CU-WATER NANOFLUID NATURAL CONVECTIVE HEAT AND FLUID FLOW IN A POROUS CAVITY

L. JINO, A. VANAV KUMAR*

Department of Basic and Applied Science, National Institute of Technology Arunachal Pradesh, Arunachal Pradesh, India

ABSTRACT

A flow behavior with the effect of magnetic field on square shaped cavity filled up with Cu-water nanofluid in porous matrix is analysed in this study. The study is carried out for uniform and sinusoidally distributed thermal left boundary. Nanofluid flow in a cavity is designed mathematically using a set of governing equations, which is non-dimensionalized and solved along with the necessary boundary conditions. Finite difference technique is incorporated to discretization of equation in a fixed domain. The flow patterns are discussed using ψ (streamlines), θ (Isotherms), Π (Heatlines) and ϕ (Iso-concentration). Variation of average Nusselt number is discussed for various values of Hartmann number, Rayleigh number, Lewis number, Brownian motion parameter, thermophoresis parameter and Darcy number. An enhancement of flow circulation and heat transfer is found in uniformly heated thermal boundary compared with the sinusoidally heated left boundary.

KEYWORDS: Natural Convection, Heatlines, Magnetic Field, Cu-Water Nanofluid, Porous

Received: Jun 08, 2020; **Accepted:** Jun 28, 2020; **Published:** Sep 17, 2020; **Paper Id.:** IJMPERDJUN20201305

INTRODUCTION

The heat transfer amplification of nanofluid plays an important role in many engineering and many other technological applications. The flow motion in a porous media help in understanding the fluid flow in a fluidized and packed beds, flow in a dendritic structures, drying and freezing of foods, phase change of materials, fabrication and extraction related works in a metallurgical industries, tissue engineering and magnetohydrodynamics flow in nuclear reactor, crystal growth (Bejan et al. (2004); Ingham and Pop (1998)). The fluid pass over a porous matrix can be described using the Carman-Kozeny equation (Hasadi (2013); Richardson et al. (2002)). A short literature survey on flow regimes, temperature distribution for various porous models in a cavity of different shape is reviewed by Kasaeian et al. (2017). An increase in an Hartmann number leads to increase in energy transfer which leads to increase the solidification rate and reduces the time for phase change, analyzed by Sheikholeslami and Mahian (2019). Javed et al. (2017) discussed the natural convection and magnetic field effect in a porous cavity filled with the ferrofluid. The results describe the domination of convective heat transfer, average Nusselt number and strength of the streamlines increased for increase in Ra , Da and Pr . Also, reverse effect is found in the increment of Ha . The magnetic field effect in a cavities of different shape filled with water based nanofluid is discussed by Sheikholeslami et al. (2020); Sheikholeslami (2017); Sheikholeslami (2018). Results indicate that, increment in heat transfer occurs for decrease in magnetic field effect and increase in the solid volume fraction of the nanoparticle, Rayleigh number, Darcy number. Rashad et al. (2017) studied the effect of internal heat generation and magnetic field effect in saturated porous cavity filled up with the Cu-water nanofluid. The study highlights that, the flow velocity in a cavity is decreased for the increase in Hartmann number, solid volume

fraction and average Nusselt number decreases for increase in Hartmann number and decrease in solid volume fraction. Gorla et al. (2017) quotes that, an addition of Cu nanoparticle gives the better heat transfer rate over the other nanoparticles in a cavity filled up with hybrid nanofluid. Increase in Hartmann number also reduces the flow field strength of hybrid nanofluid. Zahmatkesh and Ardekani (2020) conducted heat visualization study for analysing the corientation of magnetic field in a cavity. The horizontally orientated effect of magnetic field is more than oriented vertical or inclined magnetic field. Sheremet and Pop (2014); Sheremet et al. (2016) did an investigation on the effect of sinusoidally distributed temperature boundary on vertical walls of porous square cavity filled with water based nanofluid using Buongiorno's model. The Nusselt number increases with the increase in thermophoresis parameter and average Nusselt number decreases with increase in Brownian motion parameter. Basak et al. (2006) investigated the natural convective effect for uniform and non-uniformly heated boundary on porous square cavity. Kumar et al. (2019) discussed the flow patterns in a porous cavity (using Carman-Kozeny equation) due to non-uniform heated boundary and applied magnetic field.

Basak and Chamkha (2012) conducted heatline analysis on free convective in a square cavity containing the nanofluid using various thermal boundary conditions. Motlagh et al. (2016) discussed the natural convection heat transfer and fluid flow in an inclined porous cavity using Buongiorno's model. Mehryan et al. (2017) investigated the free convection effect in Al_2O_3 –water and Al_2O_3 –water–Cu nanofluid over a porous matrix. Kosti (2014) discussed the buoyancy driven convection effects in a cavity filled by Cu-water nanofluid due to heat flux boundary. Heat transfer in porous cavity filled with non-newtonian nanofluid increases due to increase in Darcy number, solid volume fraction, Rayleigh number and power-law indexes, studied by Kefayati (2016). Heat transfer in triangular, rectangular and trapezoidal shaped cavity with the non-conductive partition present in between is analysed by Almakhyoul et al. (2019). Deng (2008) discussed the heat flow in a cavity with three pairs of source-sink in a vertical walls using the heatline visualization technique. Aminossadati and Ghasemi (2011); Ghasemi et al. (2011) discussed the effect of Hartmann number for the various values solid volume fraction of nanoparticles and Rayleigh number in a square cavity. Rudraiah et al. (1995) investigated the natural convection of the electrical conducting fluid using the ADI scheme and SOR method.

The particular study involves the discussion of streamlines, isotherms, heatlines and iso-concentration in a porous cavity due to natural convective and magnetic field effects.

PROBLEM MODEL

The two-dimensional square cavity of dimension $H \times H$ is filled with nanofluid where, collision of Cu nano sized particle in water. Two horizontal walls of adiabatic temperature boundaries and vertical walls are with appropriate boundary conditions. Right sided vertical wall is uniformly distributed lower temperature and left sided with higher temperature of uniform/non-uniformly distributed boundaries as shown in Figure 1. All the walls are considered as immobile with zero velocities ($u = v = 0$). The gravitational effects (g) are considered as acting in negative vertical direction. The applied constant magnetic field effects are acts horizontally in the cavity.

Figure 1 : Physical problem

MATHEMATICAL DESCRIPTION

The flow in a cavity is assumed to be laminar, incompressible, Newtonian, neglecting the viscous dissipation and radiation effects with valid Boussinesq approximation. Which are described mathematically as continuity, momentum energy equation and iso concentration equation.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\rho_n \left[\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = \mu_n \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\partial p}{\partial x} - \rho_n \frac{A(1-\lambda)^2}{\lambda^3} u \quad (2)$$

$$\rho_n \left[\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = \mu_n \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\partial p}{\partial y} - \rho_n \frac{A(1-\lambda)^2}{\lambda^3} v + (\rho\beta)_n g(T - T_{ref}) - \sigma_n B^2 v \quad (3)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_n \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \delta \left[D_B \left(\frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + \frac{D_T}{T_c} \left\{ \left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right\} \right] \quad (4)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + D_T \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) / T_c \quad (5)$$

Here a two-dimensional flow is considered with the velocities u, v at the directions x, y . p is pressure, T in an energy equation is the temperature, C in a concentration equation is the nanoparticle concentration and co-efficients in governing equations such as ρ_n, μ_n and k_n are the nanofluids density, viscosity and thermal conductivity respectively. D_B and D_T in concentration equation represents the Brownian and thermophoresis diffusion coefficient respectively. The fluid passes over the porous matrix is defined using the Carman-Kozeny equation.

The governing equation (1-5) can be rewritten as the non-dimensional form as,

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (6)$$

$$\frac{\partial U}{\partial \tau} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\mu_n}{\rho_n \alpha_{fl}} \left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \frac{\partial P}{\partial X} - \rho_{fl} \frac{Pr}{Da} U \quad (7)$$

$$\frac{\partial V}{\partial \tau} + U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = \frac{\mu_n}{\rho_n \alpha_{fl}} \left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \frac{\partial P}{\partial Y} - \rho_{fl} \frac{Pr}{Da} V + \frac{(\rho\beta)_n}{\rho_n \beta_{fl}} Ra Pr(\theta) - Ha^2 Pr(V) \quad (8)$$

$$\frac{\partial \theta}{\partial \tau} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_n}{\alpha_{fl}} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + Nb \left(\frac{\partial \phi}{\partial X} \frac{\partial \theta}{\partial X} + \frac{\partial \phi}{\partial Y} \frac{\partial \theta}{\partial Y} \right) + Nt \left\{ \left(\frac{\partial \theta}{\partial X} \right)^2 + \left(\frac{\partial \theta}{\partial Y} \right)^2 \right\} \quad (9)$$

$$\frac{\partial \phi}{\partial \tau} + U \frac{\partial \phi}{\partial X} + V \frac{\partial \phi}{\partial Y} = \frac{1}{Le} \left(\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} \right) + \frac{Nt}{Nb, Le} \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (10)$$

where the non-dimensional variables and non-dimensional numbers used for the transformation of dimensionless governing equation are given by

$$\begin{aligned} (X, Y) &= (x, y)/H & (U, V) &= H(u, v)/\alpha_f \\ \tau &= (t\alpha_f)/H^2 & P &= (pH^2)/(\rho_n\alpha_f^2) \\ Pr &= \nu_f/\alpha_f & \phi &= C/C_0 \\ \theta &= (T - T_c)/(T_h - T_c) & Ha &= BH\sqrt{\sigma_n/(\rho_n\nu_n)} \\ Da &= \mu_f\lambda^3/A(1-\lambda)^2H^2 & Ra &= g\beta_fH^3(T_h - T_c)/\nu_f\alpha_f \\ Nt &= \delta D_T(T_h - T_c)/\alpha_fT_c & Nb &= \delta D_B C_0/\alpha_f \\ Le &= \alpha_f/D_B \end{aligned}$$

Transport properties of the nanofluids such as thermal conductivity, viscosity, density, heat capacitance and thermal expansion coefficient are given as a function of nanoparticle solid volume fraction.

$$\begin{aligned} \mu_n &= \mu_{fl}/(1 - \phi_0)^{2.5} \\ \rho_n &= (1 - \phi_0)\rho_{fl} + \phi_0\rho_p \\ (\rho c_p)_n &= (1 - \phi_0)(\rho c_p)_{fl} + \phi_0(\rho c_p)_p \\ (\rho\beta)_n &= (1 - \phi_0)(\rho\beta)_{fl} + \phi_0(\rho\beta)_p \\ k_n &= k_{fl} \left[[k_p + 2k_{fl} - 2\phi_0(k_{fl} - k_p)] / [k_p + 2k_{fl} - 2\phi_0(k_{fl} - k_p)] \right] \end{aligned}$$

No slip velocity boundary conditions are applied and the proposed dimensionless thermal, concentration boundaries of square cavity are given by,

$$\text{Left wall: } \theta = 1 \text{ (or) } \sin(\pi Y), Nb \left(\frac{\partial \phi}{\partial n} \right) + Nt \left(\frac{\partial \theta}{\partial n} \right) = 0,$$

$$\text{Right wall: } \theta = 0, Nb \left(\frac{\partial \phi}{\partial n} \right) + Nt \left(\frac{\partial \theta}{\partial n} \right) = 0,$$

$$\text{Horizontal walls: } \partial \theta / \partial n = 0, \partial \phi / \partial n = 0.$$

The average Nusselt number ($Nu = -k_n/k_{fl} \int (\partial \theta / \partial X) dY$) is used to demonstrate the heat transfer from the left wall. Heat flow in a cavity is demonstrated using the heat function (or heatlines, $\nabla^2 \Pi = \partial U \theta / \partial Y - \partial V \theta / \partial X$).

Figure 2 : Validation of current work (bottom) against published standard results by Basak et al. (2006) (top).

NUMERICAL SOLUTIONS

The equations (6-10) are formulated according to vorticity-streamfunction approach, discretized using finite difference method and solved implicitly to attain the steady state. The simulations are stopped once the convergence criteria (two consecutive iteration error of order below 10^{-5}) are satisfied.

The study is initialized after validating the work against the standard published results by Basak et al. (2006) as shown in Figure 2 . Here, streamlines and isotherms are good in agreement with validation. An evenly placed grid of 120×120 grid is selected in the study.

RESULTS AND DISCUSSIONS

The flow patterns are attained for the Rayleigh number, $Ra = 10^6$, Prandtl number for the working fluid is considered as, $Pr = 6.2$, nano particle solid volume fraction, $\phi_0 = 0.1$, Darcy number, $Da = 10^{-2}, 10^2$ and Hartmann number, $Ha = 0, 50, 100$. An average Nusselt number is discussed with the various values Ra, Ha, Da, Le (Lewis number), Nt (thermophoresis parameter), and Nb (Brownian motion parameter). The values of constant transport properties of water and Cu nano particle is listed in the Table 1.

Table 1 : Properties of water, Cu nanoparticle

	Water	Cu Nanoparticles
ρ	997.1 kg/m^3	8954 kg/m^3
μ	$8.9 \times 10^{-4} \text{ Pa s}$	—
c_p	4179 J/kg K	383 J/kg K
k	0.6 W/m K	400 W/m K
β	$2.1 \times 10^{-4} \text{ K}^{-1}$	$1.67 \times 10^{-5} \text{ K}^{-1}$

Figure 3 represents the streamlines (ψ), isotherms (θ), heatlines (Π) and iso-concentration (ϕ) for uniform heated left boundary at $Da = 10^{-2}, Le = 10, Nt = 0.1, Nb = 0.1$ and $Ra = 10^6$. A reduction in middle of the flow

circulation is found by increasing the Ha . Parallel lines found in the heatlines of the cavity denotes the reduction in convective heat transfer effect, whereas a small circulation near left bottom of the cavity. Iso-concentration contour in a cavity shows that lesser distribution of nanoparticles near the heated wall. Further lesser effect is found in the heat transfer regime during increase in the Ha .

An increase in Da to 10^2 improves the permeability and thus streamlines denotes increased flow intensity as shown in Figure 4. By increasing the Darcy number, a hike in flow circulation intensity ($\psi_{max} = -19$) is found. Also, sharp bend near the vertical wall and huge circulation in the heatlines states that, there is a domination in convection over the conduction. While increasing the Hartmann number, flow circulation intensity drops to $\psi_{max} = -4.8$ at $Ha = 100$. Reduction of bends in isotherms and decrease of circulation intensity in heatlines over increase in Ha leads to reduction in convection effects. Increase in Hartmann number makes the pull of nanoparticle concentration to right due to increase in magnetic strength.

Figure 3 : Flow patterns for $Da = 10^{-2}$, $Ra = 10^6$, a – $Ha = 0$, b – $Ha = 50$, and c – $Ha = 100$ at uniform heated boundary

Figure 4 : Flow patterns for $Da = 10^2$, $Ra = 10^6$, $a - Ha = 0$, $b - Ha = 50$, and $c - Ha = 100$ at uniform heated boundary

Figure 5 : Flow patterns for $Da = 10^{-2}$, $Ra = 10^6$, $a - Ha = 0$, $b - Ha = 50$, and $c - Ha = 100$ at non-uniform heated boundary

Figure 6 : Flow patterns for $Da = 10^2$, $Ra = 10^6$, a – $Ha = 0$, b – $Ha = 50$, and c – $Ha = 100$ at non-uniform heated boundary

Figure 5 represents the flow contours for non-uniform thermal boundary (sinusoidally heated) for various values of Hartmann number at $Da = 10^{-2}$, $Nt = Nb = 0.1$, $Le = 10^1$, $Ra = 10^6$. Maximum values of streamlines are reduced, compared to the uniformly heated boundary. Because of maximum temperature contour, $\theta = 1$ (can be visualized in Isotherm plots) originates in the middle of the wall, a secondary circulation starts to originate from both the corner of the left wall. Heatline contours shows that, the heat dissipates from the middle of the left wall. At lower Da , non-uniformly heated case, no circulation is found in the heatline and thus signifies the domination of convection is very less. Iso-concentration profile clearly denotes that, the distribution of nanoparticle near the heated zone is less. Due to lower Da , an effect of Ha is less.

Secondary circulations at the top left corner in streamline got stronger for increasing the $Da = 10^2$ is shown in Figure 6. Weakening of primary circulation and improved circulation of secondary circulation is found for higher in the Hartmann number. Isotherm bends and weakening of circulation in heatline shows that reduction in convective heat transfer effect for higher Ha . The location of circulation in heatline shows that, the maximum convection heat transfer at left middle of the cavity. Higher nanoparticle regime is found in both the left corner and right wall of the cavity.

Figure 7 : Variation of Nu with respect to Ra and Ha for $Da = 10^{-2}$ (left) and $Da = 10^2$ (right).

Figure 8 : Variation of Nu with respect to Le and Ha for $Da = 10^{-2}$ (left) and $Da = 10^2$ (right).

Figure 9 : Variation of Nu with respect to Nb and Ha for $Da = 10^{-2}$ (left) and $Da = 10^2$ (right).

Figure 10 : Variation of Nu with respect to Nt and Ha for $Da = 10^{-2}$ (left) and $Da = 10^2$ (right).

Changes in Nu due to variation in Ra and Ha at $Da = 10^{-2}, 10^2$, $Le = 10$, $Nb = 0.1$, $Nt = 0.1$ is shown in Figure 7. Nu decreases for increase in the Ra at lower Da , due to reduction in conduction dominated heat transfer with increase in convective effect. In other case, increase in Nu is occurred for increase in Ra at higher $Da = 10^2$. This is because of more permeable, increase in convective heat transfer occurs. Lesser heat transfer is found in the non-uniform heated case over the uniformly heated case. Effect of magnetic field by increasing the Ha causes the reduction in heat transfer for higher Da . By increasing the Ha , the effect of convection is suppressed and at lower $Da = 10^{-2}$, convective effect gets weaken and supports conduction to increase in Nu at higher Ha . Figure 8 shows that, reduction in Nusselt number for increase in $Le = 1$ to $Le = 100$. The concentration of nanoparticle distribution is reduced for higher Lewis number. Rate of reduction in Nu is more at lower Da compared to higher Darcy number. A small reduction in heat transfer for increase in Brownian motion parameter $Nb = 0.1$ to 0.4 is found in Figure 9 for both the cases. Figure 10 displays the heat transfer due to variation in thermophoresis parameter. An increase in Nt leads to increase in Nu due to increase in thermal diffusion.

CONCLUSIONS

Natural convection effect in a porous cavity under uniform and non-uniform heated left boundary with the magnetic field effect is analyzed. The fluid, heat, nanoparticle concentration flow patterns are obtained. Based on the results, the suppression of streamlines flow field is occurred for increasing the magnetic field strength. The flow gets strengthen by increasing the Darcy number and Rayleigh number. Heatlines describes the heat flow over the cavity is dominated by condition or convection. The change in average Nusselt number is discussed for various non-dimensional parameter such as Da, Le, Ra, Nt, Ha, Nb .

ACKNOWLEDGEMENTS

The first author acknowledges the partial financial support through TEQIP-III Project, NIT Arunachal Pradesh.

The authors acknowledge the support through DST project. Sanction number : SERB ECR/2017/001007.

REFERENCES

1. Almakhyoul, Z. M., Ali, R. A., & Aldabbagh, M. S. (2019). Effect of staggered non-conductive partition on heat transfer in a variable shape cavity. *International Journal of Mechanical and Production Engineering Research and Development*, 9(5), 381–390. <https://doi.org/10.24247/ijmperdoct201933>
2. Aminossadati, S. M., & Ghasemi, B. (2011). Natural convection of water – CuO nano fluid in a cavity with two pairs of heat source – sink. *International Communications in Heat and Mass Transfer*, 38(5), 672–678. <https://doi.org/10.1016/j.icheatmasstransfer.2011.03.013>

3. Basak, T., & Chamkha, A. J. (2012). Heatline analysis on natural convection for nanofluids confined within square cavities with various thermal boundary conditions. *International Journal of Heat and Mass Transfer*, 55(21–22), 5526–5543. <https://doi.org/10.1016/j.ijheatmasstransfer.2012.05.025>
4. Basak, T., Roy, S., Paul, T., & Pop, I. (2006). Natural convection in a square cavity filled with a porous medium: Effects of various thermal boundary conditions. *International Journal of Heat and Mass Transfer*, 49(7–8), 1430–1441. <https://doi.org/10.1016/j.ijheatmasstransfer.2005.09.018>
5. Bejan, A., Dincer, I., Lorente, S., Miguel, A., & Reis, H. (2004). *Porous and complex flow structures in modern technologies*. Springer Science & Business Media.
6. Deng, Q. (2008). *International Journal of Heat and Mass Transfer* Fluid flow and heat transfer characteristics of natural convection in square cavities due to discrete source – sink pairs. *International Journal of Heat and Mass Transfer*, 51(25–26), 5949–5957. <https://doi.org/10.1016/j.ijheatmasstransfer.2008.04.062>
7. Ghasemi, B., Aminossadati, S. M., & Raisi, A. (2011). Magnetic field effect on natural convection in a nanofluid-filled square enclosure. *International Journal of Thermal Sciences*, 50(9), 1748–1756. <https://doi.org/10.1016/j.ijthermalsci.2011.04.010>
8. Gorla, R. S. R., Siddiqua, S., Mansour, M. A., Rashad, A. M., & Salah, T. (2017). Heat source/sink effects on a hybrid nanofluid-filled porous cavity. *Journal of Thermophysics and Heat Transfer*, 31(4), 847–857. <https://doi.org/10.2514/1.T5085>
9. Hasadi, Y. M. F. El. (2013). Numerical Simulation of the Effect of the Size of Suspensions on the Solidification Process of Nanoparticle-Enhanced Phase Change Materials. May. <https://doi.org/10.1115/1.4023542>
10. Ingham, D. B., & Pop, I. (1998). *Transport phenomena in porous media*. Elsevier.
11. Javed, T., Mehmood, Z., & Abbas, Z. (2017). Natural convection in square cavity filled with ferrofluid saturated porous medium in the presence of uniform magnetic field. *Physica B: Condensed Matter*, 506, 122–132. <https://doi.org/10.1016/j.physb.2016.11.008>
12. Kasaeian, A., Azarian, R. D., Mahian, O., Kolsi, L., Chamkha, A. J., Wongwises, S., & Pop, I. (2017). Nanofluid flow and heat transfer in porous media: A review of the latest developments. *International Journal of Heat and Mass Transfer*, 107, 778–791. <https://doi.org/10.1016/j.ijheatmasstransfer.2016.11.074>
13. Kefayati, G. H. R. (2016). Heat transfer and entropy generation of natural convection on non-Newtonian nanofluids in a porous cavity. *Powder Technology*, 299, 127–149.
14. Kumar, A. V., Jino, L., Berlin, M., & Mohanty, P. K. (2019). Magnetic field effect on nanofluid suspension cavity by non-uniform boundary conditions. *AIP Conference Proceedings*, 2134(August). <https://doi.org/10.1063/1.5120205>
15. Mehryan, S. A. M., Kashkooli, F. M., Ghalambaz, M., & Chamkha, A. J. (2017). Free convection of hybrid Al₂O₃-Cu water nanofluid in a differentially heated porous cavity. *Advanced Powder Technology*, 28(9), 2295–2305. <https://doi.org/10.1016/j.appt.2017.06.011>
16. Rashad, A. M., Rashidi, M. M., Lorenzini, G., Ahmed, S. E., & Aly, A. M. (2017). Magnetic field and internal heat generation effects on the free convection in a rectangular cavity filled with a porous medium saturated with Cu–water nanofluid. *International Journal of Heat and Mass Transfer*, 104, 878–889. <https://doi.org/10.1016/j.ijheatmasstransfer.2016.08.025>
17. Richardson, J. F., Harker, J. H., & Backhurst, J. R. (2002). Flow of fluids through granular beds and packed columns. *Chemical Engineering*, 2, 191–236.
18. Rudraiah, N., Barron, R. M., Venkatachalappa, M., & Subbaraya, C. K. (1995). Effect of a magnetic field on free convection in a rectangular enclosure. *International Journal of Engineering Science*, 33(8), 1075–1084. <https://doi.org/10.1016/0020->

7225(94)00120-9

19. Sheikholeslami, M., Arabkoohsar, A., & Ismail, K. A. R. (2020). Entropy analysis for a nanofluid within a porous media with magnetic force impact using non-Darcy model. *International Communications in Heat and Mass Transfer*, 112, 104488. <https://doi.org/10.1016/j.icheatmasstransfer.2020.104488>
20. Sheikholeslami, M. (2017). CuO-water nanofluid free convection in a porous cavity considering Darcy law. *European Physical Journal Plus*, 132(1). <https://doi.org/10.1140/epjp/i2017-11330-3>
21. Sheikholeslami, M., & Mahian, O. (2019). Enhancement of PCM solidification using inorganic nanoparticles and an external magnetic field with application in energy storage systems. *Journal of Cleaner Production*, 215, 963–977. <https://doi.org/10.1016/j.jclepro.2019.01.122>
22. Sheikholeslami, Mohsen. (2018). CuO-water nanofluid flow due to magnetic field inside a porous media considering Brownian motion. 249, 921–929. <https://doi.org/10.1016/j.molliq.2017.11.118>
23. Sheremet, M A, & Pop, I. (2014). Natural Convection in a Square Porous Cavity with Sinusoidal Temperature Distributions on Both Side Walls Filled with a Nanofluid: Buongiorno ' s Mathematical. 411–429. <https://doi.org/10.1007/s11242-014-0375-7>
24. Sheremet, Mikhail A, Pop, I., & Ro, N. C. (2016). Magnetic field effect on the unsteady natural convection in a wavy-walled cavity filled with a nanofluid : Buongiorno ' s mathematical. 000, 1–12. <https://doi.org/10.1016/j.jtice.2015.12.015>
25. Siddhartha Kosti. (2014). Effect of Heat-Flux Boundary on Buoyancy-Driven Flow in a. *International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)*, 4(4), 23–36.
26. Yekani Motlagh, S., Taghizadeh, S., & Soltanipour, H. (2016). Natural convection heat transfer in an inclined square enclosure filled with a porous medium saturated by nanofluid using Buongiorno's mathematical model. *Advanced Powder Technology*, 27(6), 2526–2540.
27. Zahmatkesh, I., & Ardekani, R. A. (2020). Effect of magnetic field orientation on nanofluid free convection in a porous cavity: A heat visualization study. *Journal of Thermal Engineering*, 6(1), 170–186. <https://doi.org/10.18186/THERMAL.672297>