AN INVESTIGATION INTO FLUID FLOW, HEAT AND MASS TRANSFER IN A VERTICAL POROUS ANNULAR CYLINDER WITH OUTER WALL SUBJECTED TO NATURAL CONVECTION

MUKESH PATIL

Department of Mechanical Engineering, RNS Institute of Technology, Bengaluru- 560 098, India

ABSTRACT

Heat and mass transfer along with fluid flow in a vertical annular cylinder embedded with porous medium for the convective boundary condition of the outer wall is analysed in the present work. The fluid flow and heat transfer are assumed to take place by pure natural convection. The inner wall is subjected to uniform temperature. The species concentration is also assumed to be uniform at the inner wall. The fluid is assumed to obey Darcy's law. The governing partial differential equations are non-dimensionalised using suitable non-dimensional parameters and the solution is obtained by finite element analysis. The porous medium is divided using triangular elements of uneven size. MATLAB is used to solve the coupled energy, species and momentum equations in an iterative manner. The effects of Rayleigh number and Biot number on the fluid flow and heat transfer for different design parameters of the porous annulus are investigated. The design parameters considered for the present study are aspect ratio and radius ratio. The flow patterns and temperature distribution for the varying Rayleigh number and Biot number are also discussed in the present work. Also the effect of Lewis number on the mass transfer for the same design parameters, as mentioned above are investigated. The species concentration distribution for varying Lewis number is also discussed in the present work. The results reveal that the convection increases with the Rayleigh number. Convection also increases with the radius ratio. It is observed that the convection decreases with the increase in Aspect ratio. It is also observed that the convection increases with Biot number, however the increase in convection is not significant for higher values of Biot number. The mass transfer increases substantially with the increase in Lewis number.

KEYWORDS: Porous Medium, Vertical Annulus, Heat Transfer, Natural Convection, Radiation

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INTRODUCTION

In the last few decades porous medium has become an important area of research. Natural convection and heat transfer through porous medium have received enormous interest from the researchers. A comprehensive literature review on convection in a fluid-saturated porous medium may be found in the books by Nield and Bejan [1] and Ingham and Pop [2]. Nath and Satyamurty [3] have used finite difference method to investigate free convective heat transfer in an annular cylinder filled with porous medium. Prasad and Kulacki [4] have analyzed the natural convection in a vertical porous annulus. They concluded that the convective velocities are higher in the upper half than the lower half of the annulus and the local rate of heat transfer is much higher near the top edge of the cold wall. Prasad et al. [5] also have analyzed the natural convection in a vertical porous annulus with the inner surface maintained at constant heat flux. He concluded that the heat transfer rate is more as compared with the case of isothermal wall temperature. Rajamani et al. [6] have studied the natural convective heat transfer in an annular cylinder embedded with porous medium by assuming the thermal equilibrium model. They have made use of finite element method and discussed the effect of aspect ratio and radius ratio of the annulus on the heat transfer rate. The

problem of heat transfer in saturated porous vertical annulus with combined effect of radiation and convection has been studied by Irfan et al. [7]. Inspired by electric heating elements, Wang [8] has given a set of critical Rayleigh numbers for various size ratios of cylinder at which the convection heat transfer starts. Here the bottom surfaces of the cylinder were subjected to constant heat flux with the side walls being insulated. The problem of heat transfer in a vertical porous annulus requires attention as it has many practical applications such as gas cooled reactor vessels, bio-mass converters, insulated pipe lines, heat exchangers, geophysical applications, etc.

Yet, to the best knowledge of the authors there appears to be no study in the literature, concerned with the effect of convection (hT_∞) at the outer wall of a saturated vertical porous annular cylinder with uniform temperature ($\bar{T}_i$) and uniform concentration boundary conditions ($\bar{C}_i$) on the inner wall. The objective of the present work is to study the effect of Rayleigh number, Biot number on the fluid flow and heat transfer and also the effect of Lewis number on mass transfer for different design parameters of the vertical porous annular cylinder with natural convection at the outer wall, while maintaining uniform temperature and concentration boundary condition on the inner wall.

Nomenclatures

A	Area, m ²
Ar	Aspect ratio = $\frac{H}{L_{ref}}$
Avg.	Average
B	Buoyancy ratio
Bi	Biot number
C	Species concentration
$\bar{C}$	Non dimensional species concentration
C_i	Species Concentration at the inner wall
$\bar{C}_i$	Non dimensional species concentration at inner wall
C_o	Species Concentration at the outer wall
$\bar{C}_o$	Non dimensional species concentration at outer wall
C_∞	Ambient species concentration
C_p	Specific heat, J kg ⁻¹ K ⁻¹
D	Mass diffusivity
D_p	Particle diameter, m
g	Gravitational acceleration, ms ⁻¹
h	Heat transfer co-efficient, Wm ⁻² K ⁻¹
H	Height of the annulus, m
k	Thermal conductivity
K	Permeability of porous medium
Le	Lewis number
L_{ref}	Reference length, m
N	Shape function
$Nu.$	Nusselt number

$\overline{Nu}$	Average Nusselt number
r, z	Cylindrical co-ordinates
$\bar{r}, \bar{z}$	Non dimensional cylindrical co-ordinates
r_i	Inner radius, m
$\bar{r}_i$	Non dimensional inner radius = $\frac{r_i}{L_{ref}}$
r_o	Outer radius, m
$\bar{r}_o$	Non dimensional outer radius = $\frac{r_o}{L_{ref}}$
R	Radius ratio = $\frac{(r_o - r_i)}{r_i}$
Ra	Rayleigh number
Rd	Radiation parameter
T	Temperature, K
$\bar{T}$	Non dimensional temperature
T_i	Temperature at the inner wall, K
$\bar{T}_i$	Non dimensional temperature at inner wall
T_o	Temperature at the outer wall, K
$\bar{T}_o$	Non dimensional temperature at outer wall
T_∞	Ambient temperature, K
u	Velocity in 'r' direction, m/s
w	Velocity in 'z' direction, m/s
Greek symbols	
β	Co-efficient of thermal expansion, K^{-1}
β_C	Co-efficient of concentration expansion
β_T	Co-efficient of thermal expansion
β_R	Rosseland extinction co-efficient
ν	Kinematic viscosity, $m^2 s^{-1}$
ρ	Density, $kg m^{-3}$
κ	Thermal conductivity, $W m^{-1} K^{-1}$
Ψ	Stream function $W/m^2 K^4$
$\bar{\Psi}$	Non dimensional stream function
Φ	Porosity
μ	Dynamic viscosity, $m^2 s^{-1}$
σ	Stephan Boltzmann constant
Subscripts	
∞	Conditions at outer radius
w	Wall
i	Inner wall
o	Outer wall

2. MATHEMATICAL FORMULATION

In the present study, a saturated vertical porous annular cylinder of inner radius (r_i) and outer radius (r_o) is considered. Let the outer wall be subjected to convection as is the real case in practice. Let the inner wall be maintained at uniform temperature. Also, let the inner wall be subjected to uniform mass transfer. The two horizontal walls are insulated, thus making them adiabatic. Figure 1 shows the cross section of the vertical porous annulus for the problem being considered. The 'r' and 'z' co-ordinates indicate the radius and height of the vertical porous annulus respectively.

Figure 1 : Cross-Section and Co-ordinate System of the Vertical Porous Annular Cylinder

The following assumptions are made in the present study:

- Porous medium is saturated with fluid
- The porous medium is isotropic and homogeneous
- Fluid properties are constant except for the variation of density
- Darcy's law is assumed for the flow in the porous medium
- The radiation is assumed to be zero (pure natural convection) for the parametric studies, however it is considered to be present for the verification cases

With these assumptions, the governing equations can be written as:

$$\text{Continuity equation: } \frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \quad (1)$$

$$\text{Momentum equation: } \frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} = r \frac{gK}{\nu} \left(\beta_T \frac{\partial T}{\partial r} + \beta_C \frac{\partial C}{\partial r} \right) \quad (2)$$

$$\text{Where } K = \frac{(D_p^2 \Phi^3)}{180((1-\Phi)^2)} \quad (3)$$

The variation of density with respect to temperature can be described by Boussinesq approximation as:

$$\rho = \rho_\infty [1 - \beta (T - T_\infty)] \quad (4)$$

Energy equation is written as follows:

$$\rho C_p \left(u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) = \left[k \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right\} - \frac{1}{r} \frac{\partial}{\partial r} (r q_r) \right] \quad (5)$$

Invoking Rosseland approximation for radiation: [9, 10]

$$q_r = \frac{4n^2 \sigma}{3\beta_R} \frac{\partial T^4}{\partial r} \quad (6)$$

Concentration Equation is written as follows:

$$u \frac{\partial C}{\partial r} + w \frac{\partial C}{\partial z} = D \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) + \frac{\partial^2 C}{\partial z^2} \right\} \quad (7)$$

The continuity equation can be solved by introducing the stream function (Ψ) as follows:

$$u = -\frac{1}{r} \frac{\partial \Psi}{\partial z} \quad (8)$$

$$w = \frac{1}{r} \frac{\partial \Psi}{\partial r} \quad (9)$$

The boundary conditions used are as follows:

$$\text{at } r = r_i ; T = T_i ; C = C_i ; \Psi = 0 \quad (10a)$$

$$\text{at } r = r_o ; \left[-K \frac{\partial T}{\partial r} \right]_{r_i} = h(T_o - T_\infty) ; C = C_o ; \Psi = 0 \quad (10b)$$

The following non-dimensional parameters have been used:

$$\bar{r} = \frac{r}{L_{ref}} ; \bar{z} = \frac{z}{L_{ref}} ; \bar{\Psi} = \frac{\Psi}{\alpha L_{ref}} ; \bar{T} = \frac{(T - T_\infty)}{(T_w - T_\infty)} \quad (11a)$$

$$\bar{C} = \frac{(C - C_\infty)}{(C_w - C_\infty)} ; Rd = \frac{4\sigma T_\infty^3}{\beta_R \kappa_s} ; Ra = \frac{(g \beta_T \Delta T K L_{ref})}{\nu \alpha} \quad (11b)$$

Thermal diffusivity (α) belongs to effective porous medium and is defined as:

$$\alpha = \frac{k}{\rho C_p} \quad (12)$$

The non-dimensional parameters from the Eqs. 11 (a) and (b) are substituted in Eqs. (2), (5) and (7). The non-dimensionalised form of the equations Eqs. (2), (5) and (7) are as follows:

Non-dimensionalised momentum equation is written as follows:

$$\frac{\partial^2 \Psi}{\partial \bar{r}^2} + \bar{r} \frac{\partial}{\partial \bar{r}} \left(\frac{1}{\bar{r}} \frac{\partial \Psi}{\partial \bar{r}} \right) = \bar{r} Ra \left[\frac{\partial \bar{T}}{\partial \bar{r}} + N \frac{\partial \bar{C}}{\partial \bar{r}} \right] \quad (13)$$

Non-dimensionalised energy equation is written as follows:

$$\frac{1}{\bar{r}} \left[\frac{\partial \bar{\psi}}{\partial \bar{r}} \frac{\partial \bar{T}}{\partial \bar{z}} - \frac{\partial \bar{\psi}}{\partial \bar{z}} \frac{\partial \bar{T}}{\partial \bar{r}} \right] = \left(\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\left(1 + \frac{4Rd}{3} \right) \bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} \right) \quad (14)$$

Non-dimensionalised concentration equation is written as follows:

$$\frac{1}{\bar{r}} \left[\frac{\partial \bar{\psi}}{\partial \bar{r}} \frac{\partial \bar{C}}{\partial \bar{z}} - \frac{\partial \bar{\psi}}{\partial \bar{z}} \frac{\partial \bar{C}}{\partial \bar{r}} \right] = \frac{1}{Le} \left(\frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left(\bar{r} \frac{\partial \bar{C}}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2} \right) \quad (15)$$

For the problem under investigation, the boundary conditions can be defined as follows:

a) At the inner wall of the annulus:

$$\text{at } \mathbf{r} = \mathbf{r}_i: \bar{T} = \bar{T}_i = 1; \bar{C} = \bar{C}_i = 1; \bar{\psi} = 0 \quad (16a)$$

b) At the outer wall of the annulus:

$$\text{at } \mathbf{r} = \mathbf{r}_o: \left[-K \frac{\partial \bar{T}}{\partial \bar{r}} \right]_{r_i} = h (T_o - T_\infty); \bar{C} = \bar{C}_o = 0; \bar{\psi} = 0 \quad (16b)$$

Equations (13), (14) and (15) are the coupled equations. They are solved by finite element method to predict the fluid flow and heat transfer. A simple three noded triangular element is considered. $\bar{T}$, $\bar{\psi}$ and $\bar{C}$ varying inside the element can be expressed as:

$$\bar{T} = N_1 \bar{T}_1 + N_2 \bar{T}_2 + N_3 \bar{T}_3 \quad (17)$$

$$\bar{\psi} = N_1 \bar{\psi}_1 + N_2 \bar{\psi}_2 + N_3 \bar{\psi}_3 \quad (18)$$

$$\bar{C} = N_1 \bar{C}_1 + N_2 \bar{C}_2 + N_3 \bar{C}_3 \quad (19)$$

Where, N_1 , N_2 and N_3 are the shape functions expressed in general as:

$$N_m = \frac{a_m + b_m x + c_m y}{2A}, \text{ where } m = 1, 2, 3. \quad (20)$$

Details of finite element formulation can be obtained from [11, 12]. The coupled matrix equations are obtained after applying Galerkin's method to Eqs. (13) - (15). The matrix equations are assembled to get the global matrix for the whole domain, which is solved iteratively to obtain $\bar{T}$, $\bar{\psi}$ and $\bar{C}$ in the porous medium. In order to get accurate results, the tolerance level of the solutions for $\bar{T}$, $\bar{\psi}$ and $\bar{C}$ is set at 10^{-5} , 10^{-7} and 10^{-9} respectively. Element size in the domain is varied by having large number of elements located near the wall, where large variations in $\bar{T}$, $\bar{\psi}$ and $\bar{C}$ are expected. A sufficiently dense mesh is chosen to make the solution invariant. In the present study a mesh size consisting of 1800 elements and 961 nodes is used to solve the governing equations.

3. RESULTS AND DISCUSSION

In the present study, the hot wall is subjected to a uniform temperature boundary condition ($\bar{T}_o = 1$). The cold wall of the annular cylinder is subjected to natural convection. Results are obtained in terms of average Nusselt number ($\overline{Nu}$) at the inner wall and also average Sherwood number ($\overline{Sh}$) at the outer wall of the cylinder.

Average Nusselt number at the inner wall is given by:

$$\overline{Nu}_{in} = - \frac{\int_0^{\bar{z}} \left[\left(1 + \frac{4Rd}{3} \right) \frac{\partial T}{\partial \bar{r}} \right]_{\bar{r}=\bar{r}_i} d\bar{z}}{\bar{z}} \quad (21)$$

Average Sherwood number at the outer wall is given by:

$$\bar{Sh}_{out} = - \frac{\int_0^{\bar{z}} \left[\frac{\partial C}{\partial \bar{r}} \right]_{\bar{r}=r_0} d\bar{z}}{\bar{z}} \quad (22)$$

The temperature gradient $\left(\frac{\partial T}{\partial \bar{r}} \right)$ in Eq. (21) is evaluated using a 4-point polynomial along the nodes near the inner wall of the vertical porous annular cylinder. The concentration gradient $\left(\frac{\partial C}{\partial \bar{r}} \right)$ in Eq. (22) is evaluated using a 4-point polynomial along the nodes near the outer wall of the vertical porous annular cylinder.

3.1. Verification of the present methodology with available literature

The methodology used in the present study is validated for two cases. The variation of average Nusselt number with the aspect ratio, as is available in the literature is validated with present methodology in the first case. The second case validates the present methodology with the results related to variation of Nusselt number against the radius ratio as is available in the literature.

3.1.1. Variation of $\overline{Nu}$ with Aspect ratio (Ar)

In order to verify the accuracy of the present methodology, it is compared with the results available in the literature. One such comparison is made with the variation of $\overline{Nu}$ for different values of Ar (3, 5 and 8). It is evident from Table 1 that the present method compares well in predicting the heat transfer behaviour of the vertical porous annular cylinder as compared with that available in the literature [3, 4 and 6].

Table 1 : Validation of the Present Methodology – Comparison of average Nusselt Number with Aspect Ratio

Aspect Ratio (Ar)	Average Nusselt Number ($\overline{Nu}$)			
	Nath and Satyamurty [3]	Prasad and Kulacki [4]	Rajamani <i>et al.</i> [6]	Present
3	3.81	3.70	3.868	3.835
5	3.03	3.00	3.025	3.034
8	2.45	2.35	2.403	2.486

3.1.2. Variation of $\overline{Nu}$ with Radius Ratio (R)

To verify further, the accuracy of the present method of numerical analysis using Finite Element Method, $\overline{Nu}$ for different values of R is compared with that available in the literature. The comparison is shown in Figs. 2 and 3, at Ra =100 for three different values of Rd (0, 0.5 and 1). It is evident from these figures that the present method has good accuracy in predicting the heat transfer behaviour of the vertical porous annular cylinder as compared with that available in the literature [7].

Figure 2: Variation of $\overline{Nu}$ with $1/R$ for different values of Radiation Parameter for Hot Wall Condition

Figure 3: Variation of $\overline{Nu}$ with $1/R$ for different values of Radiation Parameter for Cold Wall Condition

3.2. Parametric study: Variation of fluid flow, heat and mass transfer for different design parameters

In the present study, the effects of Biot number and Rayleigh number on fluid flow and heat transfer for different design parameters are analyzed. Also the effect of Lewis number on mass transfer for different design parameters is investigated. The design parameters of the porous annular cylinder considered in the present study are radius ratio and aspect ratio. The range of the above mentioned numbers and parameters considered for the study are stated as follows:

- Rayleigh number: 100, 500 and 1000
- Biot number: 1 to 500
- Lewis number: 1, 5 and 10
- Aspect ratio : 1 to 10
- Radius ratio: 1 to 10

3.2.1. Effect of Biot number on fluid flow and heat transfer for different radius ratio and aspect ratio

Figure 4 shows the variation of average Nusselt number ($\overline{Nu}$) with radius ratio for four different Biot numbers (1, 10, 100 and 500). It is observed from the figure that there is a substantial increase in $\overline{Nu}$ for the variation of Bi from 1 to 10. However the increase in the value of $\overline{Nu}$ is not substantial for the variation of Bi from 100 to 500 and is not significant for any higher value of Bi beyond 500. However there is a significant increase in $\overline{Nu}$ with the increase in radius ratio for different Biot numbers considered in the present study. The increase in Biot number has a considerable effect on the fluid flow and heat transfer for Biot number up to 100 and has no significant effect beyond 500.

Figure 4: Variation of Average Nusselt Number with Radius Ratio for different values of Biot Number

Figure 5 shows the variation of average Nusselt number ($\overline{Nu}$) with aspect ratio for four different Biot numbers (1, 10, 100 and 500). It is observed from the figure that there is a substantial increase in $\overline{Nu}$ for the variation of Bi from 1 to 10. However the increase in the value of $\overline{Nu}$ is not substantial for the variation of Bi from 100 to 500 and is not significant for any higher value of Bi beyond 500. However there is a significant reduction in $\overline{Nu}$ with the increase in aspect ratio for different Biot numbers considered in the present study. The increase in Biot number has a considerable effect on the fluid flow and heat transfer for Biot number up to 100 and has no significant effect beyond Bi=500. However $\overline{Nu}$ is almost constant for the Bi=1.

Figure 5: Variation of Average Nusselt Number with Aspect Ratio for different values of Biot Number

Figure 6 shows the stream lines for two different values of Biot number (1 and 500). Similarly Fig. 7. shows the isothermal lines for the Biot numbers as mentioned above. It is observed from Fig. 6. that, at lower Biot number the streamlines appear to be nearly circular, whereas they get concentrated at outer wall at higher Biot number., as a result of enhanced convection. Similarly the isotherms show the increase in temperature with the increase in Biot number as it is observed from Fig. 7. The isotherms are sparse at Bi=1, whereas they are concentrated at Bi=500. It is observed from Fig. 7. that, the fluid flow increases as the Biot number increases from 1 to 500.

3.2.2. Effect of Rayleigh number on fluid flow and heat transfer for different radius ratio and aspect ratio

Figure 8 shows the variation of average Nusselt number ($\overline{Nu}$) with radius ratio for different values of Rayleigh number (100, 500 and 1000). It is observed that $\overline{Nu}$ increases with the Rayleigh number. It is also observed that $\overline{Nu}$ increases with the radius ratio for different values of Rayleigh numbers considered in the present study. Increase in Rayleigh number enhances the fluid movement, which in turn is due to the increase in buoyancy effect. Hence the fluid velocity is increased along with the Nusselt number.

Figure 8: Variation of Average Nusselt Number with Radius Ratio for different values of Rayleigh Number

Figure 9 shows the variation of average Nusselt number ($\overline{Nu}$) with aspect ratio for different values of Rayleigh number (100, 500 and 1000). It is observed that $\overline{Nu}$ increases with the Rayleigh number. However it is observed that $\overline{Nu}$ decreases with the increase in aspect ratio for different values of Rayleigh numbers considered in the present study. Increase in Rayleigh number enhances the fluid movement, which in turn is due to the increase in buoyancy effect.

Figure 9: Variation of Average Nusselt Number with Aspect Ratio for different values of Rayleigh Number

Figures 10 and 11 show the stream lines and isothermal lines respectively for the Rayleigh numbers as mentioned above. It is observed from Fig. 10. that the fluid flow also increases as the Rayleigh number increases from 100 to 1000. At lower Rayleigh number the streamlines appear to be sparse, whereas they get concentrated at outer wall at higher Rayleigh number. Similarly the isotherms show the increase in temperature with the increase in Rayleigh number as it is observed from Fig. 11. The isotherms are sparse at Ra=100, whereas they are concentrated at Ra=1000 as it is observed from Fig. 11.

(a) Stream Lines at Ra=100 (b) Stream Lines at Ra=1000
Figure 10: Stream Lines for different Rayleigh Numbers

(a) Isothermal Lines at Ra=100 (b) Isothermal Lines at Ra=1000
Figure 11 : Iso-Thermal Lines for different Rayleigh Numbers

3.2.3. Effect of Lewis number on mass transfer for different radius ratio and aspect ratio

Figure 12 shows the variation of average Sherwood number ($\bar{Sh}$) with radius ratio for different values of Lewis number (1, 5 and 10). It is observed that $\bar{Sh}$ increases with the Lewis number. It is also observed that $\bar{Sh}$ increases with the radius ratio for different values of Lewis numbers considered in the present study. Increase in Lewis number enhances the fluid movement, which in turn is due to the increase in buoyancy effect.

Figure 12: Variation of Average Sherwood Number with Radius Ratio for different values of Lewis Number

Figure 13 shows the variation of average Sherwood number ($\bar{Sh}$) with aspect ratio for different values of Lewis number (1, 5 and 10). It is observed that $\bar{Sh}$ increases with the Lewis number. However it is observed that $\bar{Sh}$ decreases with the increase in aspect ratio for different values of Lewis numbers considered in the present study.

Figure 13: Variation of Average Sherwood Number with Aspect Ratio for different values of Lewis Number

Figure 14 shows the stream lines and isothermal lines respectively for two different Lewis numbers (1 and 10). It is observed from Fig. 14. that the mass transfer increases as the Lewis number increases from 1 to 10. At lower Lewis number the streamlines appear to be sparse, whereas they get concentrated at outer wall at higher Lewis number. Similarly the isotherms show the increase in concentration with the increase in Lewis number as it is observed from Fig. 14. The isotherms are sparse at $Ra=100$, whereas they are concentrated at $Ra=1000$.

(a) Iso-Concentration Lines at Le=1 (b) Iso-Concentration Lines at Le=10
Figure 14: Iso-Concentration Lines for different Lewis Numbers

4. CONCLUSIONS

The fluid flow, heat transfer and mass transfer in a saturated vertical porous annular cylinder for convective boundary condition of the outer wall is considered in the present study. The inner wall temperature corresponds to a non dimensional temperature of $\bar{T}_0=1$. The concentration at the inner wall also is maintained uniform ($\bar{C}_i=1$). The governing equations are non-dimensionalised and solved using finite element method. The analysis is made for the variation of average Nusselt number ($\bar{Nu}$) with Biot number and Rayleigh number. The results are also analyzed for the variation of average Sherwood number ($\bar{Sh}$) with Lewis number.

It is observed from the results that the Nusselt number (fluid flow and heat transfer) increases with the Biot number. This increase in Nusselt number is significant for the range of Biot numbers between 1 to 100. However the increase in Nusselt number beyond the $Bi=100$ is not significant. It is also observed that the Nusselt number increases with the Rayleigh number. The effect of convection is more pronounced for higher Rayleigh numbers. Further it is observed that the Nusselt number increases with the radius ratio (width of the annular cylinder) for the given Biot numbers and Rayleigh numbers. However the Nusselt number decreases with the increase in aspect ratio.

It is further observed from the results that the Sherwood number (mass transfer) increases with the Lewis number. The effect of concentration is more pronounced for higher Lewis numbers. Further it is observed that the Sherwood number increases with the radius ratio (width of the annular cylinder) for the given Lewis numbers. However the Sherwood number decreases with the increase in aspect ratio.

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