ELASTIC STRENGTH OF HIGH PRESSURE VESSELS WITH OFFSET CIRCULAR CROSS BORE

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ABSTRACT

The study sought to establish the optimal location of a small circular cross bore with increased safe working pressures, using Von Mises and Tresca elastic failure theories. Cross bored thick cylinders with closed ends having thickness ratio ranging from 3.0 down to 1.4 were studied. The investigation was carried out on a small circular cross bore at 9 different offset locations along the longitudinal X axis plane of the cylinder. These offset location ratios were between 0 and 0.9. A total of 63 part models were created and analysed using a three-dimensional finite element analysis software. It was found that the magnitude of the working stress that can be reclaimed from the lost proportion of the pressure carrying capacity when a cross bore is located optimally ranged from 8.61% to 19.51 % and 7.99% to 17.02 % for Von Mises's and Tresca's theories, respectively. Further, the magnitude of the optimal working stress concentration factor calculated using on Mises's theory occurred at $K = 1.75$ with a magnitude of 1.975. Whereas, the respective optimal magnitude obtained using Tresca's theory was 1.837 at $K = 2.5$.

KEYWORDS: Pressure Vessels, Cross Bore, Working Stresses & Stress Concentration Factor

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INTRODUCTION

Design codes based on either elastic or elastoplastic theories are used in the design of pressure vessels (Kihui, 2002). In ductile materials, failure of elastic theories formulae occurs when the materials yield. Premature yielding of ductile materials when left uncontrolled can lead to fatigue failure (Nziu and Masu, 2019a), thus the need to design against the occurrence of premature yielding. The commonly used elastic failure theories in the pressure vessel design are Tresca's and Von mises, with the latter, being used extensively in the development of pressure vessel design codes (Kihui, 2002). Notable advantage of designing pressure vessels using elastic theories is the ability to withstand large number of cyclic loads exceeding 10^5 times (Ford and Alexander, 1977). Other benefits are less costly manufacturing processes than those from close alternative techniques such as autofrettage and shakedown (Nziu and Masu 2019b). Therefore, its importance can't be over emphasized.

Pressure vessels are weakened by drilling of holes, also known as cross bores in the wall of the cylinder. Though inevitable, drilling of cross bores results in reduction of safe working pressures (Nziu and Masu, 2019c). For instance, a study by Cole (1969) on cross bored high pressure vessels reported a reduction of 40% and 25% for Tresca and Von mises working stresses, respectively.

Numerous studies on how to reduce the additional stress generated by the introduction of a cross bore have been conducted. Most of the studies have been carried out on geometric configuration of a cross bore such as size, shape, obliquity and location (Nziu and Masu, 2019d). However, studies on effective techniques of increasing safe

working pressures are scanty.

One of the key parameters used to analyse stresses in pressure vessels is the location of the cross bore. Thus, as a stress mitigation factor, cross bores should be located in areas where stresses are low in order to reclaim considerable proportion of the reduced pressure carrying capacity (Cole et al 1976). Therefore, this study seeks to determine the optimal location of a small circular cross bore that gives improved safe working pressures.

METHODOLOGY

Seven thick walled closed cylinders having a small circular cross bore were investigated. The actual cylinder thickness ratios (K) were 1.4, 1.5, 1.75, 2.0, 2.25, 2.5 and 3.0. For effective comparison of the results, both the size of the main bore and that of the cross bore were kept constant. The actual diameter of the main bore diameter was 0.05 m. whereas, the ratio of cross bore to the main bore was 0.1.

Nine different cross bore locations along the longitudinal X axis plane of the cylinder as indicated in Figure 1 were chosen. For easier comparison, the exact locations were converted to offset location ratios by dividing the actual offset distance $\bar{x}$, with the main bore radius R_i , i.e., $\bar{x}/R_i$. The actual offset locations were 0, 0.12, 0.24, 0.36, 0.48, 0.58, 0.685, 0.79 and 0.9.

Figure 1: Configuration of the Cross Bore Location.

Modelling

3D finite element analyses (FEA) were performed using Abaqus engineering software. Only an eighth of the structure of the part was modelled owing to the symmetrical nature of the cross bored pressure vessel. A total of 63 part models were created and analysed. A brief summary of the modelling procedure adopted in this work is presented in the proceeding headings.

Part Creation

A 3D deformable solid body was created by drawing an eighth profile of the pressure vessel face. The pressure vessel face was then extruded to form the depth of the cylinder. The cut revolve tool technique was used to create the cross bore on the deformable body. Figure 2 shows a sample of the part models created.

Figure 2: A Part Sample Model.

Boundary Conditions

Symmetrical boundary conditions were applied at the cut section of the cylinder, X, Y and Z axes, due to the symmetrical nature of the created deformable structure. In addition, displacement in the Z axis direction at the far end of the cylinder was constrained from moving in order to simulate the end closures effects.

Load

The cylinder was loaded with an internal pressure at both the main bore and the cross bore.

Meshing

The meshing of the model was done by dividing the part model into small regions. Second order tetrahedral elements having 10 sided nodes were used for meshing, as they are not easily prone to distortion. The density of the mesh was more biased in the region around the cross bore resulting to mesh converging. Figure 3 shows a sample of one of the mesh profiles.

Figure 3: A Sample of the Meshed Profile.

Validation of the Finite Element Analysis Model

Two techniques namely analytical and experimental were used to validate the results from the developed FEA model. The FEA results of the regions far away from the cross bore were compared with corresponding analytical results from the same areas. Further validation of the results was done by comparing the obtained FEA results with similar experimental and analytical results presented in the reviewed literature.

RESULTS AND DISCUSSIONS

Cross Bore Offset Effects on the Elastic Working Stresses

Graphs showing the variation of elastic working stress with cross bore location and the cylinder thickness ratios are presented in figures 4 to 7.

Figure 4: Von Mises's vs Cross Bore Location.

Figure 5: Tresca's vs Cross Bore Location.

Figure 6: Von Mises's vs Thickness Ratio.

Figure 7: Tresca's vs Thickness Ratio.

In general, similar patterns with slight differences in stress magnitudes were exhibited in both Von mises's and Tresca graphs, as indicated in figures 4 to 7. These occurrences were attributed to the computational method used to determine the working stresses. For instance, Von mises criterion takes into consideration all the three principal stresses. On the other hand, Tresca's criterion uses two principal stresses only.

As shown in figures 6 and 7, it was observed that the working stress were higher in the cross bored cylinder than those of the plain cylinder. This observation was attributed to the effects of the cross bore introduction. In addition, the

working stresses were seen to reduce with increase in the cylinder thickness ratio. This occurrence confirmed that the structural integrity of the cylinder increases with increasing thickness ratio.

The analyses showing the percentage amount that can be reclaimed from the lost pressure carrying capacity of the vessel when the cross bore is located in optimal location are tabulated in tables 1 and 2 for both stress criteria. It is worthy noting that the computation of these analyses were based on the highest and the lowest magnitudes of the working stresses for each thickness ratio.

Table 1: Optimal Cross Bore Locations and their Possible Amounts that can be Reclaimed from the Lost Pressure Carrying Capacity of the Vessel using Von Mises's Criterion

K	Optimal Cross Bore Location	Reclaimed Proportion (%)
1.4	0.12	19.5
1.5	0.12	16.66
1.75	0.79	14.58
2.0	0.9	13.28
2.25	0.685	8.614
2.5	0.9	14.48
3.0	0.12	13.27

Table 2: Optimal Cross Bore Locations and their Possible Amounts that can be Reclaimed from the lost Pressure Carrying Capacity of the Vessel using Tresca's Criterion

K	Optimal Cross Bore Location	Reclaimed Proportion (%)
1.4	0.12	17.02
1.5	0.79	14.17
1.75	0.79	12.45
2.0	0.9	11.56
2.25	0.685	7.99
2.5	0.9	13.23
3.0	0.9	9.993

From table 1, it can be seen that if the cross bore is located optimally a working stress magnitude ranging from 8.61 to 19.51 % can be reclaimed from the lost proportion of the pressure carrying capacity when using Von mises criterion. Whereas, for Tresca's theory, as indicated in table 2 only working stress magnitude ranging from 7.99% to 17.02% can be reclaimed.

It is important to note that reclamation of the lost pressure carrying capacity can only occur if the pressure vessel design is within elastic region. For instance, whenever, the cross bored pressure vessel is loaded with internal pressure, the stresses in the regions around the cross bore rise rapidly until they reach the yield point. These regions around the cross bore undergo yielding resulting to plastic flow that relieves the peak stresses. This technique of stress relieving only applies when the plastically deformed region is confined within a continuous elastic stress region. In the event that, the limit of the elastic working stress is exceeded, then the material may fail due to hydrostatic tensile stress generated in the cylinder rather than yielding.

Cross Bore Offset Effects on Elastic Working Stress Concentration Factor

Theoretical stress concentration factors (SCFs) arising from a geometric discontinuity, can be defined as the ratio of localised working stresses in a cross bored cylinder to the corresponding working stresses in a plain cylinder (Ford and Alexander, 1977). SCFs allow effective comparison of stresses among different parameters such as thickness, shape or size.

The variation of elastic working stress concentration factor with cross bore location and thickness ratio are illustrated in figures 8 to 11.

Figure 8: Von Mises's vs Cross Bore Location.

Figure 9: Tresca's vs Cross Bore Location.

Figure 10: Von Mises's vs Thickness Ratio.

Figure 11: Tresca's vs Thickness Ratio.

Despite the computation of the working stress concentration factor taking into consideration, the location of the maximum stresses, the lowest and the highest magnitudes occurred in the same identified locations in the proceeding section (see tables 1 and 2). The graph showing the lowest predicted working SCFs for each thickness ratio while using the two criteria, is shown in figure 12.

Figure 12: Optimal Working Stress Concentration Factors.

From figure 12, the optimal SCF magnitude occurred at $K = 1.75$ having a magnitude of 1.98 for Von Mises's theory. On the other hand, Tresca's theory gave an optimal magnitude of 1.84 in $K = 2.5$.

Further study by Ford and Alexander (1977), developed an analytical formula for calculating working stresses in cylinders with radial circular cross bore regardless of the thickness ratio. The developed formula was to predict working stresses in pressure vessels with radial cross bore having size ratio (cross bore to main bore ratio) not exceeding 0.1. Fortunately, the cross bore size ratio studied in this work was of the same ratio as that of Ford and Alexander (1977). Another similar study was carried out by Cole (1969) on working stresses in cross bored cylinders having radial circular cross bore of size ratio 0.2 using analytical method. Results comparing among these studies are tabulated in table 3.

Table 3: Comparison of Working Stress Concentration Factors for Radial Circular Cross Bore

	Cross Bore Size Ratio	Von Mises's Theory	Tresca's Theory
Ford and Alexander (1977)	0.1	2.89	2.50
Cole (1969)	0.2	2.88	2.50
Current study	0.1	2.22 – 2.60	1.98 – 2.25

The three results provided in table 3 were found to compare favourably, taking into consideration that two different methods were used. Though the two previous studies did not take into account the effects of the cylinder thickness ratio, unfortunately, these previous studies were done only in the radial position for the circular cross bore. Thus, hindering further working stress comparison in the offset positions.

CONCLUSIONS

- Working stress magnitudes ranging from 8.61% to 19.51 % and 7.99% to 17.02 % can be reclaimed from the lost proportion of the pressure carrying capacity when a cross bore is located optimally using Von Mises's and Tresca's theories, respectively.
- The magnitude of the optimal working stress concentration factor occurred at $K = 1.75$ with a magnitude of 1.98 for the Von Mises's criterion. On the other hand, computation using Tresca's criterion gave an optimal magnitude of 1.84 at $K = 2.5$.

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