COST OPTIMIZATION STUDY OF TWO-STEP HELICAL GEARBOXES WITH FIRST STAGE DOUBLE GEAR SETS

Dr. VU NGOC PI^{1*}, Dr. NGUYEN DINH NGOC², Dr. TRAN THI HONG³ & Dr. NGUYEN HONG LINH⁴

¹Associate Professor, Mechanical Engineering Faculty, Thai Nguyen University of Technology, Thai Nguyen City, Vietnam.

²Assistant Professor, Mechanical Engineering Faculty, Thai Nguyen University of Technology, Thai Nguyen City, Vietnam.

³Associate Professor, Center of Excellence for Automation and Precision Mechanical Engineering, Nguyen Tat Thanh University, Ho Chi Minh City, Vietnam.

⁴Assistant Professor, Mechanical and Automotive Engineering Faculty, Electric Power University, Hanoi City, Vietnam

ABSTRACT

One of the important goals of the optimized gearbox design is that the gearbox cost is minimized. However, so far, there has been no research about the design of a Two-step Helical Gearbox (THG) with First Stage Double Gearsets (FSDG) and optimum gear ratios to achieve the lowest cost. This paper presents the results of research on the influence of input parameters on the optimum partial gear ratios of the mentioned gearbox. To do that, a simulation experiment was designed and performed. Moreover, a regression formula for finding the optimum gear ratio was proposed. Evaluation results show that the formula is very suitable for the experimental data.

KEYWORDS: Helical Gearbox, Cost Optimization, Gear Ratio & Optimum Gearbox Design

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1. INTRODUCTION

Up to now, the optimal gearbox design has received much attention from researchers. Various studies were conducted in several different directions, such as determining the optimal parameters to reduce gearbox vibration [1, 2], to achieve the smallest gear mass [3-8], the smallest gearbox length [3, 8-13], or the smallest gearbox cross-section [3, 8, 14-20]. Besides, the optimal design of gearboxes has carried out with the mechanical system containing a V-belt [21-26] or a chain drive [27-30]. Studies have also been done with various types of gearboxes such as helical gearboxes [6, 11, 12, 15, 17, 27, 28, 31-33], bevel gearboxes [14, 22, 23, 32, 34-36] or worm gearboxes [30, 37-40]. Recently, a cost optimization study for a three-stage helical gearbox has been introduced [41]. However, so far there has been no research on optimization design for minimum cost of two-step helical gearboxes with first stage double gearsets.

This article presents the results of optimum design of a THG with FSDG. In particular, the influence of main design parameters on the optimum gear ratios has been analyzed by designing and performing a simulation experiment. In addition, the optimum gear ratios can easily be found by proposed regression models.

2. METHODOLOGY

It is the fact that the costs of bearings, gears, shafts, casing is found to have strong influence on the cost of a given gearbox. In this study, the cost of bearings will be ignored because of complex cost determination. Consequently, the cost of a THG with FSDG, C , can be determined by the following equations:

$$C = C_g + C_{gh} + C_s \quad (1)$$

In which, C_g , C_{gh} , and C_s are the cost of gears, gearbox housing and shafts, respectively. It should be noticed that the cost of a gear contains the cost of used materials, machining process, heat treatment, operators, etc. These costs construct the final price of a gear. In terms of commerce, the price of a gear can be usually determined by unit price per kilogram which regularly changes according to markets. In the current study, these cost elements are considered as variables and calculated by the following equations:

$$C_g = c_{g,m} \cdot m_g \quad (2)$$

$$C_{gh} = c_{gh,m} \cdot m_{gh} \quad (3)$$

$$C_s = c_{s,m} \cdot m_s \quad (4)$$

Where, $c_{g,m}$, $c_{gh,m}$, and $c_{s,m}$ are the cost of gears, gearbox housing, and shafts (USD/kg); m_g , m_{gh} , and m_s are the mass of gears, gearbox housing, and shafts (kg). In this work, $c_{g,m}$, $c_{gh,m}$, and $c_{s,m}$ are variables and m_g , m_{gh} , and m_s are defined as below:

$$m_g = \rho_1 \cdot \left[2 \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w11}^2 \cdot b_{w1}}{4} + \frac{\pi \cdot e_2 \cdot d_{w21}^2 \cdot b_{w1}}{4} \right) + \left(\frac{\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2}}{4} + \frac{\pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2}}{4} \right) \right] \quad (5)$$

$$m_{gh} = \rho_2 \cdot [2 \cdot L \cdot B_1 \cdot 1.5 \cdot S_G + 2 \cdot L \cdot H \cdot S_G + 2 \cdot B_2 \cdot H \cdot S_G] \quad (6)$$

$$m_s = \rho_3 \cdot \frac{\pi}{4} \cdot (d_1^2 \cdot l_1 + d_2^2 \cdot l_2 + d_3^2 \cdot l_3) \quad (7)$$

Wherein, ρ_1 , ρ_2 , and ρ_3 are the weight density of materials of gear, gearbox housing, and shafts, respectively; $\rho_1 = \rho_3 = 7.82$; $\rho_2 = 7.2$ [42]; $e_1 = 1$ and $e_2 = 0.6$; L , S_G , H , B_1 , and B_2 are the element dimensions of gearbox housing (Figure 1) which can be found by:

$$L = (d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 22.5)/0.975 \quad [4] \quad (8)$$

$$S_G = 0.005 \cdot L + 4.5 \quad [4] \quad (9)$$

$$H = \max(d_{w21}, d_{w22}) + 6.5 \cdot S_G \quad (10)$$

$$B_1 = 2 \cdot b_{w1} + b_{w2} + 5 \cdot S_G \quad (11)$$

$$B_2 = B_1 - 2 \cdot S_G \quad (12)$$

In addition, b_{w1} , b_{w2} are the gear widths; d_{w11} , d_{w21} , d_{w12} , and d_{w22} are the pitch diameters of the pinion and the gear of the first and second stages. These parameters can be calculated by below equations [43]:

$$b_{w1} = X_{ba1} \cdot a_{w1} \quad (13)$$

$$b_{w2} = X_{ba2} \cdot a_{w2} \quad (14)$$

$$d_{w11} = 2 \cdot a_{w1} / (u_1 + 1) \quad (15)$$

$$d_{w21} = 2 \cdot a_{w1} \cdot u_1 / (u_1 + 1) \quad (16)$$

$$d_{w12} = 2 \cdot a_{w2} / (u_2 + 1) \quad (17)$$

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2 / (u_2 + 1) \quad (18)$$

$$a_{w1} = 43 \cdot (u_1 + 1) \cdot \sqrt[3]{T_{11} \cdot k_{H\beta} / ([\sigma_{H1}]^2 \cdot u_1 \cdot X_{ba1})} \quad [43] \quad (19)$$

$$a_{w2} = 43 \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{H\beta} / ([\sigma_{H2}]^2 \cdot u_2 \cdot X_{ba2})} \quad [43] \quad (20)$$

$$d_1 = \sqrt[3]{T_{11} / (0.2 \cdot [\tau])} \quad [43] \quad (21)$$

$$d_2 = \sqrt[3]{T_{12} / (0.2 \cdot [\tau])} \quad [43] \quad (22)$$

$$d_3 = \sqrt[3]{T_{13} / (0.2 \cdot [\tau])} \quad [43] \quad (23)$$

$$l_1 = B_1 + 1.2 \cdot d_1 \quad (\text{Figure 1}) \quad (24)$$

$$l_2 = B_1 \quad (\text{Figure 1}) \quad (25)$$

$$l_3 = B_1 + 1.2 \cdot d_3 \quad (\text{Figure 1}) \quad (26)$$

Figure 1: Calculated Schema.

In the equations (19) and (20), $k_{H\beta}$ is the contacting load coefficient which can be chosen by 1.1 [43].

From the above analysis, for minimizing the reducer cost, the objective function of the optimization problem can be designated as follows:

$$\text{Minimize } C \quad (27)$$

With the following constraints:

$$1 \leq u_1 \leq 9$$

$$1 \leq u_2 \leq 9 \quad (28)$$

Nevertheless, it is known that $u_t = u_1 \cdot u_2$ is the relation between total gearbox ratio and partial ratios. Hence, the optimization of u_1 is sufficient, while the optimum ratio of u_2 can be obtained by the expression: $u_2 = u_t/u_1$.

3. EXPERIMENTAL WORK

Table 1: Main Design Factors

Parameter	Code	Unit	Low	High
Total gear ratio	u_t	-	10	50
Gear face ratio of first step	X_{ba1}	-	0.3	0.35
Wheel face ratio of second step	X_{ba2}	-	0.35	0.4
Allowable contact stress of first step	AS_1	MPa	350	420
Allowable contact stress of second step	AS_2	MPa	350	420
Output torque	T_{out}	Nmm	100	10000
Gearbox housing cost	C_{gh}	USD/kg	1	5
Gear cost	C_g	USD/kg	2	9
Shaft cost	C_s	USD/kg	1.5	5

Table 2: Runs of Experiment and Values of Response

Std Order	Run Order	Center Pt	Blocks	u_t	X_{ba1}	X_{ba2}	AS_1	AS_2	T_{out}	C_{gh}	C_g	C_s	u_1
117	1	1	1	5	0.3	0.4	350	420	10000	5	9	1.5	1.56
50	2	1	1	45	0.3	0.35	350	420	10000	1	2	1.5	6.66
51	3	1	1	5	0.35	0.35	350	420	10000	1	9	5	1.24
101	4	1	1	5	0.3	0.4	350	350	10000	5	9	5	1.42
31	5	1	1	5	0.35	0.4	420	420	100	1	2	5	1.00
11	6	1	1	5	0.35	0.35	420	350	100	1	9	5	1.40
...													
74	127	1	1	45	0.3	0.35	420	350	100	5	9	5	8.99
120	128	1	1	45	0.35	0.4	350	420	10000	5	2	5	5.10

To perform the optimization problem which is defined by Equations (27) and (28), a simulation experiment was designed and conducted. For the experiment, a 2-level factorial experiment with $1/4$ fraction was selected by Minitab@19 and a number of $2^{9-2} = 128$ tests were carried out. In this case, the Taguchi method which reduces the number of test is not used. The screening experiment used herein is not only a simple way to investigate the influence of nine factors (Table 1) on the response, but also propose the mathematical models instead of the Taguchi method. Table 2 shows various input parameters and the response (the optimum gear ratio u_1).

4. RESULTS AND DISCUSSIONS

4.1. Effect of Main Design Factors

Figure 2 describes the effect of main design factors on the optimum gear ratio of the first step u_1 . From this graph, it is clear that u_1 is greatly influenced by u_t . It increases sharply when u_t increases. Besides, u_1 has a positive relationship

with AS_1 , C_g , C_{gh} while it has an inverse relation with X_{ba1} , X_{ba2} , AS_2 , T_{out} and C_s .

Figure 2: Influences of Input Factors on u_1 .

Figure 3: Relation between Factor Interactions and u_1 .

The relationship between the interactions of main design factors and u_1 can be understood by Figure 3. Indeed, it can be seen that in term of interactions, C_g and C_s have strong influences on the response in both low and high values, such as HA ($u_t * C_g$), HB ($X_{ba1} * C_g$), HC ($X_{ba2} * C_g$), HD ($AS_1 * C_g$), HE ($AS_2 * C_g$), HF ($T_{out} * C_g$), HG ($C_{gh} * C_g$), JA ($u_t * C_s$), JB ($X_{ba1} * C_s$), JC ($X_{ba2} * C_s$), JD ($AS_1 * C_s$), JE ($AS_2 * C_s$), JF ($T_{out} * C_s$), JG ($C_{gh} * C_s$), JH ($C_g * C_s$). However, u_t has the strongest impact on u_1 , but having the minor effect on the it when u_t interact with other input parameters. These

are dominant when u_t varies from 5 to 45 like AB, AC, AD, AE, AF, and AG.

The Normal plot of Standardized Effects was presented in Figure 4. This graph not only describes the relationship of the main design parameters to u_1 , but it also indicates whether the relationship is positive or inverse. From the Plot, factors u_1 , C_g , C_{gh} , AS_1 and interactions AH, HJ, AD, AG, and FH have a positive effect. Besides, factors C_s , AS_2 , F and AJ, AE, FG, and AF are inverse on u_1 .

Figure 4: Normal Plot of the Standardized Effects.

4.2. Proposed Equation for Calculating u_1

To find a model to determine u_1 , Minitab @ 19 was used for a regression process with two interactive elements and the significance of $\alpha = 0.05$. After neglecting unimportant influence factors, the predictable coefficients for u_1 are presented in Table 3. It was noted that the P-values of main design factors and interactions are less than $\alpha = 0.05$. That means these factors are robust influence on u_1 . As a result, the proposed model for calculating u_1 is given as follow:

$$\begin{aligned}
 u_1 = & 0.96 + 0.218 \cdot u_t + 0.07 \cdot X_{ba1} + 0.000119 \cdot AS_1 - 0.000444 \cdot AS_2 + 10^{-7} \cdot T_{out} + 0.128 \cdot C_{gh} \\
 & - 0.0401 \cdot C_g - 0.191 \cdot C_s - 0.0973 \cdot u_t \cdot X_{ba1} + 0.000163 \cdot u_t \cdot AS_1 - 0.000248 \cdot u_t \cdot AS_2 \\
 & - 10^{-7} \cdot u_t \cdot T_{out} + 0.002205 \cdot u_t \cdot C_{gh} + 0.004012 \cdot u_t \cdot C_g - 0.008056 \cdot u_t \cdot C_s \\
 & - 0.000008 \cdot T_{out} \cdot C_{gh} + 0.000004 \cdot T_{out} \cdot C_g - 0.01189 \cdot C_{gh} \cdot C_g + 0.0243 \cdot C_g \cdot C_s
 \end{aligned} \quad (29)$$

It is found that the test data is very suitable with the proposed equation as all of the values of R-square are more than 98% (Table 3). Therefore, this regression equation is great to use to determine u_1 . After having u_1 , the optimal gear ratio u_2 can easily be found by $u_2 = u_t/u_1$.

Table 3: Coded Coefficients for Proposed Model

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		4.4532	0.0193	231.04	0.000	
ut	6.1430	3.0715	0.0193	159.35	0.000	1.00
Xba1	-0.1183	-0.0591	0.0193	-3.07	0.003	1.00
AS1	0.2933	0.1466	0.0193	7.61	0.000	1.00
AS2	-0.4648	-0.2324	0.0193	-12.06	0.000	1.00
Tout	-0.1702	-0.0851	0.0193	-4.41	0.000	1.00
Cgh	0.3042	0.1521	0.0193	7.89	0.000	1.00
Cg	0.8508	0.4254	0.0193	22.07	0.000	1.00
Cs	-0.9055	-0.4527	0.0193	-23.49	0.000	1.00
ut*Xba1	-0.0973	-0.0487	0.0193	-2.53	0.013	1.00
ut*AS1	0.2280	0.1140	0.0193	5.91	0.000	1.00
ut*AS2	-0.3470	-0.1735	0.0193	-9.00	0.000	1.00
ut*Tout	-0.0980	-0.0490	0.0193	-2.54	0.012	1.00
ut*Cgh	0.1764	0.0882	0.0193	4.58	0.000	1.00
ut*Cg	0.5617	0.2809	0.0193	14.57	0.000	1.00
ut*Cs	-0.5639	-0.2820	0.0193	-14.63	0.000	1.00
Tout*Cgh	-0.1636	-0.0818	0.0193	-4.24	0.000	1.00
Tout*Cg	0.1236	0.0618	0.0193	3.21	0.002	1.00
Cgh*Cg	-0.1664	-0.0832	0.0193	-4.32	0.000	1.00
Cg*Cs	0.2977	0.1488	0.0193	7.72	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.218068	99.61%	99.54%	99.45%

Table 4: ANOVA for u_1

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	19	1303.42	68.60	1442.60	0.000
Linear	8	1270.96	158.87	3340.85	0.000
ut	1	1207.55	1207.55	25393.56	0.000
Xba1	1	0.45	0.45	9.41	0.003
AS1	1	2.75	2.75	57.88	0.000
AS2	1	6.91	6.91	145.41	0.000
Tout	1	0.93	0.93	19.48	0.000
Cgh	1	2.96	2.96	62.28	0.000
Cg	1	23.16	23.16	487.08	0.000
Cs	1	26.24	26.24	551.71	0.000
2-Way Interactions	11	32.46	2.95	62.06	0.000
ut*Xba1	1	0.30	0.30	6.38	0.013
ut*AS1	1	1.66	1.66	34.97	0.000
ut*AS2	1	3.85	3.85	81.04	0.000
ut*Tout	1	0.31	0.31	6.46	0.012
ut*Cgh	1	1.00	1.00	20.94	0.000
ut*Cg	1	10.10	10.10	212.33	0.000
ut*Cs	1	10.18	10.18	213.98	0.000
Tout*Cgh	1	0.86	0.86	18.01	0.000
Tout*Cg	1	0.49	0.49	10.28	0.002
Cgh*Cg	1	0.89	0.89	18.63	0.000
Cg*Cs	1	2.84	2.84	59.62	0.000
Error	108	5.14	0.05		
Total	127	1308.55			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.218068	99.61%	99.54%	99.45%

4.3. Analysis of Variance

The Analysis Of Variance (ANOVA) is carried out and the results are revealed in Table 4 where the weak influences are eliminated. From the table, it is visualized that the parameters of A, D, E, G, H, J, AE, AH, AJ, and HJ exhibit the F-value superior to 50. These have the static importance as all of the values of R-square are higher than 99%.

4.4. Validating Regression Equation

The error valuation between the experimental data and the regression equation of u_1 is designated in Figure 5. It can be seen from Figure 5a that the error contribution is very similar to the normal distribution. Figure 5b shows the relation between the residual and model fitted values. It is found that they are arbitrary data. That means the observation order depends on the control parameters. Furthermore, the random connection between the residual and the observation data is also random.

Figure 5: Estimating Errors between Experiments and Regression Equation of u_1 .

5. CONCLUSIONS

The present work deals with cost optimization design of THG with FSDG. In this work, the effects of key design factors on the optimal gear ratios were evaluated by designing and performing a simulation experiment. In addition, optimal gear ratios can easily be found using the suggested regression equations. Also, several conclusions are given as follows:

- The effect of u_t on the optimal gear ratio of the first step u_1 is much more than that of other main design factors.
- The F-value of main design factors and their interactions of A, D, E, G, H, J, AE, AH, AJ, and HJ are significant for u_1 .
- The suggested regression equation for calculating u_1 is extremely fit with the data of the experiment (all of the values of R-square are higher than 99%).

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