EFFECTS OF EXPERIMENTAL PARAMETERS ON THE OUTPUT RESPONSES FOR THREE-STAGE BEVEL HELICAL GEARBOXES

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ABSTRACT

In gearbox design, selecting reasonable design parameters for gearboxes has been challenges since this affects the size, mass, and cost of gearboxes. This paper presents the evaluation of the effects of the experimental parameters on the output responses based on the gearbox mass of three-stage bevel helical gearboxes. The gearbox mass is determined by the main components of the gearbox such as gears, shafts, and gearbox housing. The output responses are obtained by minimizing the gearbox mass based on the condition of the moment equilibrium of a mechanical system. A screening experiment was carried out with eight experimental parameters. This experiment selected the parameters and the interactions that have significant influences on the output responses. By using the regression analysis, mathematical models of the responses were established and carefully evaluated. The results demonstrate that the proposed models are highly fit to the experiment data. For this, these models can be simply applied for calculating the transmission ratios of the three-stage helical gearbox with more economic efficiency.

KEYWORDS: Bevel Helical Gearboxes, Gearbox Mass & Output Responses.

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1. INTRODUCTION

Gearboxes are commonly used in many mechanical drive systems such as conveyor chain systems and conveyor systems. In a process of gearbox design, calculating gear ratios play an important role. Therefore there are several methods to compute gear ratios such as the graphical method, practical methods, and model methods. In the graphical method, gear ratios are determined by using graphs [1–5]. For the practical method, partial transmission ratios are commonly given in a table by the professional experiences from gearbox factories [6]. With the model method, mathematical models are commonly used for calculating gear ratios. These models are obtained from optimization problems such as minimum volume of gears [1], minimal gearbox mass [7,8], the minimum gearbox cross-section area [9–13], and the minimum gearbox length or height [14–18]. In regard to the study type of gearboxes, gear ratios have been considered for two-step gearboxes [12,15,17,19–21], three-step gearboxes [14,22–25] and four-step gearboxes [7,8,26].

Referring the three-stage gearbox, Vu and Nguyen [27] presented optimal gear ratios based on minimizing the cross-section dimension function for a three-step helical gearbox. The gear ratios of the three steps were shown as follows

$$u_1 = 0.8527 \cdot u_g^{0.5714}; u_2 = 1.0324 \cdot u_g^{0.2857}; u_3 = 1.136 \cdot u_g^{0.1429} \quad (1)$$

Recently, Ovidiu [28] used genetic algorithms for determining the gear ratios with minimizing the length of gearboxes

$$u_1 = 0.9126 \cdot u_g^{0.3731}; u_2 = 0.7414 \cdot u_g^{0.4188}; u_3 = 1.486 \cdot u_g^{0.2023} \quad (2)$$

In recent years, several researches proposed the determination of gear ratios of mechanical driven systems, e.g., a V-belt drive and a three-step bevel helical gearbox [23], a chain drive and a three-stage bevel helical gearbox [24], and a V-belt drive and a three-step helical gearbox [25]. In these studies, the objective functions of the cross-section and the height of the systems were used to obtain the optimal gear ratios.

This study deals with the influences of the experimental parameters on the optimum transmission ratios of the second and the third stages of a three-stage bevel helical gearbox based on the gearbox mass. A screening experiment was conducted in order to investigate the main effects of the experimental parameters and their interactions on the output responses. By using the regression and variance analysis, the models of the output responses were found.

2. GEARBOX MASS

Gearboxes are commonly used in many mechanical drive systems such as conveyor chain systems and conveyor systems. In a process of gearbox design, calculating gear ratios play an important role. Therefore there are several methods to compute gear ratios such as the graphical method, practical methods, and model methods. In the graphical method, gear ratios are determined by using graphs [1–5]. For the practical method, partial transmission ratios are commonly given in a table by the professional experiences from gearbox factories [6]. With the model method, mathematical models are commonly used for calculating gear ratios. These models are obtained from optimization problems such as minimum volume of gears [1], minimal gearbox mass [7,8], the minimum gearbox cross-section area [9–13], and the minimum gearbox length or height [14–18]. In regard to the study type of gearboxes, gear ratios have been considered for two-step gearboxes [12,15,17,19–21], three-step gearboxes [14,22–25] and four-step gearboxes [7,8,26].

$$m_g = m_{g\Box} + m_g + m_s \quad (3)$$

Where $m_{g\Box}$, m_g , and m_s are respectively the mass of gearbox housing, gears and shafts.

2.1. Gearbox Housing Mass

The mass of gearbox housing $m_{g\Box}$ can be computed as Eq. (4) in which $\rho_{g\Box}$ is representative for the weight density of gearbox housing material (kg/m³), with cast iron material $\rho_{g\Box} = 7,2$ [29], and $V_{g\Box}$ is the volume of the gearbox housing (m³).

$$m_{g\Box} = \rho_{g\Box} \cdot V_{g\Box} \quad (4)$$

The volume of the gearbox housing can be separated into three parts i.e., V_{A1} is the volume of front and back sides of the gearbox; V_{A2} is the volume of the left and right sides, and V_b is the volume of the top and bottom sides (see figure 1). Therefore, $V_{g\Box}$ can be expressed as

$$V_{g\Box} = 2 \cdot V_b + 2 \cdot V_{A1} + 2 \cdot V_{A2} \quad (5)$$

In the above equations, $c_{g,m}$, $c_{g\Box,m}$ and $c_{s,m}$ are the cost per kilogram of gears, gearbox house and shafts

(USD/kg); m_g , $m_{g\Box}$, and m_s are the mass of gears, gearbox house, and of all shafts (kg).

From the above equations, to calculate the gearbox cost C_{gb} , it is needed to determined m_g , $m_{g\Box}$ and m_s .

Figure 1: Schema for Determination of Gearbox Mass.

As seen in Figure 1, V_b , V_{A1} and V_{A2} can be calculated as

$$V_b = L \cdot B \cdot S_G \quad (6)$$

$$V_{A1} = L \cdot H \cdot S_G \quad (7)$$

$$V_{A2} = B_1 \cdot H \cdot S_G = (B - 2 \cdot S_G) \cdot H \cdot S_G \quad (8)$$

where L , H , B_1 and S_G can be determined by:

$$L = (2 \cdot l_0/3 + d_{e21}/2 + d_{w12}/2 + d_{w22}/2 + d_{w13}/2 + d_{w23} + 22.5)/0.975[30] \quad (9)$$

$$H = \max(d_{e21}; d_{w22}; d_{w23}) + 6.5 \cdot S_G \quad (10)$$

$$B = \max(b_{w2} + b \cdot \cos\delta_2; b_{w2} + b_{w3}) + 6 \cdot S_G \quad (11)$$

$$S_G = 0.005 \cdot L + 4.5[30] \quad (12)$$

Where, $l_0 = 3 \cdot d_{s1}[3]$ with d_{s1} is the diameter of the first shaft which will be determined in the next part; δ_2 is gear pitch cone angle; $\tan \delta_2 = u_1$.

2.2. Gear Mass

There are three pairs of gears, i.e., a pair of bevel gears, two pairs of helical gears in the second and third stages as shown in Figure 1. m_{bg} , $m_{\square g1}$, and $m_{\square g2}$ are respectively symbolized for the mass of the three pairs of gears (kg). Thus, the mass of gears in the gearbox can be written as

$$m_g = m_{bg} + m_{\square g2} + m_{\square g3} \quad (13)$$

The mass of a pair of bevel gears, m_{bg} , can be expressed as

$$m_{bg} = m_{bg1} + m_{bg2} \quad (14)$$

In which, m_{bg1} and m_{bg2} can be determined by

$$m_{bg1} = \rho_g \cdot \{(\pi/3) \cdot [d_{e11}^2 \cdot R_e \cdot \cos\delta_1/4 - d_{e11}^2 \cdot (R_e - b) \cdot \cos\delta_1/4]\} \quad (15)$$

$$m_{bg2} = \rho_g \cdot \{(\pi/3) \cdot [d_{e21}^2 \cdot R_e \cdot \cos\delta_2/4 - d_{e21}^2 \cdot (R_e - b) \cdot \cos\delta_2/4]\} \quad (16)$$

In Eq. (13), $m_{\square g2}$ and $m_{\square g3}$ are calculated as

$$m_{\square g2} = \rho_g \cdot (\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2}/4 + \pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2}/4) \quad (17)$$

$$m_{\square g3} = \rho_g \cdot (\pi \cdot e_1 \cdot d_{w13}^2 \cdot b_{w3}/4 + \pi \cdot e_2 \cdot d_{w23}^2 \cdot b_{w3}/4) \quad (18)$$

Where, ρ_g is the weight density of gear material (kg/m^3); with gear material is steel, $\rho_g = 7.82$ [29];

The parameters in Eqs. (15) and (16) can be presented as the following:

b is the face width of bevel gears, $b = k_{be}R_e$;

d_{e11} and d_{e21} are the outer pitch diameters of the pinion and the gear; with straight bevel gear set, they can be computed as [3]

$$d_{e11} = 2 \cdot R_e / (1 + u_1^2)^{1/2} \quad (19)$$

$$d_{e21} = 2 \cdot u_1 \cdot R_e / (1 + u_1^2)^{1/2} \quad (20)$$

R_e is the cone distance (mm) and it can be expressed by

$$R_e = k_R \cdot \sqrt{u_1^2 + 1} \cdot \sqrt[3]{T_{11} \cdot k_{\square\beta 1} / [(1 - k_{be}) \cdot k_{be} \cdot u_1 \cdot [AS_1]^2]} \quad (21)$$

For straight bevel gears made by steel materials, k_R is the face width coefficient and $k_R = 50$ [3]; $k_{H\beta 1}$ is representative for the contacting load ratio for pitting resistance and $k_{H\beta 1} = 1.11$; k_{be} is the face width coefficient; T_{11} called the torque on the pinion is calculated as

$$T_{11} = T_{out} / (u_t \cdot \eta_{bg} \cdot \eta_{\square g}^2 \cdot \eta_{be}^4) \quad (22)$$

Wherein, u_t is the total gearbox ratio; η_{bg} , $\eta_{\square g}$ and η_{be} are efficiencies, accordingly chosen in [3] and their values expressed in table 1

Table 1: Efficiency of Gearbox Components

Type of Efficiencies	Symbol	Unit	Value
Efficiency of bevel gear set	η_{bg}	-	0.96
Efficiency of helical gear set	$\eta_{\square g}$	-	0.97
Efficiency of a rolling bearing pair	η_{be}	-	0.992

In Eqs. (17) and (18), e_1 and e_2 are volume coefficients of the pinion and the gear of the helical gear set; e_1 and e_2 are the volume coefficients of the drive gear and the driven gear, respectively. In practice, these coefficients can be chosen as $e_1 = 1$ and $e_2 = 0.6$; b_{w2} and b_{w3} are the gear widths of the second and the third stages (mm); $b_{w2} = \psi_{ba2} \cdot a_{w2}$ and

$$b_{w3} = \psi_{ba3} \cdot a_{w3} \text{ (mm)}$$

The pitch diameters of the second and the third stage can be determined by:

$$d_{w12} = 2 \cdot a_{w2} / (u_2 + 1) \quad (23)$$

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2 / (u_2 + 1) \quad (24)$$

$$d_{w13} = 2 \cdot a_{w3} / (u_3 + 1) \quad (25)$$

$$d_{w23} = 2 \cdot a_{w3} \cdot u_3 / (u_3 + 1) \quad (26)$$

Where, a_{w2} and a_{w3} are the center distances of the second and the third which are calculated by [3]

$$a_{w2} = k_a \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{H\beta} / ([AS_2]^2 \cdot u_2 \cdot X_{ba2})} \quad (27)$$

$$a_{w3} = k_a \cdot (u_3 + 1) \cdot \sqrt[3]{T_{13} \cdot k_{H\beta} / ([AS_3]^2 \cdot u_3 \cdot X_{ba3})} \quad (28)$$

With

$$T_{12} = T_{out} / (u_2 \cdot u_3 \cdot \eta_{\square g}^2 \cdot \eta_{be}^3) \quad (29)$$

$$T_{13} = T_{out} / (u_3 \cdot \eta_{\square g} \cdot \eta_{be}^2) \quad (30)$$

In which, $k_{H\beta}$ is the contacting load ratio for pitting resistance, $k_{H\beta} = 1.08$ [3]; k_a is the material coefficient, with using steel materials for the helical gears, $k_a = 43$; $\eta_{\square g}$ and η_{be} are shown in Table 1; $[AS_2]$ and $[AS_3]$ are the allowable contact stress of the second and third stages (MPa); X_{ba2} and X_{ba3} are the coefficients of wheel face widths of the second and the third stages.

2.3. Shaft Mass

The mass of all shaft components denoted by m_s can be expressed as

$$m_s = m_{s1} + m_{s2} + m_{s3} + m_{s4} \quad (31)$$

Where, m_{s1} , m_{s2} , m_{s3} and m_{s4} are representative for the mass of the shaft 1, 2, 3, and 4 of the gearbox, respectively (see figure 1). Each shaft mass is normally computed based on multiplying the weight density of shaft material ρ_s and the volume of the shaft. Thus, m_{s1} , m_{s2} , m_{s3} and m_{s4} can be written as

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s1}^2 \cdot l_{s1} / 4 \quad (32)$$

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s2}^2 \cdot l_{s2} / 4 \quad (33)$$

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s3}^2 \cdot l_{s3} / 4 \quad (34)$$

$$m_{s4} = \rho_s \cdot \pi \cdot d_{s4}^2 \cdot l_{s4} / 4 \quad (35)$$

In here, l_{s1} , l_{s2} , l_{s3} and l_{s4} are respectively the length of shaft 1, 2, 3 and 4 of the gearbox. As shown in figure 1, the length of these shafts can be described by

$$l_{s1} = 1.3 \cdot l_0 + 1.2 \cdot d_{s1} \quad (36)$$

$$l_{s2} = B \quad (37)$$

$$l_{s3} = B \quad (38)$$

$$l_{s4} = B + 1.2 \cdot d_{s4} \quad (39)$$

In which, d_{s1} , d_{s2} , d_{s3} and d_{s4} are preliminary diameters of the shaft 1, 2, 3, and 4 respectively. They can be calculated as

$$d_{s1} = [T_{11}/(0.2 \cdot [\tau])]^{1/3} \quad (40)$$

$$d_{s2} = [T_{12}/(0.2 \cdot [\tau])]^{1/3} \quad (41)$$

$$d_{s3} = [T_{13}/(0.2 \cdot [\tau])]^{1/3} \quad (42)$$

$$d_{s4} = [T_{14}/(0.2 \cdot [\tau])]^{1/3} \quad (43)$$

In Eqs. (40)-(43), $[\tau]$ is the allowable shear stress (MPa) and can be chosen as 17 Mpa [3].

2.4. Optimization Problem

As the above procedure, the gearbox mass depending on the transmission ratios of the three stages i.e., u_1 , u_2 , and u_3 is considered as the objective function

$$\text{minimize } m_{gb} = m_{gb}(u_1, u_2, u_3) \quad (44)$$

subjected to

$$1 \leq u_1 \leq 6, 1 \leq u_2 \leq 9 \text{ and } 1 \leq u_3 \leq 9 \quad (45)$$

As known that the relation between the partial ratios u_1 , u_2 , and u_3 can be described as

$$u_g = u_1 \cdot u_2 \cdot u_3 \quad (46)$$

Where, u_g can be changed from the interval [20, 50]. Due to this, the objective function in Eq. (44), can be rewritten as

$$\text{minimize } m_{gb} = m_{gb}(u_2, u_3) \quad (47)$$

3. EXPERIMENTAL PLAN

Table 2: Design Experimental Parameters and their Level

Factor	Name in Minitab	Code	Unit	Low	High
Total gearbox ratio	A	u_g	-	20	50
Coefficient of the face width of bevel gear set	B	K_{be}	-	0.25	0.3
Coefficient of wheel face width of stage 2	C	X_{ba2}	-	0.3	0.4
Coefficient of wheel face width of stage 3	D	X_{ba3}	-	0.3	0.4
Allowable contact stress of stage 1	E	AS_1	Mpa	360	420
Allowable contact stress of stage 2	F	AS_2	Mpa	360	420
Allowable contact stress of stage 3	G	AS_3	Mpa	360	420
Output torque	H	T_{out}	Nmm	10^3	10^4

Table 3: Factorial Design of the Input Parameters and Output Responses

Test Runs No.	CenterPt	Blocks	u_g	K_{be}	X_{ba2}	X_{ba3}	AS_1	AS_2	AS_3	T_{out}	u_2	u_3
1	1	1	50	0.25	0.4	0.3	420	360	420	1000	7.11	6.52
2	1	1	50	0.25	0.3	0.3	360	360	360	10000	7.99	5.81
3	1	1	20	0.25	0.4	0.3	360	360	360	10000	3.78	4.90
4	1	1	50	0.3	0.4	0.3	360	420	420	10000	7.43	6.23
5	1	1	50	0.3	0.4	0.4	420	360	360	10000	7.69	6.02
6	1	1	20	0.3	0.4	0.3	420	360	360	10000	3.78	4.90
...	...	...										
126	1	1	50	0.3	0.3	0.3	360	420	420	1000	7.94	5.84

The optimum transmission ratios of the second and the third stages i.e., u_2 and u_3 are obtained by minimizing Eq. (47), and the first stage is also achieved from Eq. (46). As shown in the above procedure, the eight parameters including the total gearbox ratio u_g , the coefficient of the face width of the bevel gear set K_{be} , the coefficient of wheel face width of the second and the third stages, i.e., X_{ba2} and X_{ba3} , the allowable contact stress of the first, second and third stages, i.e., AS_1 , AS_2 and AS_3 and the output torque T_{out} were examined to exhibit their influences on the optimum ratios. The level of these parameters is presented in Table 2. Therefore, a screening experiment was conducted. Minitab®19 was used for the purpose of the experimental plan with the full factorial design 2^8 . Due to the available ability of Minitab®19, the 2-level factorial design with $1/2$ fraction was applied; thus, the number of experiments is 2^{8-1} , i.e., 128 test runs. The factorial design of the parameters and the output responses u_2 and u_3 are shown in Table 3.

4. ANALYSIS OF EXPERIMENTAL RESULTS

4.1. Evaluating the Influences of the Experimental Parameters and their Interactions

From the screening experiment as reported in section 3, the main effects of the experimental parameters on the responses are presented in figure 2. The slope angles counted by the first and the end points of these lines are arranged in descending i.e., 2,109765 (u_g), 0,303205 (AS_3), 0,194765 (AS_2), 0,099765 (X_{ba2}), 0,034435 (T_{out}), 0,01586 (X_{ba3}), 0,001955 (AS_1), 0,000575 (K_{be}) as shown in figure 2a. For this analysis, the effect's order of the parameters on the output response u_2 is respectively u_g , AS_3 , AS_2 , X_{ba2} , T_{out} , X_{ba3} , AS_1 , and K_{be} . For regarding the main effects on u_3 as seen in figure 2b, the slopes of the lines are similarly calculated as in figure 2a. The results show that the influences of the parameters on u_3 are orderly sorted from the strongest to the smallest effects, i.e., u_g , AS_3 , AS_2 , X_{ba2} , T_{out} , X_{ba3} , AS_1 , and K_{be} . It is observed that the total gearbox ratio has the largest significant effects on the output responses both u_2 and u_3 .

Figure 2: Diagram of the Effects of Experimental Parameters on (a) u_2 and (b) u_3 .

Figure 3 presents the normal plot of the standardized effects at the statistical significance $\alpha = 0.05$ in order to show more clearly the tendency of the effect the experimental parameters as well as their interactions on the output responses. From the observation of this figure, the single parameters and their interactions with square signs have the significant effects on the responses; otherwise, the circle signs have insignificant effects. As seen in figure 3a, the positive factors and their interactions including A (u_g), F(AS_2), AF ($u_g * AS_2$) and CF ($X_{ba2} * AS_2$) accelerate the response u_2 when they are changed from the low value to high value. Inversely, the negative factors and the interactions such as G (AS_3), AG ($u_g * AS_3$), C (X_{ba2}), H (T_{out}), AC ($u_g * X_{ba2}$), D (X_{ba3}), AH ($u_g * T_{out}$) and CG ($X_{ba2} * AS_3$) decrease the response u_2 . Concerning the influence on the response u_3 as shown in Figure 3b, the factors and the interactions i.e., A (u_g), C (X_{ba2}), H (T_{out}), AG ($u_g * AS_3$), CG ($X_{ba2} * AS_3$), D (X_{ba3}), AH ($u_g * T_{out}$), and CH ($X_{ba2} * T_{out}$) have positive influences on the response u_3 . Whereas, the negative factors and the interactions contain F (AS_2), AF ($u_g * AS_2$), FG ($AS_2 * AS_3$), CF ($X_{ba2} * AS_2$), FH ($AS_2 * T_{out}$), HD ($T_{out} * X_{ba3}$), and AD ($u_g * X_{ba3}$).

Figure 3: The Tendency of the Effects of the Experimental Parameters and their Interactions on (a) u_2 and (b) u_3 .

Figure 4 can more legibly show the influences of the interactions on the responses. As seen that the lines with the blue circle signs at the start and the end points are built by the combination of one factor with keeping constant value and other factors, for example, the blue line in the first row of the figure with $u_g = 20$ and the other factors in which their values are varied from the low to high values. Whereas the red lines with square signs are generated at the highest value of one factor, for instance, $u_g = 50$. The influences of the interactions are evaluated by the difference of slope angles between the blue line and the red line. It is observed from Figure 4a that the interactions e.g., $u_g * X_{ba2}$, $u_g * AS_2$, $u_g * AS_3$, $u_g * T_{out}$, $AS_2 * X_{ba2}$, $AS_2 * X_{ba3}$ have the significant effects on the response u_2 , and the interactions e.g., $u_g * K_{be}$, $X_{ba3} * K_{be}$, $AS_1 * K_{be}$, $AS_2 * K_{be}$, etc. do not have significant influences on u_2 . Similarly, in the case of the response u_3 (see Figure 4b), It can be seen that the interactions, such as $u_g * AS_2$, $u_g * \square S_3$, $u_g * T_{out}$, $u_g * X_{ba3}$, $X_{ba2} * AS_3$, $X_{ba2} * T_{out}$, $X_{ba2} * AS_2$, $AS_2 * AS_3$, $AS_2 * T_{out}$ have the significant influences on u_3 .

4.2. Analysis of Regression and Variance

The analysis of regression and variance is needed to exhibit the quantity of the effects of the parameters. Table 4 and table 5 present the effect detail of each parameter on the responses u_2 and u_3 . In these tables, eliminating all factors having in significant influences on the responses, the parameters and the interactions in which P-Values are smaller than the statistically significant value, i.e., $\alpha = 0.05$ bring the great effect on the output responses (see P-Value in table 4 and table 5). Additionally, the positive and negative values show more clearly the increasing and decreasing influences of the input parameters and the interactions as seen in the second column the tables.

Besides, the coefficients of the input parameters and their interactions are also depicted in the third column of these tables. In order to simply compute the optimum transmission ratio of the second and the third stages i.e., u_2 and u_3 , the regression equations can be expressed as

$$\begin{aligned} u_2 = & 1.874 + 0.17635 u_g + 0.24 X_{ba2} - 0.3172 X_{ba3} - 0.001055 AS_2 + 0.000867 AS_3 \\ & - 0.000008 T_{out} - 0.02677 u_g * X_{ba2} + 0.000158 u_g * AS_2 - 0.000222 u_g * AS_3 \\ & 0.000000 u_g * T_{out} + 0.00578 X_{ba2} * AS_2 - 0.00911 X_{ba2} * AS_3 \end{aligned} \quad (48)$$

$$\begin{aligned} u_3 = & -0.749 + 0.03817 u_g - 0.311 X_{ba2} + 0.709 X_{ba3} + 0.00781 AS_2 + 0.01126 AS_3 \\ & + 0.000007 T_{out} - 0.00743 u_g * X_{ba3} - 0.000052 u_g * AS_2 + 0.000056 u_g * AS_3 \\ & 0.00691 X_{ba2} * AS_2 + 0.01209 X_{ba2} * AS_3 + 0.000025 X_{ba2} * T_{out} - 0.000025 X_{ba3} * T_{out} \\ & 0.000023 AS_2 * AS_3 - 0.000000 AS_2 * T_{out} + 0.000000 AS_3 * T_{out} \end{aligned} \quad (49)$$

From the observation of the model summary as shown in table 4 and table 5, all the correlation coefficients are larger than 99.81%. Due to this, the proposed model in Eqs. (48) and (49) presents a good fit to the experimental data. Therefore, based on these models, the optimum transmission ratio of the second and the third stages (u_2, u_3) are simply obtained. According to Eq. (46) **Error! Reference source not found.**, the optimum transmission ratio of the first stage is also determined.

4.3. Evaluating the Reliability of the Regression Models

The regression models are more evaluated by the normal probability and versus order in order to validate the reliability of the models as shown in Eqs. (48) and (49). It is observed from the normal probability plot in Figure 5 that the residuals closely distribute the reference line. Moreover, in the versus order, the residuals also exhibit the random distribution both

for u_2 and u_3 . These results demonstrate that the regression models are consistent with the experiment data. Therefore, these models are highly reliable for computing the optimum transmission ratios.

Table 4: Quantitatively Evaluating the Effects of the Parameters on u_2

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		6.19820	0.00369	1680.21	0.000	
ug	4.21953	2.10977	0.00369	571.92	0.000	1.00
Xba2	-0.19953	-0.09977	0.00369	-27.04	0.000	1.00
Xba3	-0.03172	-0.01586	0.00369	-4.30	0.000	1.00
AS2	0.38953	0.19477	0.00369	52.80	0.000	1.00
AS3	-0.60641	-0.30320	0.00369	-82.19	0.000	1.00
Tout	-0.13734	-0.06867	0.00369	-18.62	0.000	1.00
ug*Xba2	-0.04016	-0.02008	0.00369	-5.44	0.000	1.00
ug*AS2	0.14203	0.07102	0.00369	19.25	0.000	1.00
ug*AS3	-0.20016	-0.10008	0.00369	-27.13	0.000	1.00
ug*Tout	-0.02797	-0.01398	0.00369	-3.79	0.000	1.00
Xba2*AS2	0.01734	0.00867	0.00369	2.35	0.020	1.00
Xba2*AS3	-0.02734	-0.01367	0.00369	-3.71	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0417357	99.97%	99.96%	99.96%

Table 5: Quantitatively Evaluating the Effects of the Parameters on u_3

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		5.42207	0.00236	2294.49	0.000	
ug	1.11873	0.55936	0.00236	236.71	0.000	1.00
Xba2	0.18478	0.09239	0.00236	39.10	0.000	1.00
Xba3	0.03100	0.01550	0.00236	6.56	0.000	1.00
AS2	-0.34142	-0.17071	0.00236	-72.24	0.000	1.00
AS3	0.53450	0.26725	0.00236	113.09	0.000	1.00
Tout	0.12665	0.06332	0.00236	26.80	0.000	1.00
ug*Xba3	-0.01115	-0.00557	0.00236	-2.36	0.020	1.00
ug*AS2	-0.04674	-0.02337	0.00236	-9.89	0.000	1.00
ug*AS3	0.05073	0.02536	0.00236	10.73	0.000	1.00
Xba2*AS2	-0.02074	-0.01037	0.00236	-4.39	0.000	1.00
Xba2*AS3	0.03628	0.01814	0.00236	7.68	0.000	1.00
Xba2*Tout	0.01140	0.00570	0.00236	2.41	0.018	1.00
Xba3*Tout	-0.01133	-0.00566	0.00236	-2.40	0.018	1.00
AS2*AS3	-0.04136	-0.02068	0.00236	-8.75	0.000	1.00
AS2*Tout	-0.01482	-0.00741	0.00236	-3.14	0.002	1.00
AS3*Tout	0.01983	0.00992	0.00236	4.20	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.0267353	99.86%	99.83%	99.81%

5. CONCLUSIONS

In this paper, a screening experiment was conducted based on the experimental parameters that are obtained from the procedure of the gearbox mass analysis of the three-stage bevel helical gearboxes. The optimal transmission ratios are achieved by minimizing the gearbox mass. The influences of the experimental parameters on the optimum transmission ratios were investigated. The experimental parameters comprise the total gearbox ratio u_g (A), the coefficient of the face width of the bevel gear set K_{be} (B), the coefficient of wheel face width of the second and the third stages X_{ba2} (C) and X_{ba3} (D), the allowable contact stress of the first, second and third stages AS_1 (E), AS_2 (F) and AS_3 (G) and the output torque T_{out} (H). The results of this paper can be summarized as follows:

- 2-level factorial with eight experimental parameters was used for the screening experiment that eliminates the insignificant influences on the output responses.
- The experimental parameters and their interactions, i.e., A, G, F, AG, C, AF, H, AC, D, AH, CG, and CF are the significant influences on u_2 ; the effect weight of the parameters and the interactions on u_3 are arranged from the large to small values, i.e., A, G, F, C, H, AG, AF, FG, CG, D, CF, GH, FH, CH, DH and AD.
- The reliability of the proposed models is carefully evaluated. Therefore, these models are highly applied for computing the optimum transmission ratios.

Figure 5: Normal Probability and Versus Order for (a) u_2 and (b) u_3 .

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