93292	1.95996	2.00490
	∞	∞	∞	∞	∞	∞	∞	∞
F^*	0	0.15278	0.14683	0.15276	0.146811	0.22971	0.38675	0.56367
	0.01	0.15278	0.14683	0.15278	0.14683	0.22972	0.38682	0.56391
	0.1	0.15304	0.14697	0.15304	0.14697	0.23058	0.39207	0.57945
	0.5	0.15839	0.14997	0.15840	0.14997	0.24158	0.43883	0.69763
	1.0	0.17329	0.15813	0.17329	0.15813	0.26111	0.49077	0.81282
	10	2.04956	1.17291	2.04890	1.17288	2.14120	2.41737	2.87527
	∞	∞	∞	∞	∞	∞	∞	∞
C	0	5.49999	8.22224	5.50000	8.22222	6.78725	8.21446	9.02998
	0.01	5.49982	8.22196	5.49955	8.22196	6.78665	8.21370	9.02906
	0.1	5.45799	8.19673	5.45800	8.19689	6.73354	8.15428	8.96688
	0.5	4.74313	7.71921	4.74313	7.71922	5.92378	7.38495	8.29420
	1.0	3.58843	6.72666	3.58843	6.72666	4.61684	6.18797	7.24011
	10	1.06495	2.20929	1.06497	2.20929	1.10775	1.23338	1.43412
	∞	1.00000	2.00000	1.00000	2.00000	1.00000	1.00000	1.00000

Figure 4 Variation of non dimensional load carrying capacity W^* with h_1^* for different values M_0 and l^* with $L_I^*=0.7$.

Figure 5 Variation of non dimensional load carrying capacity W^* with L_I^* for different values M_0 and l^* with $h_1^*=2$.

Frictional Force

Figure 6 displays the variation of non-dimensional frictional force F^* with step height h_1^* for different values of M_0 and l^* with fixed values $L_I^*=0.7$. It is observed that frictional force increases with increasing values of M_0 and l^* as compared with Non-magnetic case and Newtonian case. For the bearing with a Non-magnetic lubricant the frictional force decreases with increasing value of the step height h_1^* and the bearing with a magnetic lubricant the frictional force increases with increasing value of the step height h_1^* . Figure 7 shows the variation of non-dimensional frictional force F^* with L_I^* for different values of M_0 and l^* with fixed values $h_1^*=2$. It is seen that frictional force decreases with an increase of L_I^* for nonmagnetic case and for magnetic case frictional force increases with an increase of L_I^* until a maximum is obtained, and there after decreases with L_I^* .

Coefficient of Friction

Figure 8 shows the variation of coefficient of friction C with step height h_1^* for different values of M_0 and l^* with fixed values $L_I^*=0.7$. It is observed that coefficient of friction increases with an increase in Hartmann number M_0 and decreases due to increase in couplestress parameter l^* . Figure 9 depicts, the deviation of coefficient of friction C with L_I^* for different values of M_0 and l^* with fixed values $h_1^*=2$. It is noted that the coefficient of friction C decreases with L_I^* until a minimum is obtained, and there after increases with L_I^* .

The relative percentage increase in the non-dimensional load carrying capacity R_W^* , non-dimensional frictional force R_F^* and coefficient of friction R_C are defined by

$$R_W^* = \left\{ \left(W_{\text{magnetic}}^* - W_{\text{non-magnetic}}^* \right) / W_{\text{non-magnetic}}^* \right\} \times 100 \quad R_F^* = \left\{ \left(F_{\text{magnetic}}^* - F_{\text{non-magnetic}}^* \right) / F_{\text{non-magnetic}}^* \right\} \times 100$$

$$\text{and } R_C = \left\{ \left(C_{\text{magnetic}} - C_{\text{non-magnetic}} \right) / C_{\text{non-magnetic}} \right\} \times 100$$

The values of R_W^* , R_F^* and R_C are listed in Table 2 for various values of l^* , M_0 with $L_I^*=0.7$, $h_1^*=2$. It is clear that an increase of nearly 66.43%, 185% and 71.23% in W^* , F^* and C is observed for $l^*=0.2$ and $M_0=4$.

Figure 6 Variation of non dimensional frictional force F^* with h_j^* for different values M_0 and l^* with $L_j^* = 0.7$.

Figure 7 Variation of non dimensional frictional force F^* with L_j^* for different values M_0 and l^* with $h_j^* = 2$.

CONCLUSIONS

Using the continuity equation and the MHD motion equations the characteristics of Rayleigh step slider bearings with an electrically conducting fluid in the presence of a transverse magnetic field are investigated. According to the results and discussion, conclusions can be drawn as follows.

- The Pressure, load carrying capacity, frictional force and coefficient of friction of Rayleigh step slider bearing is significant for larger values of couples stress parameter l^* as compared to Newtonian case.
- The applied magnetic field increases the load carrying capacity, the frictional force and the coefficient of friction as compared with the corresponding non-magnetic case.
- The relative load R_{W^*} , the relative frictional force R_{F^*} and the relative coefficient of friction R_C increases with increasing values M_0 , an increase of about 66.43% in W^* , 184.97% in F^* and 71.23% in C is observed for $l^* = 0.2$ and $M_0 = 4$.

Figure 8 Variation of non dimensional coefficient of friction C with h_j^* for different values M_0 and l^* with $L_j^* = 0.7$.

Figure 9 Variation of non dimensional coefficient of friction C with L_j^* for different values M_0 and l^* with $h_j^* = 2$.

Numerical example of Rayleigh step slider bearing with MHD and couple stress is given in table 3.

Table 2: Variation of R_W^* , R_F^* and R_C for different values l^* and M_0 with $L_1^* = 0.7$, $h_1^* = 2$.

l^*	M_0	R_W^*	R_F^*	R_C
0	2	19.62295	59.77411	33.56485
	4	61.86164	175.7124	70.33877
	6	110.8811	302.3946	90.81629
0.2	2	20.31353	60.6829	33.55357
	4	66.42942	184.9719	71.22706
	6	123.4867	330.3289	92.55259
0.4	2	20.47991	61.57736	34.11076
	4	69.50104	196.5752	74.96881
	6	132.3145	363.4397	99.48705
0.6	2	19.5771	61.44175	35.0109
	4	68.58107	204.0893	80.38196
	6	132.9851	386.3871	108.7643

Table 3: Numerical example of Rayleigh step slider bearing with MHD and couple-stress fluid

Physical parameter	Notation	Range of values chosen
Length of the bearing	L	1.0×10^{-1} m
Minimum film thickness	h_2	0.5×10^{-4} m
Maximum film thickness	h_1	$(0.6 \text{ to } 1.4) \times 10^{-4}$ m
Electrical conductivity	σ	1.07×10^6 mh ₀ / m
Applied magnetic field	B_0	0, 1.52, 3.04, 4.56 Wb / m ²
Lubricant viscosity	μ	1.55×10^{-3} Pa.s
Couple stress material	η	0, 0.3875, 1.55, 6.2×10^{-13} N.s

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