ON I_2 - STATISTICALLY CONVERGENT AND I_2 - STATISTICALLY CAUCHY SEQUENCE OF DOUBLE SEQUENCES IN TWO NORMED SPACES OVER ULTRAMETRIC FIELDS

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ABSTRACT

Let K be a complete, locally compact, non-trivially valued non-Archimedean field. In this paper, I_2 - Statistically Convergent of double sequences and I_2 - Statistically Cauchy Sequence of double sequences are discussed and investigated in two normed spaces over ultra metric fields.

KEYWORDS: Non-Archimedean fields, Double Sequences, I_2 - Statistically Convergent, I_2 - Statistically Cauchy

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1. INTRODUCTION

Fast [8] introduced the statistical convergence concept to sequences of real numbers. Several authors (see [7,14]) have investigated the concept of generalizations, applications of real numbers and the concept of ideal convergence was introduced by P.Kostyrko et.al [13]. The notion of Ideal convergence, Ideal Cauchy and I^* -Cauchy double sequences in a linear metric space were studied by E. Dundar and B.As an extension many authors were discussed more concepts on ideal convergence and ideal Cauchy sequences [2,3,4,5,6,11,14,16]. The generalization of statistical convergence was established as an ideal convergence by P.Kostyrko et.al [13]. S. Gahler [10] introduced the idea of 2-normed spaces in 1960's. As a classical analogue of normed spaces, this was further investigated by many mathematicians. Many researchers [17,11,5] studied the ideal convergence, I -Cauchy sequences respectively and I_2 -Cauchy for double sequence in 2-normed spaces. The concepts of ideal statistical convergence and ideal statistically Cauchy sequence for a single sequence in a 2-normed space was studied by Gurdal [20]. Now, studying the concepts of I - Statistically Convergent and I - Statistically Cauchy Sequences seems to be an important aspect. Here these two concepts are investigated over Two Normed Spaces.

As an analogous S.Sangeetha and V.Srinivasan have recently investigated I - convergence, I -Cauchy, I^* -convergence and I^* -Cauchy sequences in 2-normed spaces X over non-Archimedean field K and they studied some basic properties of these concepts and relation between them in non-Archimedean normed spaces X over K [19].

In this paper, we will discuss the idea I_2 - Statistically convergent of double sequences, I_2 - Statistically Cauchy sequence of double sequences in non-Archimedean two normed spaces over non-Archimedean field K and we are going to prove some basic properties of these concepts and relation between them in non-Archimedean two normed spaces over K .

Let us consider all through K denotes a complete, locally compact, non-trivially valued, non-archimedean field. For non-archimedean fields one can refer [1], and any general reference of classical analysis can be found [8].

A non-empty subset I of a ring R of subsets of $\mathbb{N} \subset X$ is an ideal in R if and only if

(i) $A, B \in I$ implies $A \cup B \in I$,

(ii) $A \in I, B \in R, B \subset A$ implies $B \in I$.

An ideal is called non-trivial if $I \neq \emptyset$ and $\mathbb{N} \notin I$, while a non-trivial ideal I in $\mathbb{N}$ is said to be an admissible if $\{x\} \in I$ for any $x \in \mathbb{N}$, where $\mathbb{N}$ denotes the set of positive integers.

Given a non-trivial ideal I , the sequence $(x_m), x_m \in K$, is said to be I -convergent to $\ell \in K$ if for every $\varepsilon > 0$ the set $\{m \in \mathbb{N} : |x_m - \ell| \geq \varepsilon\} \in I$. In this case, we write,

$$I\text{-}\lim_{m \rightarrow \infty} x_m = \ell$$

For an admissible ideal I and X being a non-archimedean normed space, the sequence $(x_m), x_m \in X$, is called I -Cauchy sequence in X if for all $\varepsilon > 0$,

$$\{m \in \mathbb{N} : \|x_{m+1} - x_m\| \geq \varepsilon\} \in I.$$

2. I_2 -STATISTICAL CONVERGENCE IN TWO NORMED SPACES

Definition 2.1

Let X be a non-archimedean vector space with dimension d , where $2 \leq d < \infty$. The Two norm on X is a function

$\|\cdot, \cdot\| : X \times X \rightarrow K$ Which satisfies

- $\|x, y\| = 0$ if x and y are linearly independent,
- $\|x, y\| = \|y, x\|$
- $\|\alpha x, y\| = |\alpha| \|x, y\|$, $\alpha \in K$
- $\|x, y + z\| \leq \max\{\|x, y\|, \|x, z\|\}$.

And the pair $(X, \|\cdot, \cdot\|)$ denotes a two normed space.

Definition 2.2

The sequence $\{x_m\}$ of X is said to be I -statistically convergent to l in two normed space for a non-trivial ideal I , if for any $\varepsilon > 0$ and $u \in X$ the set

$$\lim_{m \rightarrow \infty} \frac{1}{m} |\{n \in \mathbb{N} : m \leq n, \|x_n - l, u\| \geq \varepsilon\}| = 0$$

If $\{x_m\}$ is I -statistically convergent to l in two normed space. Let us denote

$$I\text{-stat.} \lim_{m \rightarrow \infty} \|x_m - l, u\| = 0 \text{ or } I\text{-}\lim_{m \rightarrow \infty} \|x_m, u\| = \|l, u\| \text{ for all } u \in X.$$

Here l is the I -statistical limit of the sequence $\{x_m\}$.

Definition 2.3

The sequence $\{x_m\}$ of X is said to be I -statistically Cauchy sequence in two normed space, if for every $\varepsilon > 0$ and $u \in X$ the set

$$\lim_{m \rightarrow \infty} \frac{1}{m} |\{n \in N: m \leq n, \|x_m - x_n, u\| \geq \varepsilon\} \in I| = 0$$

If $\{x_m\}$ is I -statistically Cauchy sequence in two normed space then we can write

$$I\text{-stat.} \lim_{m \rightarrow \infty} \|x_{m+1} - x_m, u\| = 0, \text{ for all } u \in X.$$

Definition 2.4

Let K be a complete, non-trivially valued non-archimedean field. A double sequence $\{x_{ij}\}, x_{ij} \in K$ is said to be I_2 -statistically convergent to a limit l in two normed space for non-trivial ideal I_2 , if for any $\varepsilon > 0$ and $u \in X$ the set

$$\lim_{m, n \rightarrow \infty} \frac{1}{mn} |\{(i, j) \in N: i \leq m, j \leq n, \|x_{ij} - l, u\| \geq \varepsilon\} \in I_2| = 0$$

Symbolically, we write $I_2\text{-stat.} \lim_{m, n \rightarrow \infty} \|x_{m, n} - l, u\| = 0, \text{ for all } u \in X.$

Definition 2.5

Let K be a complete, non-trivially valued non-archimedean field. A double sequence $\{x_{ij}\}, x_{ij} \in K$ is called as I_2 -statistically Cauchy double sequence in two normed space for non-trivial ideal I_2 , if for any $\varepsilon > 0$ and $u \in X$, the set

$$\lim_{m, n \rightarrow \infty} \frac{1}{mn} |\{(i, j) \in N: i \leq m, j \leq n, \|x_{ij} - x_{m, n}, u\| \geq \varepsilon\} \in I_2| = 0$$

Theorem 2.6

Let $\{x_{mn}\}$ be the double sequence for each $u \in X$, $I_2\text{-stat.} \lim_{m, n \rightarrow \infty} \|x_{mn}, u\| = \|l_1, u\|$

$I_2\text{-stat.} \lim_{m, n \rightarrow \infty} \|x_{mn}, u\| = \|l_2, u\|$ then $l_1 = l_2$, where I_2 denoted as admissible ideal.

Proof

For any $u \in X$, let us assume that,

$$A_1(\varepsilon, u) = \lim_{m, n \rightarrow \infty} \frac{1}{mn} |\{(i, j) \in N: i \leq m, j \leq n, \|x_{ij} - l_1, u\| \geq \varepsilon\} \in I_2| = 0 \quad (1)$$

$$A_2(\varepsilon, u) = \lim_{m, n \rightarrow \infty} \frac{1}{mn} |\{(i, j) \in N: i \leq m, j \leq n, \|x_{ij} - l_2, u\| \geq \varepsilon\} \in I_2| = 0 \quad (2)$$

Consider,

$$\begin{aligned}
 & \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n, \|l_1 - l_2, u\| \geq \varepsilon \right\} \in I_2 \right| \\
 &= \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n, \|l_1 - x_{ij} + x_{ij} - l_2, u\| \geq \varepsilon \right\} \in I_2 \right| \\
 &= \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n, \|(l_1 - x_{ij}) + (x_{ij} - l_2), u\| \geq \varepsilon \right\} \in I_2 \right| \\
 &\leq \max \left\{ \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n, \|x_{ij} - l_1, u\| \geq \varepsilon \right\} \in I_2 \right|, \right. \\
 &\quad \left. \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n, \|x_{ij} - l_2, u\| \geq \varepsilon \right\} \in I_2 \right| \right\} \\
 &\rightarrow 0, \text{ as } n \rightarrow \infty \quad \text{[using (1) and (2)]}
 \end{aligned}$$

Hence, $l_1 = l_2$.

Theorem 2.7

If I is an admissible Ideal and if $I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|x_{mn}, u\| = \|l_1, u\|$ and $I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|y_{mn}, u\| = \|l_2, u\|$ then,

- $I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|x_{mn} + y_{mn}, u\| = \|l_1 + l_2, u\|$;
- $I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|\alpha x_{mn}, u\| = \|\alpha l_1, u\|$ for every $u \in X$ and $\alpha \in K$.

Proof

- Let l_1 and l_2 be the I_2 -statistical limit of the sequence $\{x_{mn}\}$ and $\{y_{mn}\}$ respectively.

For all $\varepsilon > 0$ and $u \in X$ the set $A_1, A_2 \in I_2$, where

$$A_1(\varepsilon, u) = \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n, \|x_{ij} - l_1, u\| \geq \varepsilon \right\} \in I_2 \right| = 0 \quad (3)$$

$$A_2(\varepsilon, u) = \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n, \|y_{ij} - l_2, u\| \geq \varepsilon \right\} \in I_2 \right| = 0 \quad (4)$$

Consider,

$$\lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n, \|(x_{ij} + y_{ij}) - (l_1 + l_2), u\| \geq \varepsilon \right\} \in I_2 \right|$$

$$\begin{aligned}
 &= \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|(x_{ij} - l_1) + (y_{ij} - l_2), u\| \geq \varepsilon \right\} \in I_2 \right| \\
 &\leq \max \left\{ \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|x_{ij} - l_1, u\| \geq \varepsilon \right\} \in I_2 \right|, \right. \\
 &\quad \left. \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|y_{ij} - l_2, u\| \geq \varepsilon \right\} \in I_2 \right| \right\} \\
 &= 0,
 \end{aligned}$$

using [3] & [4]

Hence,

$$I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|x_{mn} + y_{mn}, u\| = \|l_1 + l_2, u\|$$

• Let $I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|x_{mn}, u\| = \|l_1, u\|$

For, $\alpha \in K$ and $\alpha \neq 0$, we have

$$\lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|x_{ij} - l_1, u\| \geq \frac{\varepsilon}{|\alpha|} \right\} \in I_2 \right| = 0 \quad (5)$$

Now, we shall prove that,

$$I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|\alpha x_{mn}, u\| = \|\alpha l_1, u\| \text{ for every } u \in X \text{ and } \alpha \in K.$$

That is, to prove $I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|\alpha x_{mn} - \alpha l_1, u\| = 0$

It is given that,

$$\begin{aligned}
 &\lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|\alpha x_{ij} - \alpha l_1, u\| \geq \varepsilon \right\} \in I_2 \right| \\
 &= \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|\alpha(x_{ij} - l_1), u\| \geq \varepsilon \right\} \in I_2 \right| \\
 &= \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : |\alpha| \|(x_{ij} - l_1), u\| \geq \varepsilon \right\} \in I_2 \right| \\
 &= \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|x_{ij} - l_1, u\| \geq \frac{\varepsilon}{|\alpha|} \right\} \in I_2 \right|
 \end{aligned}$$

Therefore, $I_2 - \text{stat. lim}_{m+n \rightarrow \infty} \|\alpha x_{mn} - \alpha l_1, u\| = 0$, for every $u \in X$.

Hence the proof is complete.

Let us prove the theorem by considering a basis $v = \{v_1, v_2, v_3, \dots, v_d\}$ for X .

Theorem 2.8

If I_2 is an admissible ideal then the double sequence $\{x_{mn}\}$ in X is I_2 -statistically convergent to l in X if and only if $I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, v_i\| = 0$

For every $i = 1, 2, \dots, d$.

Proof

Let us assume that the double sequence $\{x_{mn}\}$ in X is I_2 -statistically convergent to l in X .

That is,

$$I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, z\| = 0 \quad (6)$$

Where $z = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_d v_d, \alpha_1, \alpha_2, \dots, \alpha_d \in K$

To prove

$$I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, v_i\| = 0$$

Consider

$$\begin{aligned} & \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i, j) \in N : i \leq m, j \leq n : \|x_{ij} - l, z\| \geq \varepsilon \right\} \in I_2 \right| \\ &= \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i, j) \in N : i \leq m, j \leq n : \|x_{ij} - l, \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_d v_d\| \geq \varepsilon \right\} \in I_2 \right| \\ &\leq \max \left\{ \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i, j) \in N : i \leq m, j \leq n : \|x_{ij} - l, \alpha_1 v_1\|, \|x_{ij} - l, \alpha_2 v_2\|, \dots, \|x_{ij} - l, \alpha_d v_d\| \geq \varepsilon \right\} \in I_2 \right| \right\} \\ &\leq \max_{1 \leq i \leq d} \left\{ \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i, j) \in N : i \leq m, j \leq n : \|\alpha\| \|x_{ij} - l, v_i\| \geq \varepsilon \right\} \in I_2 \right| \right\} \end{aligned}$$

Since

$$I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, z\| = 0$$

$$I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, v_i\| = 0, \text{ for every } i = 1, 2, \dots, d.$$

Conversely, we now assume that if $I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, v_i\| = 0$, for every $i = 1, 2, \dots, d$.

Let us now prove that

$$I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, z\| = 0$$

For all non-zero $z \in X$.

$$\begin{aligned} & \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|x_{ij} - l, z\| \geq \varepsilon \right\} \in I_2 \right| \\ &= \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|x_{ij} - l, \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_d v_d\| \geq \varepsilon \right\} \in I_2 \right| \\ &\leq \max \left\{ \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|x_{ij} - l, \alpha_1 v_1\|, \|x_{ij} - l, \alpha_2 v_2\|, \dots, \|x_{ij} - l, \alpha_d v_d\| \geq \varepsilon \right\} \in I_2 \right| \right\} \\ &\leq \max_{1 \leq i \leq d} \left\{ \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|\alpha\| \|x_{ij} - l, v_i\| \geq \varepsilon \right\} \in I_2 \right| \right\} \\ &\leq \max_{1 \leq i \leq d} \left\{ \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|x_{ij} - l, v_i\| \geq \frac{\varepsilon}{|\alpha_i|} \right\} \in I_2 \right| \right\} \end{aligned}$$

$$\text{Since, } I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, v_i\| = 0,$$

$$\text{We have } \lim_{m+n \rightarrow \infty} \frac{1}{mn} \left| \left\{ (i,j) \in N : i \leq m, j \leq n : \|x_{ij} - l, v_i\| \geq \varepsilon \right\} \in I_2 \right| = 0, \text{ for every } \varepsilon > 0.$$

Therefore,

$$\Rightarrow I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l, z\| = 0$$

Hence the theorem

We now define a norm on X with respect to the basis $v = \{v_1, v_2, v_3, \dots, v_d\}$ and is denoted by

$$\|x\|_{\alpha} = \max\{\|x, v_i\| : i = 1, 2, \dots, d\}.$$

Theorem 2.9

If I_2 is an admissible ideal then the double sequence $\{x_{mn}\}$ in X is I_2 -statistically convergent to l in X if and only

$$\text{If } I_2 - \text{stat.} \lim_{m+n \rightarrow \infty} \|x_{mn} - l\|_{\alpha} = 0.$$

Theorem 2.10

Any given I_2 -statistically Cauchy sequence $\{x_{mn}\}$ in X is I_2 -statistically convergent with respect to the norm $\|\cdot\|_{\alpha}$

If and only if it is I_1 -statistically convergent with respect to the norm $\|\cdot\|_\infty$, where $I_1\text{-stat.}\lim_{m+n\rightarrow\infty}\|x_{mn}-l\|_\infty=0$, being and admissible ideal.

Proof

From Theorem 3 and 4, we have,

$$I_1\text{-stat.}\lim_{m+n\rightarrow\infty}\|x_{mn}-l\|=0, u \text{ in } X \text{ if and only if } I_2\text{-stat.}\lim_{m+n\rightarrow\infty}\|x_{mn}-l\|_\infty=0.$$

Also we have,

$\{x_{mn}\}$ is an Ideal statistically Cauchy double sequence with respect to $\|\cdot\|$ if and only if

$$I_2\text{-stat.}\lim_{m+n\rightarrow\infty}\|x_{mn}-x_{pq}, v\|=0 \text{ For all } v \text{ in } X$$

$$\Leftrightarrow I_2\text{-stat.}\lim_{m\rightarrow\infty}\|x_{mn}-x_{pq}, v_i\|=0, i=1,2,\dots,d,$$

Where v is the basis

$$\Leftrightarrow I_2\text{-stat.}\lim_{m\rightarrow\infty}\|x_{mn}-x_{pq}, v_i\|_\infty=0.$$

Therefore, $\{x_{mn}\}$ is an I_2 -statistically Cauchy sequence with respect to $\|\cdot\|_\infty$.

Hence the theorem is proved.

REFERENCES

1. G. Bachman, *Introduction to p-adic numbers and valuation theory*, Academic Press, 1964.
2. P. Das, P. Kostyrko, W. Wilczynski, P. Malik, "I and I* Convergence of double sequences", *Math Slovaca*, Vol 58, No.5, 2008, pp.605-620
3. P. Das, P. Malik, "On extremal I-limit points of double sequences", *Tatra Mt. Math*, 40, 2008, pp. 91-102.
4. K. Demirci, "I-limit superior and I-limit inferior", *Math. Commun.*, 6, 2001, pp. 165-172.
5. E. Dundar, B. Altay, *On some properties of I2-convergence and I2-Cauchy of double sequences*, *Gen. Math. Notes*, 7(1) (2011), 1-12.
6. E. Dundar, B. Altay, *I2-convergence and I2-Cauchy of double sequences*, *Acta Mathematica Scientia*, 34B(2) (2014), 343-353.
7. Wagh, Mamta Amol, and Sanjeev Kumar. "Köthe Töeplitz Duals of Certain Bicomplex Sequence Spaces." *Inter. J. Math. & Comp. App. Research (IJMCAR)* 4.3 (2014).
8. M. Eshaghi Gorgy, F. AmoueiArani, S. "Statistical convergence of double sequences in paranormed spaces", *JMCS*, 2014, 47-53.
9. H. Fast, "Sur la Convergence statistique", *Colloq. Math.* 2, 1951, 241-244.
10. J. A. Fridy, "On Statistical Convergence", *Analysis*, 5, 1985, pp. 301-313.

11. Raad, S A, and M M Alatawi. "A Numerical Treatment of Two-Dimensional Quadratic Volterra Integral Equation of the Second Kind." *International Journal of Mathematics and Computer Applications Research (IJMCAR)* 9. 2, Dec 2019, 53–62
12. S.Gahler, "2- metrischeRaume und ihre topologische Struktur", *Math. Nachr.*, 26, 1963, pp. 115-148.
13. M. Gurdal, I.Acik, "On I – Cauchy Sequences in 2-normed spaces, *Math. Inequal. Appl.*, 11, no.2, 2008 ,pp 349-354
14. G.H. Hardy, *Divergent Series*, Oxford, 1949.
15. Powar, P. L., and G. R. K. Sahu. "Serendipity fixed point." *International Journal of Mathematics and Computer Applications Research (IJMCAR)* ISSN (2013): 2249-6955.
16. P. Kostyrko, T. Salat, W. Wilczyński, "I-Convergence", *Real Analysis Ex-change.*, 26(2), 2001, pp. 669-686.
17. Mursaleen, Osama, H. H. Edely, Statistical convergence of double sequences, *J. Math. Anal. Appl.*, 288, 2003, 223–231.
18. A. Nabiev, S. Pehlivan, M. Gurdal, "On I -cauchy sequences", *Taiwanese Journal of Mathematics*, 11(2), 2007, pp. 569-576.
19. S.Sarabadian, S.Talebi, Statistical convergence and Ideal convergence of sequences of functions in 2-normed spaces, *Inter. J. Math. Math. Sci.*, (2011), 10 pages.
20. Abed, Salwa Salman, and Rana Fadhil Abbas. "S-iteration for general quasi multi valued contraction mappings." *Int. J. Appl. Math. Stat. Sci.* 5 (4), 9 22 (2016).
21. S. Saraban, F.A. Arani, S. Khalehghli, "A condition for the equivalence of I and I^* convergence in 2-normed spaces, *Int.J. Comtemp. Math. Sci.* 43,no.6, 2011, pp.2147-2156.
22. A.Sahine, M. Gurdal, S.Saltan and H. Gunawan, " Ideal convergence in 2-normed spaces", *Taiwanese Journal of Mathematics* , 11(5), 2007, pp.1477-1484.
23. S. Sangeetha, V.Srinivasan, "I-Cauchy sequence and I-core of a sequence in Ultrametric Fields", *Int. J. of Pure and Applied Mathematics*, 113(10), 2017, pp.17-19.
24. K. Suja and V. Srinivasan, "On Statistically Convergent and Statistically Cauchy Sequences in Non-Archimedean Fields", *Journal of Advances in Mathematics*, 6(3), 2014, 1038–1043.
25. U. Yamanc and M. Gurdal, " I-Statistical convergence in 2-normed spaces", *Arab Journal of Mathematical Sciences*, 20(1), 2014, pp. 41-47.