MATHEMATICAL MODELS OF THE OPTIMAL DISTRIBUTION OF WATER IN THE CHANNELS OF IRRIGATION SYSTEMS

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ABSTRACT

In the article the problem of optimal water distribution in the canals of irrigation systems is solved in the case of using as a mathematical model of un steady water movement in the canal sections the system of full differential equations of Saint-Venant, and also numerical decisions on a method of a finite difference of systems of differential equations of Saint-Venant for system of automatic regulation of a water level in a channel site are resulted.

KEYWORDS: *Optimal Management, Methods, Channel, Water Distribution, Necessary Conditions of Optimality*

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1. INTRODUCTION

Optimal water distribution in irrigation canals requires the use of modern systems of optimal water distribution, which in combination with the means of mathematics, physics and modern computer technology to obtain close to the optimal operating modes of canal areas, hydraulic structures, significantly reduce non-productive losses of water resources.

The modern theory of optimal water distribution in irrigation systems, based on the differentiability of coefficients of equation of water movement in irrigation systems and their right parts, does not allow to formulate the necessary conditions of optimality on the selected criterion of water distribution between water consumers [1,2].

The articles [3,4] presents several methods for solving the complete system of Saint - Venant differential equations describing the unsteady movement of water in the canals of irrigation systems and recommend a method of sequential approximation based on the method of projection of the gradient for the problems of optimal water distribution between consumers in conditions of discrete water supply.

2. METHODS AND RESULTS

Mathematical models of unit hydraulic structures for the automation system are selected from the main technological requirements of pumping station operation mode control [5].

The Main Tasks of Managing The Operating Modes of Nodal Hydraulic Engineering Structures Are As Follows:

- determination of actual water consumption at the moment of observation for all structures;
- observance of dispatching modes of operation of hydraulic engineering hubs;
- calculation of actual water volume values for all structures.. (Normal).

The Main Measured Technological Parameters of Atypical Node Hydraulic Engineering Structure Are:

- gate openings of hydraulic structures;
- water level of up stream and down stream;
- electric conductivity of water as an indirect parameter for assessment of water resources quality.

To implement the function of telemetry of these parameters are installed technological sensors. Water flows through hydraulic structures are calculated by the water level on graduated hydropaths, as well as by a hydraulic formula.

Processing of information received from these sensors is carried out by intelligent process controllers, which through their input and output modules process and store the information received in the computer.

Functions of automatic regulation of water levels and discharges of hydraulic structures are realized by developed programs on controllers with the help of received information from sensors.

Radio stations for data transmission and voice communication are installed for continuous communication of the hydraulic engineering structure with the control center.

Automation of control processes of irrigation canals in conditions of changing water supply and water consumption requires the use of modern systems of automatic regulation, which in combination with telemechanics and computer technology to obtain close to the optimal modes of operation of canal areas, hydraulic structures, pumping stations and significantly reduce non-productive losses of water resources.

In the system of automatic upstream water level control, feedback is provided by measuring the upstream water level of the blockage structure. The measured signal is fed into a comparison unit where deviations between the water level set point and the measured value are determined. Then the deviation signal is fed to the water level regulator, where according to the chosen regulation law the regulator generates a control effect on the opening or closing of the gate of the blocking structure.

Let's consider the description of the presented system of water level regulation of the upstream of blocking structures on the channel sections (Figure 1).

Channel section-1. Initial conditions at $x = 0$ plot-1 [6, 7]

$$Q_1(0, t) = F_0(t) = \mu_0 \omega_0(t) \sqrt{2g[z_{us}(t) - z_1(0, t)]}, \quad (1)$$

The equation of the regulator P_0 level (flow rate) of the downstream water

$$\omega_0(t) = W_{r0}(t, H_0(b_0, t), H_0^*(t)), \quad (2)$$

where W_{r0} is a controller regulation operator (law) 1, $H_0(b_0, t)$ —water level in section 1, where the water level sensor stands,

b_0 is the distance from the beginning of the site, $H_1^*(t)$ – assignments to the regulator P_0 [8,9].

Boundary conditions at $x = l_1$ plot–1[10]

$$\begin{aligned} Q_1(l_1, t) &= F_1(t) + Q_2(t) + Q_3(t), \\ F_2(t) &= \mu_1 \omega_1(t) \sqrt{2g[z_1(l_1, t) - z_2(0, t)]} \end{aligned} \quad (3)$$

Figure 1: Upstream Automatic Control System.

Figure 2: Structural Diagram of the Automatic Water Level Control System.

The equations of regulators P_1 , P_2 and P_3 , level (flow rate) of upstream[11]

$$\omega_2(t) = W_{r2}(H_2(b_2, t), H_2^*(t)), \quad (4)$$

where W_{r0} is the operator (law) of regulator 1, $H_1(b_1, t)$ is the water level mark on site 1 where the water level sensor stands, b_1 is the distance from the beginning of the site, $z_i^*(t)$ is the set point for regulator P_0 .

The UpstreamControl System works as follows. If, for some reason, for example, the upstream water level increases compared to the set point of the water level, the regulator generates a signal to open the gate by a certain value calculated by the regulator's regulation law. After the shutter is opened, the water level in the upstream begins to decrease as the water flow in the blocking structure is higher than its previous value. The regulator will add just the opening value of the shutter of the blocking structure until the water level in the upstream is close to the set value [12,13].

In the upstream control system, water flow regulation systems are used at the side discharge outlets located in front of the blocking plants. Usually, flow rate control in outlets is implemented by regulating the water level in gauging stations located below the outlet with unambiguous flow characteristics.

In automatic water level control systems, various water level regulators are usually used at the canal sections. In the

practice of water automation systems, hydraulic, electrical and electronic water level regulators are used. Hydraulic regulators basically have laws of regulation of continuous operation, while electric and electronic regulators have discrete operation.

The laws regulating hydraulic regulators depend mainly on their structural elements [12, 13], in most cases these laws are proportional or proportional-integral. Electric regulators [14, 15] basically realize proportional-integral differential (PID) laws of regulation. Application of programmable logic controllers in a control loop of water management system allows to apply rather complex laws of regulation in a control loop.

Proportional law of regulation, when the control action, i.e. the value of opening of gates, is determined proportionally by the value of deviation of actual value of water level from its set value.

The Proportional Law of Regulation is recorded as follows [16]

$$Q(t) = Q_0 + k\theta(t), \quad (5)$$

where Q_0 – initial value of regulatory impact; k a coefficient of proportionality; $\theta(t)$ is an adjustable value mismatch value.

In the proportional-integral law of regulation, the control action, i.e. the value of the gate opening, is determined in proportion to the value of the deviation of the actual value of the water level from its present value and the integral time deviation.

The proportional-integral law of regulation is recorded as follows [17]

$$Q(t) = Q_0 + k \left(\theta(t) + \frac{1}{T_i} \int_0^t \theta(\tau) d\tau \right) \quad (6)$$

where Q_0 is the initial value of the control effect; k is the proportionality coefficient, T_i is the integration time constant; $\theta(t)$ is the value of the mismatch of the controlled value.

Let's consider application of PID laws of regulation by means of programmable controllers. The proportional-integral-differential law is the standard algorithm used for the control of unpredictably changing values.

The Proportional-Integral-Differential Law is written as follows [18].

$$Q(t) = Q_0 + k \left(\theta(t) + T_d \frac{d\theta(t)}{dt} + \frac{1}{T_i} \int_0^t \theta(\tau) d\tau \right), \quad (7)$$

where Q_0 is the initial value of the manipulated value; k is the coefficient, T_d is the time constant of differentiation; T_i is the integration time constant; $\theta(t)$ is the value of mismatch of the manipulated value.

Here is the implementation principle on the PID controller [19].

$$Q_{out}^{k+1} = Q_0 + \left(k(P^k + I^k + D^k) \right) \quad (8)$$

where Q_{out}^{k+1} is the output value of the regulator; Q_0 is the constant offset value; k is the controller proportionality factor; P^k is the proportional member; I^k is the integral member; D^k is the differential member.

Proportional term is calculated by the following formula [20]

$$P^k = H_{set}^k - H_{act}^k \quad (9)$$

where P^k is the proportional member; H_{set}^k is the set point value of the upstream level; H_{act}^k is the actual upstream level.

The integral term is calculated by the following formula [21]

$$I^k = I^{k-1} + \frac{(P^k + P^{k-1}) dt}{T_i} \quad (10)$$

where I^k - integral member; I^{k-1} - integral member calculated in the previous time point (in the previous step); P^k - proportional member; P^{k-1} - proportional member calculated in the previous time point (in the previous step); dt - time step of the controller (time interval with which the controller performs calculations); T_i - integration time constant.

To calculate the differential term, the following expression is used [22]

$$D^k = \frac{(H^{k-1} - H^k) T_i}{2 dt} \quad (11)$$

where D^k is a differential member; H^{k-1} is the actual upstreamwater level measured in the previous step; T_i is the time constant for differentiation.

In the considered algorithms if we do not take into account the member on differentiation we will get the algorithm proportional-integral law of regulation, or if we do not take into account the members of differentiation and integration we will get the proportional law of regulation.

The main parameters of the law regulating the PID regulator are k , T_i and T_d . These parameters are determined by the quality of the transition process in the automatic control system. Since objects in the water system refer to non-linear objects with distributed parameters, the analytical definition of the separameters from the condition that the best quality of the transition process is not currently developed, so the definition of parameters k , T_i and T_d is carried out on the basis of mathematical modeling or on a real object experimentally during the adjustment of the regulation system.

In [7, 8] there has been developed the modeling method of the complicated system with distributed parameters on the computer on the base of their structure image.

For modeling of dynamic processes in automatic regulation systems of water object on the basis of their structural representation the following tasks are solved: separation of regulation system by sections of canals and channels; drawing up a structural scheme of inter connected sections by types of conjugation conditions; finding a matrix of discrete presentation by its structural scheme and matrixes of discrete presentation of each block. This task is solved by transforming the initial structural scheme to simpler structural schemes.

The Sequence of Actions is as Follows:

- A mathematical description of the system of automatic regulation of a water object (the system of equations of

sections of an irrigation canal with initial and boundary conditions and conjugation conditions, the regulator equation, etc.) is made.

- More blocks, describing typical water objects with modeling algorithms, are distinguished.
- Selects the algorithms of discrete presentation of selected blocks
- A structural diagram of the system of automatic regulation of a water management facility is drawn up and input and output signals are highlighted.
- The elements of the matrix of discrete representation of the system by means of sequential transformation of its structural scheme are calculated using algorithms for calculating the characteristics of connections of distributed blocks.
- The specified input signals are used to determine the output signals.
- According to the developed algorithms program modules for calculation of dynamic processes of water level automatic control system are composed.

Following this sequence, we make up the structural representation of the system of regulation by the upstream.

Drawing up the structural diagram (points 1-4). In the structural approach to modeling the system in question, it should be divided into two parts:

- a system with distributed parameters, described by equations (1) with initial conditions (2) and boundary conditions (3) - (4) in [22];
- a concentrated system that represents its own link with the transfer coefficient K . This division is convenient because there is already a discrete presentation of the corresponding distributed block.

Considering expression $W(t_k) \left[(z(b, t_{k+1}) - z_0(b)) - h^*(t) \right] + g_1(t)$ as a new boundary effect on the object applied at the point $x = l$, we will find expression for vector $w_N^{k+1} = \{w_{1N}^{k+1}, w_{2N}^{k+1}\}$:

$$\begin{aligned} w_{1N}^{k+1} &= s_{11}^k Q_N^k + s_{11}^k z_N^k - \tau \left[\left(\frac{\partial f_1}{\partial Q} \right)_N^k Q_N^k + \left(\frac{\partial f_1}{\partial z} \right)_N^k z_N^k \right] + \tau f_1^{k+1}, \\ w_{2N}^{k+1} &= W(t_k) \left[(z(b, t_{k+1}) - z_0(b)) - h^{*k+1} \right] + g_2^{k+1} \end{aligned} \quad (12)$$

or

$$\begin{aligned} w_{1N}^{k+1} &= s_{11}^k Q_N^k + s_{11}^k z_N^k - \tau \left[\left(\frac{\partial f_1}{\partial Q} \right)_N^k Q_N^k + \left(\frac{\partial f_1}{\partial z} \right)_N^k z_N^k \right] + \tau f_1^{k+1}, \\ w_{2N}^{k+1} &= W^k \left[(\delta_{ri} (z_i^{k+1} - z_{0i}) - h^{*k+1}) \right] + g_2^{k+1}, \end{aligned} \quad (13)$$

where δ_{ri} is the Kronecker symbol, r is the number of the spatial grid node closest to the point a ; a_{nm} is the object parameters; Q_{ni}^{k+1} ($n=1, 2; i=1, \dots, N$) is the discrete value of the distributed object state function corresponding to the n -

output in i - spatial point on the $k+1$ - time layer; f_{mi}^{k+1} ($m=1,2;i=1,...,N$) is the discrete value of the external distributed influence applied to m - input of the system at i space point on $k+1$ - time layer.

In the case of the PID controller, the expression for W^k looks like

$$W^k = k(P^k + I^k + D^k) \quad (14)$$

Taking into account (13) and the structural scheme of the object (Figure 3.c [22]), we build the structural scheme of the system under consideration (Figure 3). The obtained structural scheme allows to define formulas for elements of G1 matrix of discrete representation of the system. Transformation of the structural scheme (point - 5). Let's find the expression for the elements of G_{21}^{1k} matrix of discrete representation of the structural scheme channel from the first input signal w_{1j}^{k+1} to the second output signal z_i^{k+1} . The structural scheme of the part of the system linking the first input signal to the second output signal at $h^*=0$ is shown in Fig.3.b. It consists of blocks described by matrices G_{21}^k , G_{12}^k and V^k . The elements of matrix V^k are as follows:

$$V_{ji}^k = \delta_{ji} W^k \delta_{ji} \quad (i, j = 1, ..., N). \quad (15)$$

Further, it is necessary using the formulas of transformation of connections of discretely represented distributed blocks, to consistently transform the structural scheme shown in Figure 3.b), until the structural scheme consisting of one block is obtained. The order of transformation of the structural scheme is shown in Figure 3.b-d. The sequential connection of blocks (Figure 3.b) described by the matrixes with elements G_{22ij}^k and V_{ji}^k has been replaced by one block (Fig. 3.c) described by the matrix with elements V_{ji}^{1k} defined by the formula [4, 22].

$$V_{ij}^{1k} = G_{22il}^k V_{ij}^k = G_{22il}^k \delta_{il} W^k \delta_{rj} = G_{22il}^k W^k \delta_{rj} \quad (i, j = 1, ..., N). \quad (16)$$

The block described by matrix V^{2k} (Fig.3, g) replaces the connection of the blocks shown with a dotted line in Figure 3.c.

Figure 3: Discrete Representation of the Upstream Control System in a Channel Section.

Formula for the elements of this matrix has the following form

$$V_{ij}^{2k} = \delta_{ij} + \frac{W^k G_{22il}^k \delta_{rj}}{I - W^k G_{22rl}^k} \quad (i, j = 1, ..., N). \quad (17)$$

Finally, the elements of matrix G_{21}^k are defined in Fig. 3.g by formula (17)

$$G_{21ij}^{1k} = \left[\delta_{il} + \frac{W^k G_{22il}^k \delta_{rj}}{1 - W^k G_{22r1}^k} \right] G_{21ij}^k = G_{21ij}^k + \frac{G_{22il}^k W^k G_{21rj}^k}{1 - W^k G_{22r1}^k} \quad (i, j = 1, \dots, N). \quad (18)$$

Similarly, there are formulas for calculating matrix elements of discretedetermination of other structural schemechannels (Figure 3.a):

$$G_{11ij}^{1k} = G_{11ij}^k + \frac{G_{12il}^k W^k G_{21rj}^k}{1 - W^k G_{22r1}^k}; \quad (19)$$

$$G_{12ij}^{1k} = G_{12ij}^k + \frac{G_{12il}^k W^k G_{22rj}^k}{1 - W^k G_{22r1}^k}; \quad (20)$$

$$G_{22ij}^{1k} = G_{22ij}^k + \frac{G_{22il}^k W^k G_{22rj}^k}{1 - W^k G_{22r1}^k}; (i, j = 1, \dots, N). \quad (21)$$

Calculation of the output signal (point-5). Further, using (18)-(21) calculate the discrete values of the output signal at the given discrete values of the generalized input signals:

$$Q_i^{k+1} = G_{11ij}^{1k} W_{ij}^{k+1} + G_{12ij}^{1k} W_{2j}^{k+1}, \quad (22)$$

$$z_i^{k+1} = G_{21ij}^{1k} W_{ij}^{k+1} + G_{22ij}^{1k} W_{2j}^{k+1}, \quad (i = 1, \dots, N; \quad k = 0, 1, \dots). \quad (23)$$

According to the developed algorithms are composed of software modules for calculation of dynamic processes of the system of automatic regulation of water levels

CONCLUSIONS

A method for solving the problem of optimal water distribution in the canals of irrigation systems, a mathematical model of unsteady water movement in the canal sections as a system of full differential equations of Saint - Venant, and a numerical solution of the problem of optimal water distribution in the canals of irrigation systems by the method of finite difference of the differential equation of Saint - Venant for automatic regulation of water level in the canal section were developed. On the basis of the developed algorithms and program modules the systems of regulation on water object were modeled.

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