FLUID FIELD TEMPERATURE OVER A STRETCHING SHEET WITH UNIFORM HEAT FLUX BY MODIFIED HOMOTOPY PERTURBATION TRANSFORM TECHNIQUE

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ABSTRACT

In this paper, A modified form of Homotopy perturbation transformed method is proposed to work out a pair of equations during the run of a tacky incompressible liquid caused by means of the stretching sheet. This study shows that a series solution is obtained for velocity and the temperature distribution profile of the fluid by the given methods. The projected procedure finds the result without any off-putting assumptions and avoids the work on errors. This MHPTM for the coupled equation is yet not available in the literature survey. Finally, we reveal how the software enables investigating velocity and heat flux.

KEYWORDS: *Boundary Layer; Heat Transfer; Stretching Sheet; Prandtl Number; Homotopy Perturbation Method*

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INTRODUCTION

Many researchers have paid more concentration on new techniques for solving boundary layer equations. The learning of laminar boundary stream of an incompressible watery has a numeral essential manufacturing submission such as smooth extrusion of synthetic sheets, the cooling of an endless tinny shield in a cooling bath, the boundary layer beside with runny pictures strengthening development, beaker and polymer industries. A joint shape of the Laplace transform scheme through a homotopy perturbation method is used in the direction of cracking the nonlinear equations [1].

The boundary layer run of a nanofluid precedent a stretching sheet has been analyzed numerically [2]. An analytic solution called the homotopy analysis method is applied for the MHD boundary layer stream of a sticky incompressible liquid in excess of an exponentially stretching sheet [3]. The scheme of third array kdv equations also the system of joined Burger's equations within single and double magnitudes are solved by the homotopy perturbation transform method [4]. Elzaki transform homotopy perturbation technique is worned in favor of the temperature ground of a liquid over a stretching sheet by means of uniform heat flux [5]. Incomplete Gamma function is applied for the temperature division in the stream of a thick incompressible fluid caused via the stretching sheet which issues from a cut addicted to the solution [6]. An elementary concept of the homotopy perturbation method is considered [7]. An estimated logical clarification of the magnetohydrodynamic (MHD) margin level current of an Upper-convected Maxwell (UCM) liquefied more than a porous stretching sheet is obtained using homotopy perturbation process [8]. Heat and mass transfer on a stretching sheet with suction or

blowing is analyzed [9]. The revision is executed for the MHD run owed to a linearly stretching sheet through an convinced magnetic field [10]. A hypothetical scrutiny of the laminar bound-layer flow and heat transport of power-law non-Newtonian fluids over a stretching sheet is investigated by a new procedure based on the homotopy analysis method [11]. Temperature pasture of a flowing in excess of a stretching sheet with uniform heat flux urbanized next to variational homotopy perturbation method [12]. The MHD incompressible glutinous fluid pour ended a non-linear stretching sheet among trip circumstance being studied by the process modified homotopy perturbation transform way [13]. A wide separation of analytical solution methods was used to knob boundary layer problems [14-25].

Brilliant and provoked by the constant examine within this region, we employ the modified homotopy perturbation method tied in the company of the Laplace transform for solving the coupled equations in this paper. The proposed technique supplied the solution in a quickly convergent series which may lead to the solution in a closed -form.

The advance of unidentified way is its ability to merge dua influential methods for obtaining accurate solutions for non-

Mathematical Model

Let us think the case of a smooth sheet supplying from a slim split on $x=0, y=0$, and then being extended, while in a polymer processing function. The course made by the stretching of this sheet is guessed to be laminar.

Figure 1: A Draft of Material Problem.

The equations of continuity, momentum, and heat transfer in the boundary layer estimations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1) \quad u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\nu}{\sigma} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

where u and v refer the fluid speed mechanism in the x and y directions respectively. σ stands for Prandtl number, ν is the kinematic viscosity.

The boundary settings take the form

$$u = \alpha x \text{ at } y = 0, v = 0 \text{ at } y = 0 \quad (4a)$$

$$-\lambda \frac{\partial T}{\partial y} = A' \text{ at } y = 0 \quad (4b)$$

$$u = 0 \text{ at } y \rightarrow \infty, T = T_{\infty} \text{ at } y \rightarrow \infty \quad (4c)$$

where α -thermal diffusivity, λ - thermal conductivity, T_∞ - ambient temperature and

A' (uniform surface heat flux).

Let us introduce the dimensionless variable f and g and the similarity variable η as

$$u = \alpha x f'(\eta), v = -(\alpha \nu)^{1/2} f(\eta) \quad (5a)$$

$$g = \frac{T - T_\infty}{T_w - T_\infty}, \psi = -(\alpha \nu)^{1/2} x f(\eta) \quad \text{and} \quad \eta = \left(\frac{\alpha}{\nu} \right)^{1/2} y \quad (5b)$$

where ψ is the stream function.

By taking the relations (4a) – (5b), the boundary layer equation for velocity and temperature fields can be obtained

$$f'''(\eta) = f'^2(\eta) - f(\eta)f''(\eta) \quad (6)$$

$$g''(\eta) + \sigma f(\eta)g'(\eta) = 0 \quad (7)$$

The boundary settings are

$$f(0) = 0, f'(0) = 1, f'(\infty) = 0, f''(0) = \alpha \quad (8)$$

$$g'(0) = -1, g(\infty) = 0, g(0) = 1 \quad (9)$$

Solution Procedure for MHPTM

Applying the MHPTM to the Eq.(6) and(7), we get,

$$L(f) = \frac{1}{s^2} + \frac{\alpha}{s^3} + \frac{1}{s^3} L\left(-ff'' + f'^2\right) \quad (10)$$

$$L(g) = 1 - \frac{1}{s^2} - \frac{\sigma}{s^2} L(fg') \quad (11)$$

By homotopy perturbation transform method

$$\sum_{m=0}^{\infty} p^m f_m(\eta) = \eta + \frac{\alpha \eta^2}{2} + p \left(L^{-1} \left(\frac{1}{s^3} L \left(\sum_{m=0}^{\infty} p^m H_{1m}(\eta) - \sum_{m=0}^{\infty} p^m H_{2m} \right) \right) \right) \quad (12)$$

$$\sum_{m=0}^{\infty} p^m g_m(\eta) = (1 - \eta) - p \sigma L^{-1} \left[\frac{1}{s^2} L \left(\sum_{m=0}^{\infty} p^m H_{3m}(\eta) \right) \right] \quad (13)$$

$$G(\eta) = \eta + \frac{\alpha\eta^2}{2} = f_0(\eta) + f_1(\eta) \quad (14)$$

as a result of this reflection, we primary set adapted recursive relation in the type,

$$p^0 : f_0(\eta) = \eta \quad (15)$$

$$\begin{aligned} p^1 : f_1(\eta) &= \frac{\alpha\eta^2}{2} + L^{-1}\left(\frac{1}{s^3}L[H_{10} - H_{20}]\right) \\ &= \frac{\alpha\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha\eta^4}{24} + \frac{\alpha^2\eta^5}{120} \end{aligned} \quad (16)$$

$$\begin{aligned} p^2 : f_2(\eta) &= L^{-1}\left(\frac{1}{s^3}L[H_{11} - H_{21}]\right) \\ &= -\frac{\alpha\eta^6}{720} - \frac{\alpha^2\eta^7}{5040} - \frac{\alpha^3\eta^8}{40320} \end{aligned} \quad (17)$$

$$f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + \dots \quad (18)$$

$$f(\eta) = \eta + \frac{\alpha\eta^2}{2} + \frac{\eta^3}{6} + \frac{\alpha\eta^4}{24} + \frac{\alpha^2\eta^5}{120} + \frac{\alpha\eta^6}{720} + \frac{\alpha^2\eta^7}{5040} + \frac{\alpha^3\eta^8}{40320} + \dots \quad (19)$$

$$p^{(0)} : g_0 = 1 - \eta \quad (20)$$

Using modified homotopy perturbation transform method

$$G_1(\eta) = 1 - \eta = g_0(\eta) + g_1(\eta) \quad (21)$$

$$p^0 : g_0(\eta) = 1 \quad (22)$$

$$p^{(1)} : g_1 = -\eta + \frac{\alpha\eta^3}{6} + \frac{\sigma\alpha\eta^4}{24} \quad (23) \quad p^{(2)} : g_2(\eta) = -\frac{\alpha^2\eta^6}{240} - \frac{\sigma^2\alpha\eta^7}{840} - \frac{\sigma^2\alpha\eta^7}{1260} \quad (24)$$

The series solution is given by

$$g(\eta) = \lim_{n \rightarrow \infty} g_n \quad (25)$$

$$g(\eta) = (1 - \eta) + \frac{\sigma\eta^3}{6} + \frac{\sigma\alpha\eta^4}{24} - \frac{\sigma^2\eta^6}{240} - \frac{\sigma\alpha^2\eta^7}{504} - \frac{\sigma^2\alpha^2\eta^8}{4032} + \frac{\sigma\alpha\eta^6}{720} + \frac{\sigma\alpha\eta^7}{5040} + \frac{\sigma\alpha^2\eta^8}{40320} \dots \quad (26)$$

The wall temperature is specified by

$$\begin{aligned}
 T_w - T_\infty &= \frac{A'}{\lambda} \left(\frac{\gamma}{\alpha} \right)^{1/2} g(0) \\
 &= \frac{A'}{\lambda} \left(\frac{\gamma}{\alpha} \right)^{1/2}
 \end{aligned}
 \tag{27}$$

Figure 2: Velocity Outline Designed for Positive Principles of α

Figure 3: Velocity Outline Designed for Negative Principles of α

Figure 4: Temperature Distribution with Diverse Ethics of α

Figure 5: Temperature Distribution with Diverse Ethics of σ

Table 1: Wall Temperature for Various Values of λ and α

$\lambda=0.6$		$\alpha=0.7$	
α	$T_w - T_\infty$	λ	$T_w - T_\infty$
0.2	1.1429	0.2	1.333
0.4	0.5714	0.4	0.6667
0.6	0.3810	0.6	0.4444
0.8	0.2857	0.8	0.3333

RESULTS AND DISCUSSIONS

Figures.2,3,4 and 5 have been made in to see the impact of thermal diffusivity and Prandtl number in velocity profile moreover temperature distribution. In Fig.2 velocity increases when thermal diffusivity increases. Fig.3 shows the increase in velocity when we considered the negative value of thermal diffusivity. But initially, the velocity profile is in steady - state. It means that fluid flow is not there. After some time we can realize the increase in velocity. In Fig.4 we can see the decreasing trend of the temperature profile with unusual values of α and the Prandtl number be assumed as one. Fig.5 exhibits the nature of the Prandtl number. To facilitate for the large value of Prandtl number temperature starting from a steady state and to decrease. Fluid temperature decreases owed to the increase in the thermal conductivity parameter.

Table 1 exhibits to facilitate the wall temperature reduces when α increases through the fixed value of λ . Also, wall temperature decreases with the increasing value λ with $\alpha=0.7$. It is seen that from table decreasing trend of wall temperature is more for various values of α and compared with different values of λ . From the table, it can be concluded that wall temperature is significantly high with the effect of thermal diffusivity.

CONCLUSIONS

In general, the outcomes show that the projected approach is steady as well as convergent. This method provides an easy way to control the convergence region. Edge layer breadth decreases because the fluid velocity rises, higher thermal diffusion means the heat will cover more area and the thermal boundary layer will be thicker. The consequence of expanding the values of the Prandtl number (Pr) is to enhance temperature mostly close to the stretching sheet and the thermal boundary layer width. The Computation of this method (MHPTM) depends on the suitable initial guess.

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