IN-PLANE FREE VIBRATION OF TWO-MEMBER OPEN FRAME WITH TRANSVERSE CRACK WITH END MASS

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ABSTRACT

This paper focus on addressing the problem of In-plane free vibration analysis of two-member open frame with transverse crack. A two-member open frame is modelled using Euler-Bernoulli beam theory. The angle between the beams is 90° and payload is attached at the end of the second beam. The linearized equation of motion is derived using Hamilton's principle and the general frequency equations are presented with crack with two member open frame. Transverse vibration of a frame in the presence of a normal edge crack is considered. The crack is modelled using the concept of rotational spring. The determination of natural frequencies for a given crack location and size is a straightforward problem and is the aim of project work. Numerical results are tabulated for both the cases (i.e without and with end mass) and graphs are drawn.

KEYWORDS: Dynamic Behavior, Crack, Modal Parameter, Vibration

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INTRODUCTION

In a structural member crack introduces local flexibility due to strain energy concentration under load at the vicinity of the crack tip. This depends on crack depth. the dynamic behavior of the structure changes due to flexibility and from this the position of crack and magnitude can be identified. It is known that vibrational parameters changes when there is a crack in the structure e.g. reduction in stiffness, increase in the damping etc, and these parameters are mode dependent. They are natural frequencies, modal damping values and mode shapes etc.

LITERATURE SURVEY

Using Hu-Washizu-Barr variational formulation the differential equation and the boundary conditions of the cracked beams and compared the local flexibility method. The crack location and depth was measured using vibration amplitude by Rizos et al [2]. A Dado [3] developed an algorithm to predict the depth and location of cranked shaft. Here Frobenius technique is used to model the crack.

Jialou Hu and Robert [4] have used massless springs to represent cracks and the continuum damage concept is used for crack detection.

B P. Nandwana and S. K. Maiti[5] have used measurement of natural frequencies for the detection of crack size and location in a stepped cantilever beam.

B P. Nandwana and S. K. Maiti[6] have used rotational spring concept for modeling inclined and internal crack in a slender beam.

DESCRIPTION OF THE SYSTEM

The system is mainly composed of 2 flexible beam members and a payload which are arranged as shown in figure (1). The first beam is vertical, while the second beam is attached to the tip of the first beam such that it is inclined at an angle of 90° from the vertical. The payload is attached at the end of the second beam and the motion of the system is assumed to be a planar and the case (1) crack is located in vertical segment and case (2) crack is located in vertical segment and case (2) crack is located in horizontal segment.

Figure 1: Two Frame Open Frame (L-frame) Structure with End Masses and Crack is in Vertical Segment.

Case 1: Crack Located in Vertical Segment

Using Euler Bernoulli theory the equations of motion can be written as

$$EI \partial^4 v_i / \partial x_i^4 + p L_i \partial^2 v_i / \partial t^2 = 0 \quad (1)$$

In which E is Young's Modulus of elasticity, I is the second moment of area of cross-section p is the density of material, v_i is the transverse displacement and x_i is the axial coordinate for the segment i . In the above the torsional oscillation of a segment has been ignored.

The general solution of equation (1) can be written in the form.

$$v_i(x_i, t) = L_i v_i(\xi_i) \cos \omega t \quad (2)$$

In which ω is the natural frequency, $\xi_i = x_i / L_i$ and L_i is the length of the segment i .

Using equation (1) and (2)

The equations of motion can be written as

$$\partial^4 V_1 / \partial \xi_1^4 - \lambda_1^4 V_1 = 0 \quad \dots \dots \dots 0 \leq \xi_1 \leq \beta_1 \quad (3a)$$

$$\partial^4 V_2 / \partial \xi_1^4 - \lambda_1^4 V_2 = 0 \quad \dots \dots \dots \beta_1 \leq \xi_1 \leq 1 \quad (3b)$$

$$\partial^4 V_3 / \partial \xi_2^4 - \lambda_2^4 V_3 - \epsilon_2 V_2(1) = 0 \quad \dots \dots \dots 0 \leq \xi_2 \leq 1 \quad (3c)$$

Where, $\beta_1 = L_{C1}/L_1$, $\varepsilon_1 = x/L_1$, $\varepsilon_2 = x/L_2$

$$\lambda_1 = \rho L_1 4\omega^2 / EI, \lambda_2 = \rho L_{12} 4\omega^2 / EI,$$

The general solution to the equation of motion is given by

$$V_1(\varepsilon_1) = A_1 \sin \lambda_1 \varepsilon_1 + B_1 \cos \lambda_1 \varepsilon_1 + C_1 \sin \lambda_1 \varepsilon_1 + D_1 \cos \lambda_1 \varepsilon_1 \quad (4a)$$

$$V_2(\varepsilon_1) = A_2 \sin \lambda_1 \varepsilon_1 + B_2 \cos \lambda_1 \varepsilon_1 + C_2 \sin \lambda_1 \varepsilon_1 + D_2 \cos \lambda_1 \varepsilon_1 \quad (4b)$$

$$V_3(\varepsilon_2) = A_3 \sin \lambda_2 \varepsilon_2 + B_3 \cos \lambda_2 \varepsilon_2 + C_3 \sin \lambda_2 \varepsilon_2 + D_3 \cos \lambda_2 \varepsilon_2 - v_2 - (1) \quad (4c)$$

Where A_i , B_i , C_i and D_i are arbitrary constants, which are determined from the boundary conditions.

The boundary conditions are at the fixed and free end, continuity of displacement, moment, shear forces at the crack location, jump condition in the slope and compatibility conditions can be written as follows.

The boundary conditions at the fixed end slope are zero.

$$V_1(0) = 0 \quad (5)$$

$$V_1(0) = 0 \quad (6)$$

The compatibility conditions of the displacement, slope, moment and shear forces at the crack location are

At $\varepsilon_1 = \beta_c$

The continuity of the deflection at the crack is given by

$$V_1(\beta_1) - V_2(\beta_1) = 0 \quad (7)$$

At the crack location, the slope of deflection is discontinuous; the values of the slope at both sides of the crack are related to the crack stiffness by.

$$V_{1c}(\beta_1) + (E_1 I_1 / K_c L_1) V_1(\beta_1) - V_2(\beta_1) = 0 \quad (8)$$

From the theory of linear elastic fracture mechanics, the rotational spring stiffness K_c is given by

$$K_c = E_1 b h^2 / 72 \pi (a/h)^2 f(a/h)$$

Where,

$$f(a/h) = 0.6384 - 1.035(a/h) + 3.7201(a/h)^2 - 5.1773(a/h)^4 - 7.3324(a/h)^5 + 2.4909(a/h)^6$$

Where,

$$K = (K_c L_1 / E_1 I_1)$$

Is the non-dimensional stiffness of the rotational spring representing the crack.

The internal moment at the crack location is same for the two segments and that leads to

$$V_1(\beta_1) - V_2(\beta_1) = 0 \quad (9)$$

The shear compatibility equation at the crack location is given by

$$V_1(\beta_1) - V_2(\beta_1) = 0 \quad (10)$$

The boundary conditions at

At $(\xi_1)=1$ and $(\xi_2)=0$ with end mass is given by

$$V_3(0) = 0 \quad (11)$$

$$V_3'(0) = 0 \quad (12)$$

$$\lambda_1^4 \rho L \int_0^1 \xi_2 \{V_3 + \xi_2 V_2(1)\} d\xi_2 + \lambda_1^4 M_e L^2 \{V_2(1) + (V_3(1)) - V_2''(1) = 0 \quad (V_3(1)) - V_2''(1) = 0 \quad (13 \& 14)$$

$$\lambda_1^4 \rho L^2 \int_0^1 \{1/L V_2(1)\} d\xi_2 + \lambda_1^4 M_e \{1/L V_2(1)\} + V_2''(1) = 0 \quad (15)$$

The boundary conditions at

At $\xi_2 = 1$ with end mass is given by

$$V_3''(1) = 0 \quad (16)$$

$$\lambda_1^4 M_e \{V_3(1) + V_2(1)\} + \rho L \{V_3(1) = 0 \quad (17)$$

Using the above boundary and compatibility conditions twelve homogeneous equations involving the unknown arbitrary constants are obtained. These can be expressed conveniently in the following form.

$$[A]_{12 \times 12} \{q\}_{12 \times 1} = \{0\}_{12 \times 1}$$

$$\text{Where } \{q\} = [A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2, A_3, B_3, C_3, D_3]^T$$

The non-zero elements of the matrix $[A]$ are listed below. The terms contain the natural frequency parameters λ .

For a non-trivial solution for $\{q\}$

$$\text{Det } [A] = 0$$

This gives the characteristic equation.

Case: 2: Crack Located in Horizontal Segment

Figure 2: Two Frame Open Frame (L-Frame) Structure with End Masses and Crack is in Horizontal Segment.

Fig. 2 Two Frame Open Frame (L-Frame) structure with end mass and crack is in horizontal segment

Here it is assumed the crack is being located in horizontal segment.

Using Euler – Bernoulli beam theory the equations of motion can be written as

$$E_i I_i \partial^4 v_i / \partial x_i^4 + \rho_i L_i \partial^2 v_i / \partial t^2 = 0 \quad (18)$$

In which E is Young's Modulus of Elasticity, I is the second moment of area of cross section, ρ is the density of material, v_i is the transverse displacement and x_i is the axial co-ordinate for the segment i . In the above the torsional oscillation of a segment has been ignored.

The general solution of equation (18) can be written in the form

$$V_i(x_i, t) = L_i v_i(\epsilon_i) e^{j\omega t} \quad (19)$$

In which ω is the natural frequency, $\epsilon_i = x_i/L_i$ and L_i is the length of the segment i

Using equations (18) and (19)

The equations of motion can be written as

$$\partial^4 V_1 / \partial \epsilon_1^4 - \lambda_1^4 V_1 = 0 \quad \text{-----} \quad 0 \leq \epsilon_1 \leq 1 \quad (20a)$$

$$\partial^4 V_2 / \partial \epsilon_2^4 - \lambda_1^4 V_2 - \epsilon_2 V_1(1) = 0 \quad \text{-----} \quad 0 \leq \epsilon_2 \leq \beta_2 \quad (20b)$$

$$\partial^4 V_3 / \partial \epsilon_2^4 - \lambda_2^4 V_3 - \epsilon_2 V_1(1) = 0 \quad \text{-----} \quad \beta_2 \leq \epsilon_2 \leq 1 \quad (20c)$$

$$\text{Where } \beta_2 = \frac{L_{c2}}{L_2}, \quad \xi_1 = \frac{x}{L_1}, \quad \xi_2 = \frac{x}{L_2}$$

$$\lambda_1 = \rho L_1^4 \frac{\omega^2}{EI}, \quad \lambda_2 = \rho L_2^4 \frac{\omega^2}{EI}$$

The general solution to the equation of motion is given by

$$V_1(\xi_1) = A_1 \sin \lambda_1 \xi_1 + B_1 \cos \lambda_1 \xi_1 + C_1 \sinh \lambda_1 \xi_1 + D_1 \cosh \lambda_1 \xi_1 \quad (21a)$$

$$V_2(\xi_2) = A_2 \sin \lambda_2 \xi_2 + B_2 \cos \lambda_2 \xi_2 + C_2 \sinh \lambda_2 \xi_2 + D_2 \cosh \lambda_2 \xi_2 - \xi_2 v_1(1) \quad (21b) \quad V_3(\xi_2) = A_3$$

$$\sin \lambda_2 \xi_2 + B_3 \cos \lambda_2 \xi_1 + C_3 \sinh \lambda_2 \xi_2 + D_3 \cosh \lambda_2 \xi_2 - \xi_3 v_1(1) \quad (21c)$$

Where A_i , B_i , C_i and D_i are arbitrary constants, which are determined from the boundary conditions.

The boundary conditions are at the fixed and free end, continuity of displacement moment, shear forces at the crack location, jump condition in the slope and compatibility conditions can be written as follows

The boundary conditions at the fixed end are as follows,

At $\xi_1 = 0$ both displacement and slope are zero.

$$V_1(0) = 0 \quad (22)$$

$$V_1(0) = 0 \quad (23)$$

The boundary conditions

At $\xi_1 = 1$ and $\xi_1 = 0$ is given by

$$V_2(0) = 0 \quad (24)$$

$$V_2(0) = 0 \quad (25)$$

$$\lambda_1^4 \rho L \int_0^1 \varepsilon_2 \{V_2 + \varepsilon_2 V_1(1)\} d\varepsilon_2 + \lambda_1^4 M_1 L^2 \{V_1(1) + (V_2(1)) - V_1''(1) = 0 \quad (26)$$

$$\lambda_1^4 \rho L^2 \int_0^1 \{1/L V_2(1)\} d\varepsilon_2 + \lambda_1^4 M_1 L^2 \{1/L V_2(1)\} - V_2''(1) = 0 \quad (27)$$

The compatibility conditions of the displacement, slope, moment and shear forces at the crack location are

At $\xi_1 = \beta$

The continuity of the deflection at the crack is given by

$$V_2(\beta_2) - V_2(\beta_2) = 0 \quad (28)$$

At the crack location, the slope of deflection is discontinuous, the values of the slope at both sides of the crack are related to the crack stiffness by

$$V_2(\beta_2) + (E_2 I_2 / K_1 L_2) V_2''(\beta_2) - V_3''(\beta_2) = 0 \quad (29)$$

From the theory of linear elastic fracture mechanics, the rotational spring stiffness K_t is given by $K_t = E_2 b h^2 / 72 \pi (a/h)^2 f(a/h)$

Where

$$F(a/h) = 0.6384 - 1.035 (a/h) + 3.720 I (a/h)^2 - 5.1773 (a/h)^3 + 7.553 (a/h)^5 + 2.4909 (a/h)^6$$

Where $K = (K_t L_2 / E_2 I_2)$ is the non-dimensional stiffness of the rotational spring representing the crack.

The internal moment at the crack location is same for the two segments and that leads to

$$V_2''(\beta_2) - V_3''(\beta_2) = 0 \quad (30)$$

The shear compatibility equation at the crack location is given by

$$V_2''(\beta_2) - V_3''(\beta_2) = 0 \quad (31)$$

The boundary conditions at

At $\varepsilon_2 = 1$ with end mass is given by

$$V_3''(1) = 0 \quad (32)$$

$$\lambda_2^4 M_t \{V_3(1) + V_1(1)\} + \rho L V_3''(1) = 0 \quad (33)$$

Using the above boundary and compatibility conditions twelve homogeneous equations involving the unknown arbitrary constants are obtained. These can be expressed conveniently in the following form

$$[A]_{12 \times 12} \{q\}_{12 \times 1} = \{0\}_{12 \times 1}$$

Where

$$\{q\} = [A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2, A_3, B_3, C_3, D_3]^T$$

$$\lambda_1^4 \rho L \int_0^1 \varepsilon_2 \{V_3 + \varepsilon_2 V_2(1)\} d\varepsilon_2 + \lambda_1^4 M_t L^2 \{V_2(1) + (V_3(1)) - V_2''(1) = 0 \quad (V_3(1)) - V_2''(1) = 0$$

$$\lambda_1^4 \rho L^2 \int_0^1 \{1/L V_2(1)\} d\varepsilon_2 + \lambda_1^4 M_t \{1/L V_2(1)\} + V_2''(1) = 0$$

The boundary conditions at

At $\varepsilon_2 = 1$ with end mass is given by

$$V_3(1) = 0$$

$$\lambda_1^4 M_t \{V_3(1) + V_2(1)\} + \rho L \{V_3(1) = 0$$

Using the above boundary and compatibility conditions twelve homogeneous equations involving the unknown arbitrary constants are obtained. These can be expressed conveniently in the following form.

$$[A]_{12 \times 12} \{q\}_{12 \times 1} = \{0\}_{12 \times 1}$$

Where,

$$\{q\} = [A_1, B_1, C_1, D_1, A_2, B_2, C_2, D_2, A_3, B_3, C_3, D_3]^T$$

The non-zero elements of the matrix $[A]$ are listed below. The terms contain the natural frequency parameters λ .

For a non-trivial solution for $\{q\}$

$$\text{Det } [A] = 0$$

This gives the characteristic equation.

RESULTS AND DISCUSSIONS

The characteristic equations are solved using MAT Lab 7.0.1 programming computational package to obtain the natural frequencies.

For numerical case study the below material properties and dimensions of the structure are taken up. The dimensions of the beam structure are length $L_1 = 0.9\text{m}$, $L_2 = 0.6\text{ m}$, width $b = 0.03\text{m}$, depth $h = 0.05\text{m}$. The material properties used for calculation material as aluminum with modulus of elasticity $E_1 = E_2 = 70\text{ Gpa}$, density $\rho_1 = \rho_2 = 260\text{ kg/m}^3$

The system without and with end mass and the angle between the beams is 90° . The natural frequencies are obtained by solving for the roots of the frequency equation are tabulated in tables 1 to 6.

The frequency ratio is evaluated for different sets of crack parameters and a series of end mass ratios. The frequency ratio in percentage is defined as

$$r\omega = \omega(\text{cracked}) - \omega(\text{uncracked}) / \omega(\text{uncracked}) \times 100$$

Table 1: Natural Frequencies of L-Frame with Crack on Segment Number1 (Vertical Segment Without End Mass)

Crack Location (L_c/L_i)	Crack Size (a/h)	Natural Frequencies (rad/sec)							
		ω_1	% Diff	ω_2	% Diff	ω_3	% Diff	ω_4	% Diff
No Crack		149.730		487.830		1892.850		3714.510	
0.20	0.10	149.081	-0.43	487.51	-0.066	1892.623	-0.012	3714.233	-0.007
0.20	0.20	147.279	-1.634	486.625	-0.247	1891.987	-0.046	3713.467	-0.028
0.20	0.30	144.201	-3.689	485.159	-0.548	1890.927	-0.102	3712.166	-0.063
0.20	0.40	139.561	-6.783	483.058	-0.979	1889.392	-0.183	3710.225	-0.115
0.20	0.50	133.264	-10.994	480.39	-1.526	1887.414	-0.287	3707.623	-0.185
0.20	0.60	125.598	-16.114	477.42	-2.134	1885.175	-0.406	3704.529	-0.269
0.40	0.10	149.341	-0.256	487.822	-0.002	1888.945	-0.206	3711.099	-0.092
0.40	0.20	149.256	-0.981	487.795	-0.008	1878.04	-0.783	3701.632	-0.347
0.40	0.30	146.368	-2.242	487.749	-0.017	1859.522	-1.761	3685.724	-0.775
0.40	0.40	143.439	-4.199	487.678	-0.031	1831.94	-3.218	3662.448	-1.402
0.40	0.50	139.278	-6.977	487.581	-0.051	1795.073	-5.166	3632.184	-2.216
0.40	0.60	133.938	-10.544	487.46	-0.076	1751.515	-7.467	3597.764	-3.143
0.60	0.10	149.528	-0.132	487.35	-0.099	1887.381	-0.289	3714.495	-0.0001
0.60	0.20	148.967	-0.506	485.99	-0.378	1872.192	-1.091	3714.46	-0.001
0.60	0.30	147.975	-1.169	483.626	-0.862	1846.669	-2.44	3714.402	-0.003
0.60	0.40	146.395	-2.224	479.966	-1.612	1809.253	-4.417	3714.316	-0.005
0.60	0.50	144.064	-3.781	474.794	-2.673	1760.293	-7.003	3714.203	-0.008
0.60	0.60	140.916	-5.884	468.214	-4.021	1703.871	-9.984	3714.074	-0.012
0.80	0.10	149.645	-0.053	468.448	-0.284	1892.549	-0.016	3709.491	-0.135
0.80	0.20	149.416	-0.206	482.563	-1.08	1891.697	-0.061	3695.384	-0.515
0.80	0.30	149.005	-0.481	475.878	-2.45	1890.234	-0.138	3671.091	-1.169
0.80	0.40	148.339	-0.926	465.701	-4.536	1888.017	-0.255	3634.181	-2.163
0.80	0.50	147.324	-1.603	451.645	-7.418	1884.974	-0.416	3583.672	-3.522
0.80	0.60	145.893	-2.559	434.274	-10.979	1881.25	-0.613	3522.635	-5.166

Table 2: Natural Frequencies of L-Frame with Crack on Segment Number 1 (Vertical Segment) with End Mass = 0.4 Kg

Crack Location (L _c /L _i)	Crack Size (a/h)	Natural Frequencies (rad/sec)							
		ω ₁	% Diff	ω ₂	% Diff	ω ₃	% Diff	ω ₄	% Diff
No Crack		111.827		318.9163		1733.4130		2943.8490	
0.20	0.10	114.455	-0.383	318.574	-0.107	1733.089	-0.019	2943.82	-0.01
0.20	0.20	110.253	-1.457	317.63	-0.403	1732.192	-0.07	2943.738	-0.04
0.20	0.30	108.187	-3.303	316.066	-0.894	1730.698	-0.157	2943.601	-0.008
0.20	0.40	105.046	-6.11	313.822	-1.597	1728.535	-0.281	2943.4	-0.015
0.20	0.50	100.715	-9.982	310.97	-2.492	1725.752	-0.442	2943.137	-0.024
0.20	0.60	95.365	-14.763	307.794	-3.487	1722.604	-0.624	2942.833	-0.035
0.40	0.10	111.596	-0.256	318.91	-0.002	1730.481	-0.169	2941.21	-0.09
0.40	0.20	110.787	-0.979	318.894	-0.007	1722.268	-0.643	2933.907	-0.338
0.40	0.30	109.379	-2.238	318.865	-0.016	1708.227	-1.453	2921.714	-0.752
0.40	0.40	107.194	-4.191	318.821	-0.03	1687.094	-2.672	2904.036	-1.352
0.40	0.50	104.091	-6.964	318.761	-0.049	1658.444	-4.325	2881.308	-2.124
0.40	0.60	100.108	-10.524	318.688	-0.072	1624.01	-6.311	2855.764	-2.992
0.60	0.10	111.707	-0.157	318.754	-0.051	1728.274	-0.297	2943.307	-0.018
0.60	0.20	111.207	-0.604	318.296	-0.195	1713.978	-1.121	2941.821	-0.069
0.60	0.30	110.326	-1.391	317.501	-0.444	1689.865	-2.512	2939.38	-0.152
0.60	0.40	108.932	-2.637	316.272	-0.829	1654.308	-4.564	2935.924	-0.269
0.60	0.50	106.892	-4.461	314.541	-1.372	1607.401	-7.27	2931.609	-0.416
0.60	0.60	104.17	-6.893	312.346	-2.06	1552.794	-10.42	2926.906	-0.576
0.80	0.10	111.787	-0.086	318.222	-0.218	1732.557	-0.049	2941.505	-0.08
0.80	0.20	111.511	-0.332	316.272	-0.829	1730.149	-0.188	2934.934	-0.303
0.80	0.30	111.021	-0.77	312.913	-1.883	1725.006	-0.427	2923.676	-0.685
0.80	0.40	110.229	-1.478	307.79	-3.489	1719.701	-0.791	2906.684	-1.262
0.80	0.50	109.036	-2.545	300.699	-5.712	1711.003	-1.293	2883.563	-2.048
0.80	0.60	107.378	-4.026	291.921	-8.465	1700.295	-1.911	2855.68	-2.995

Table 3: Natural Frequencies of L-Frame with Crack on Segment Number 1 (Vertical Segment) with End Mass = 0.8 Kg

Crack Location (L_c/L_i)	Crack Size (a/h)	Natural frequencies (rad/sec)							
		ω_1	% Diff	ω_2	% Diff	ω_3	% Diff	ω_4	% Diff
No Crack		92.7908		262.0426		1689.0628		2852.0465	
0.20	0.10	91.514	-1.38	260.798	-0.47	1689.713	-0.02	2851.982	-0.00220 -0.00506
0.20	0.20	89.89	-3.13	259.283	-1.05	1686.061	-0.08	2851.902	0.00506
0.20	0.30	87.41	-5.80	257.105	-1.88	1683.67	-0.18	2851.785	-0.01
0.20	0.40	83.968	-9.51	254.329	-2.94	1680.593	-0.32	2851.633	-0.01
0.20	0.50	79.687	-14.12	251.23	-4.13	1677.113	-0.50	2851.457	-0.01
0.20	0.60	92.556	-0.25	262.028	-0.01	1686.357	-0.71	2849.453	-0.02
0.40	0.10	91.892	-0.97	261.985	-0.02	1678.773	-0.16	2842.27	-0.090
0.40	0.20	90.736	-2.21	261.913	-0.05	1665.796	-0.61	2830.269	-0.34
0.40	0.30	88.941	-4.15	261.804	-0.09	1646.23	-1.38	2812.85	-0.76
0.40	0.40	86.391	-6.90	261.654	-0.15	1619.642	-2.54	2790.417	-1.37
0.40	0.50	83.114	-10.43	261.47	-0.22	1587.586	-4.11	2765.151	-2.16
0.40	0.60	92.637	-0.197	261.945	-0.04	1686.024	-6.01	2851.396	-3.05
0.60	0.10	92.199	-0.64	261.669	-0.14	1670.003	-0.30	2849.613	-0.02
0.60	0.20	91.428	-1.47	261.19	-0.33	1646.335	-1.13	2846.69	-0.09
0.60	0.30	90.211	-2.278	260.452	-0.61	1611.396	-2.53	2842.565	-0.19
0.60	0.40	88.436	-4.69	259.412	-1.00	1565.232	-4.60	2837.434	-0.33
0.60	0.50	86.078	-7.23	258.095	-1.51	1511.385	-7.33	2831.867	-0.51

0.60	0.60	92.698	-0.10	261.532	-0.19	1688.058	-10.52	2849.949	-0.71
0.80	0.10	92.433	-0.39	260.096	-0.74	1685.232	-0.06	2844.072	-0.07
0.80	0.20	91.961	-0.89	257.623	-1.69	1680.37	-0.23	2834.016	-0.28 28-0.
0.80	0.30	91.203	-1.71	253.852	-3.13	1672.969	-0.51	2818.862	-0.63
0.80	0.40	90.066	-2.94	248.634	-5.12	1662.762	-0.95	2798.29	-1.16
0.80	0.50	88.499	-4.63	242.178	-7.58	1650.197	-1.56	2773.539	-1.88
0.80	0.60	88.499	-4.63	242.178	-7.58	1650.197	-2.30	2773.539	-2.75

Table 4: Natural Frequencies of L-Frame with Crack on Segment Number 2 (Horizontal Segment) Without End Mass

Crack Location (L_c/L_2)	Crack Size (a/h)	Natural frequencies (rad/sec)							
		ω_1	% Diff	ω_2	% Diff	ω_3	% Diff	ω_4	% Diff
No Crack		149.730		487.830		1892.850		3714.510	
0.20	0.10	149.725	-0.0040	487.596	-0.05	1880.419	-0.66	3650.576	-1.72
0.20	0.20	149.724	-0.0040	486.915	-0.19	1843.363	-2.61	3462.691	-6.78
0.20	0.30	149.723	-0.0047	485.682	-0.44	1773.119	-6.33	3171.275	-14.62
0.20	0.40	149.722	-0.0054	483.636	-0.86	1656.4	-12.49	2873.851	-22.63
0.20	0.50	149.719	-0.0073	480.421	-1.52	1495.099	-21.01	2657.861	-28.45
0.20	0.60	149.715	-0.100	475.675	-2.49	1319.216	-30.31	2530.081	-31.89
0.40	0.10	149.725	-0.0040	487.738	-0.02	1887.024	-0.31	3639.148	-2.03
0.40	0.20	149.725	-0.0040	487.468	-0.07	1869.965	-1.21	3462.091	-6.80
0.40	0.30	149.724	-0.0040	486.981	-0.17	1838.237	-2.89	3234.656	-12.92
0.40	0.40	149.724	-0.0040	486.18	-0.34	1787.336	-5.73	2995.328	-19.36
0.40	0.50	149.723	-0.0047	484.94	-0.59	1700.343	-10.17	2781.651	-25.11
0.40	0.60	149.721	-0.0060	483.145	-0.96	1589.834	-16.17	2620.325	-29.46
0.60	0.10	149.725	-0.0040	487.809	-0.00	1891.158	-0.09	3679.872	-0.93
0.60	0.20	149.725	-0.0040	487.743	-0.02	1886.252	-0.35	3590.555	-3.34
0.60	0.30	149.725	0.0040	487.625	-0.04	1877.28	-0.82	3457.879	-6.91
0.60	0.40	149.725	0.0040	487.431	-0.08	1862.205	-1.62	3290.696	-11.41
0.60	0.50	149.724	0.0040	487.134	-0.14	1838.14	-2.89	3104.412	-16.42
0.60	0.60	149.724	0.0040	486.709	-0.23	1802.186	-4.79	2920.714	-21.37
0.80	0.10	149.725	0.0040	487.83	-0.0010	1892.703	-0.01	3710.345	-0.11
0.80	0.20	149.725	0.0040	487.825	-0.0028	1832.272	-0.03	3698.542	-0.43
0.80	0.30	149.725	0.0040	487.816	-0.00	1891.497	-0.07	3677.842	-0.99
0.80	0.40	149.725	0.0040	487.802	-0.01	1890.226	-0.14	3645.321	-1.86
0.80	0.50	149.725	0.0040	487.779	-0.01	1888.261	-0.24	3598.209	-3.13
0.80	0.60	149.725	0.0040	487.747	-0.02	1885.425	-0.39	3536.024	-4.81

Table 5: Natural Frequencies of L-Frame with Crack on Segment Number 2 (Horizontal Segment) Without End Mass = 0.4 kg

Crack Location (L_c/L_2)	Crack Size (a/h)	Natural frequencies (rad/sec)							
		ω_1	% Diff	ω_2	% Diff	ω_3	% Diff	ω_4	% Diff
No Crack		111.8827		318.9163		1733.411		2943.8627	
0.20	0.10	111.882	-0.0006	318.665	-0.08	1700.356	-1.191	2920.604	-0.79
0.20	0.20	111.879	-0.0033	317.94	-0.31	1602.769	-7.54	2824.436	-4.06
0.20	0.30	111.874	-0.0177	316.627	-0.72	1437.838	-17.05	2623.609	-10.88
0.20	0.40	111.866	-0.0149	314.451	-1.40	1232.622	-28.89	2452.751	-16.68
0.20	0.50	111.854	-0.0356	311.043	-2.47	1035.193	-40.28	2358.166	-19.90
0.20	0.60	111.836	-0.0417	306.05	-4.03	873.499	-49.61	2308.641	-21.58
0.40	0.10	111.882	-0.0006	318.788	-0.04	1711.846	-1.24	2865.814	-2.65
0.40	0.20	111.881	-0.0015	318.417	-0.04	1650.001	-4.81	2705.492	-8.10
0.40	0.30	111.878	-0.0042	317.75	-0.37	1543.956	-10.93	2543.982	-13.58
0.40	0.40	111.874	-0.0077	316.652	-0.71	1396.452	-19.44	2420.963	-17.76

0.40	0.50	111.868	-0.0131	314.949	-1.24	1229.102	-29.09	2342.229	-20.44
0.40	0.60	111.859	-0.0002	312.48	-2.02	1069.656	-38.29	2295.873	-22.01
0.60	0.10	111.883	-0.0002	318.867	-0.02	1723.109	-0.59	2888.524	-1.88
0.60	0.20	111.882	-0.0006	318.724	-0.06	1693.496	-2.30	2767.262	-6.00
0.60	0.30	111.881	-0.0015	318.467	-0.14	1640.913	-5.34	2627.407	-10.75
0.60	0.40	111.88	-0.0024	318.047	-0.27	1559.512	-10.03	2498.128	-15.14
0.60	0.50	111.877	-0.0050	317.4	-0.48	1449.917	-16.35	2397.164	-18.57
0.60	0.60	111.874	-0.0077	316.471	-0.77	1324.481	-23.59	2328.251	-20.91
0.80	0.10	111.883	-0.0002	318.906	-0.00	1730.959	-0.14	2927.019	-0.57
0.80	0.20	111.883	-0.0002	318.877	-0.01	1723.904	-0.55	2882.855	-2.07
0.80	0.30	111.882	-0.0006	318.825	-0.03	1711.197	-1.28	2815.679	-4.35
0.80	0.40	111.882	-0.0006	318.739	-0.06	1690.411	-2.48	2729.385	-7.29
0.80	0.50	111.882	-0.0006	318.608	-0.10	1658.669	-4.31	2632.507	-10.58
0.80	0.60	111.881	-0.0015	318.421	-0.16	1614.289	-6.87	2537.552	-13.80

Table 6: Natural Frequencies of L-Frame with Crack on Segment Number 2 (Horizontal Segment) Without End Mass = 0.8 kg

Crack Location (L _c /L ₂)	Crack Size (a/h)	Natural Frequencies (rad/sec)							
		ω ₁	% Diff	ω ₂	% Diff	ω ₃	% Diff	ω ₄	% Diff
No Crack		92.7908		262.0426		1689.0609		2852.0674	
0.20	0.10	92.79	-0.0008	261.802	-0.09	1638.072	-3.02	2829.036	-0.81
0.20	0.20	92.786	-0.0051	261.108	-0.36	1490.225	-11.77	2702.154	-5.26
0.20	0.30	92.779	-0.0127	259.851	-0.84	1269.807	-24.82	2472.189	-13.32
0.20	0.40	92.768	-0.0245	257.769	-1.63	1042.76	-38.26	2337.847	-18.03
0.20	0.50	90.752	-0.0418	254.508	-2.88	853.316	-49.48	2273.275	-20.29
0.20	0.60	92.727	-0.0687	249.741	-4.69	710.482	-57.94	2240.645	-21.44
0.40	0.10	92.79	0.0008	261.914	-0.05	1654.346	-2.06	2746.335	-3.71
0.40	0.20	92.788	-0.0030	261.546	-0.19	1556.823	-7.83	2558.989	-10.28
0.40	0.30	92.785	-0.0062	260.881	-0.44	1403.375	-16.91	2406.823	-15.61
0.40	0.40	92.779	-0.0127	259.786	-0.86	1218.672	-27.85	2312.008	-18.94
0.40	0.50	92.77	-0.0224	258.088	-1.51	1037.208	-38.59	2258.682	-20.80
0.40	0.60	90.758	-0.0435	255.627	-2.45	881.957	-47.78	2229.16	-21.84
0.60	0.10	92.791	-0.0002	261.99	-0.02	1671.431	-1.04	2773.515	-2.75
0.60	0.20	92.79	-0.0008	261.839	-0.08	1621.433	-4.00	2622.603	-8.04
0.60	0.30	92.788	-0.0030	261.567	-0.18	1536.524	-9.03	2477.034	-13.15
0.60	0.40	92.786	-0.0051	261.122	-0.35	1416.702	-16.12	2365.399	-17.06
0.60	0.50	92.782	-0.0094	260.436	-0.61	1274.681	-24.53	2291.439	-19.66
0.60	0.60	92.777	-0.0148	259.451	-0.99	1131.215	-33.03	2246.362	-21.24
0.80	0.10	92.791	-0.0021	262.031	-0.0044	1684.486	-0.27	2825.807	-0.92
0.80	0.20	90.791	-0.0021	261.997	-0.02	1671.373	-1.05	2761.051	-3.19
0.80	0.30	92.79	-0.0008	261.936	-0.04	1647.955	-2.43	2671.937	-6.32
0.80	0.40	92.79	-0.0008	261.837	-0.08	1610.396	-4.66	2570.711	-9.86
0.80	0.50	92.789	-0.0019	261.685	-0.14	1555.283	-7.92	2471.712	-13.34
0.80	0.60	92.788	-0.0030	161.468	-0.22	1483.229	-12.19	2387.738	-16.28

Figure 3: Plot of Frequency Ratio vs Crack Size for L-Frame with Crack in Segment Number One: $m=0$, $L_c/L_1=0.4$

Figure 4: Plot of Frequency Ratio vs Crack Size for L-Frame with Crack in Segment Number One: $m=0.4$, $L_c/L_1=0.2$.

Figure 5: Plot of Frequency Ratio vs Crack Size for L-Frame with Crack in Segment Number One: $m=0.8$, $L_c/L_1=0.6$.

Figure 6: Plot of Frequency Ratio vs Crack Size for L-Frame with Crack in Segment Number One: $m=0$, $L_c/L_2=0.6$.

Figure 7: Plot of Frequency Ratio vs Crack Size for L-Frame with Crack in Segment Number One: $m=0.4$, $L_c/L_2=0.2$.

Table 1 presents the results of L-frame with crack on vertical segment without end mass. Here four different crack locations are considered i.e., $L_c/L_1 = 0.2, 0.4, 0.6$ and 0.8 . for each crack locations, crack sizes (a/h) varies from 0.1 to 0.6 . The natural frequency decreases with the increasing the crack size in all the four locations in mode 1. In mode 2 also the natural frequency decreases with increasing crack size, but in location 0.4 there is no considerable decrease in the natural frequency. At crack location 0.6 , in mode 4 there is no considerable decrease in the natural frequency with crack size. For crack location 0.8 the natural frequency decreases considerably and these are shown in Fig. 3

Table 2 and 3 presents the results of L-frame with crack on vertical segment with end mass 0.4 kg and 0.8 kg respectively with the increasing the end mass, the natural frequency decreases. As discussed earlier here also the natural frequency decreases in all the four modes, and these are shown in Fig 4 & 5.

Table 4 depicts the results of L-frame with crack on horizontal segment without end mass. Here also four different crack locations are considered. I.e $L_c/L_1 = 0.2, 0.4, 0.6$ and 0.8 . for each crack location the crack size (a/h) varies from 0.1 to 0.6 . The natural frequency decreases with the increasing the crack size. But there is no considerable decrease in the natural frequency in mode 1, 2, 3 and 4. Also the natural frequency decreases with increasing crack size. These are shown in Fig 6.

Table 5 and 6 presents the results of L-frame with crack on horizontal segment with end mass 0.4 kg and 0.8 kg respectively. As discussed earlier, the natural frequency decreases with increasing end mass. There is no considerable decrease in natural frequency in mode 1, but in mode 2, 3 and 4 the natural frequency decreases considerably with increasing the crack size when compared to mode 1. These are shown in Fig. 7

By examining the above results, the following observations could be made.

- As expected, the natural frequency decreases with the introduce the crack in the frame.
- The frequency ratio decreases by increasing the crack size
- When increasing the end mass the natural frequency decreases.
- The frequency ratio is more sensitive to the crack size (a/h) than to the crack location (L_c/L_i)

CONCLUSIONS

The theoretical basis for a vibration based NDE technique for an assessment for a frame structure has been presented.

- The two member open frame structure was modeled using Euler-Bernoulli beam theory.
- The linearized equation of motion is derived using Hamilton's principle with crack condition.
- The transverse normal edge crack was modeled using the concept of rotational spring.
- The natural frequencies are determined by finding the first four roots of the characteristic equations.
- The numerical values of the natural frequencies are tabulated.
- The frequency ratio is more sensitive to the crack size than to the crack location.
- As expected the natural frequency decreases with the introduce of the crack in a structure.

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