DYNAMIC ANALYSIS OF COAXIAL ROTOR SYSTEMS

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ABSTRACT

Unmanned aerial vehicles (UAV's), also known as drones, are becoming more popular from being used for military purposes to space exploration. While the current airframes face difficulty in speed, agility, and conformance to purpose, this paper explores the dynamics & forces experienced by the rotor system of a coaxial drone which is still under limited research exposure. The prototype designed here can be used to carry generic payloads that can be adapted for a wide range of applications such as Remote Sensing, Identification of objects, Monitoring events, traffic surveillance, delivery, and even for agricultural purposes. This system has better dynamic stability, compact in design, and reduces energy swirl losses.

KEYWORDS: Coaxial drones, Figure of merit (FM), Disc loading, Inter-rotor Separation, Newton-Euler Equation & Dynamic force Analysis

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1.INTRODUCTION

Coaxial propeller systems have been used successfully in commercial and military aircraft for many years due to their inherent efficiency savings (3–9%) over conventional single propeller systems [1]. The aim of this paper is to both review and analyze coaxial rotor systems with special reference to applications of this innovative propulsion system within small UAVs. Applying coaxial rotors, the single biggest advantage is the redundancy of a tail rotor due to the inherent yaw (torque) cancellation effect. A conventional tail rotor has been estimated to consume 5–10% of the total power supplied by the engines, sometimes up to 20% at the extremes of the flight envelope [2].

Configuration of Coaxial Rotor Systems: Coaxial rotors or coax rotors are a pair of helicopter rotors mounted one above the other on concentric shafts, with the same axis of rotation, but turning in opposite directions (contra-rotating) as shown in Figure 1

Figure 1: Open Section View of a Coaxial Drones.

Theoretical Principle - It was theorized and proven that a helicopter with two counter-rotating rotors could be built such that the rotational force of one rotor counteracts the rotational reaction force of the other, leaving the helicopter body stable without the need for a perpendicularly acting tail rotor.[4]

THEORETICAL ANALYSIS OF COAXIAL ROTOR SYSTEMS

As aerodynamics and aeromechanics have the greatest influence on SUAV during the overall performance in-flight operating condition, therefore this section would briefly discuss the necessary parameters for the determination of flight efficiency and all the core components that influence the coaxial rotor system in the flight condition of hover [2]. The Figure of Merit (FM) is a non-dimensional efficiency metric that provides a basis to conduct a relative comparison of rotor performance. So it uses the ratio of “ideal power to hover” to “actual power used to hover” [3].

$$FM = \frac{\text{Real power Required to hover}}{\text{hoverActual power required to hover}}$$

Where FM is,

An equation for the Figure of Merit by Leishman [3] is given as follows:

$$FM = \frac{1.2657 \frac{C_{Tl}^{3/2}}{\sqrt{2}} \left[\left(\frac{C_{Tu}}{C_{Tl}} \right)^{3/2} + 1 \right]}{C_{P_{meas}}}$$

In terms of measured coaxial systems power, the definition for FM is:

$$FM = \frac{1.2657 \frac{C_{Tl}^{3/2}}{\sqrt{2}} \left[\left(\frac{C_{Tu}}{C_{Tl}} \right)^{3/2} + 1 \right]}{K_{int} K \frac{C_{Tl}^{3/2}}{\sqrt{2}} \left[\left(\frac{C_{Tu}}{C_{Tl}} \right)^{3/2} + 1 \right] + \frac{\sigma C_{d0}}{4}}$$

Where,

$C_{Tu} + C_{Tl}$ = Rotor Thrust Coefficient (Upper and Lower)

$C_{P_{meas}}$ = Rotor Power Coefficient (measured)

σ^+ = Rotor solidity

C_{d0} = Minimum or zero-lift Drag Coefficient

- **The Formula of Efficiency of Motors and ESC's** - Motor and ESC convert electrical power into mechanical power, delivered at the shaft of the motor.

The efficiency of motor and ESC combined is therefore calculated as:

$$\eta_{\text{motor} + \text{ESC}} = \frac{\omega Q}{U I} = \eta_{\text{motor}} \eta_{\text{ESC}}$$

where ω = angular speed,

Q = torque measured at the motor shaft,

U = voltage,

I = Current at power sources

• **In hovering condition, the efficiency of the propeller, η_{plr}** - This is also called the Figure of merit. The following mentioned equation comes from the *Rankine-Froude momentum* equation [10]

$$V_0 = 0, V_1 = v_i$$

$$T = 2\rho A(v_i)^2, \quad v_i = \sqrt{\frac{T}{2\rho A}}$$

$$P_i = T V_1$$

$$\eta_{plr} = \frac{P_i}{P_{mech}} = \frac{T^{3/2}}{Q \omega \sqrt{2\rho A}}$$

where,

V_0 = the airspeed ahead of the propeller,

V_1 = the airspeed at of the propeller,

V_2 = the airspeed behind the propeller, after the contraction of the flow tube as illustrated in (Figure -2).

T = thrust produced by the propeller,

v_i = is induced velocity,

The efficiency of η_{plr} is calculated by the ratio of P_i and P_{mech}

Figure 2: Momentum Theory Applied to a propeller in Hover Condition.

• **The Total Efficiency of the Total Propulsion system is calculated as:**

$$\eta_{propulsion} = \frac{P_i}{P_{electrical}} = \frac{T^{3/2}}{IU\sqrt{2\rho A}} = \eta_{motor}\eta_{ESC}\eta_{plr}$$

Here, U = voltage of the power source

I = Delivered current to the propulsion system.

In this project report, the disk area A is defined as the projected surface of the turning propeller. For a coaxial pair of propellers, this area is the same as for a single propeller of the same diameter.

- **The Total Losses in the Coaxial System:** Figure given below shows losses in coaxial rotors during operations

Figure 3: Overview of Losses from Energy Sources to Kinetic Energy in the Air.

- **Disc Loading of a Hovering Craft-** It is the ratio of its (weight) to the (total main rotor disc area). When a helicopter is being maneuvered, its disc loading changes. The higher the loading, the more power needed to maintain rotor speed. A low disc loading is a direct indicator of high lift thrust efficiency. Increasing the weight of a helicopter increases disk loading. For a given weight, a helicopter with shorter rotors will have higher disk loading and will require more engine power to hover. A low disk loading improves autorotation performance in rotorcraft. Typically, an autogyro (or gyroplane) has a lower rotor disc loading than a helicopter, which provides a slower rate of descent in autorotation.

Further investigation of the coaxial rotor system will primarily revolve around Five-testing variables:

1.1.1 Inter-Rotor Spacing

The separation distance (H) between the coaxial rotor system discs. Inter-rotor spacing is one of the fundamental components of the UAV co-axial system which has been tested due to the associated aerodynamic effects; interference-induced power losses, wake contractions, and rotors vena contracta[2].

The H/D ratio is used as a non-dimensional figure to enable a comparison of multiple systems across a range of scales[2].

Aircraft Model	Rotor Dia.	Disc Area	Rotor Spacing	Disc Loading	H/D
Kamov Ka-50	14.50 m	330.3 m ²	1.42 m	30 kg/m ²	0.098
Cierva CR-Twin	10.06 m	74.7 m ²	0.76 m	21 kg/m ²	0.076
Sikorsky X2 [†]	8.04 m	51.0 m ²	0.46 m	54 kg/m ²	0.057
Gyrodyne QH-50	6.10 m	29.2 m ²	0.84 m	35 kg/m ²	0.138
Westland Wideye	2.30 m	4.2 m ²	0.31 m	30 kg/m ²	0.135
ML Aviation	1.60 m	2.0 m ²	0.20 m	18 kg/m ²	0.125

(SPRITE - Surveillance Patrol Reconnaissance Intelligence Target Designation Electronic Warfare)

$$\text{H/D Ratio} = \frac{\text{Separation distance}(m)}{\text{Rotor diameter}(m)}$$

So generally, it is seen that the H/D ratio lies between 0.08–0.14, and average rotor separation is of about 0.11.

1.1.2 Propeller Pitch

The propellers used in the coaxial configuration are fixed pitch, but unlike full-scale rotor blades which have an almost uniform pitch throughout the diameter due to the design preference of a symmetrical blade section [5]; the test propellers have a varying pitch.

There has been little research into the twist distribution and pitch of commercially available propellers used in a co-axial arrangement, the majority of investigations focus on traditional helicopter planforms and blade design. Results

from certain research papers indicate that “a higher blade twist is desired on the upper rotor to reduce induced losses of the co-axial unit, whereas a high blade twist on the lower rotor increases the induced losses” [7]. On the contrary, commercially available co-axial rotor systems such as the Himax CR2816-1100 unit recommend an increase in the lower rotor pitch. The airfoil characteristics of a small-scale coaxial rotorcraft were also analyzed by Samuel et al, where it was shown that profile drag accounts for 45% of the losses compared to 30% in full-scale helicopters [6].

Figure 4: Flow Model of a Coaxial Rotor System[8]

1.1.3 Propeller Diameter (upper and lower)

The diameter of a propeller is one of the most important characteristics in determining the induced power of a rotor system:

$$P_i = \frac{T^{2/3}}{\sqrt{2\rho A}}$$

It has been shown in studies that the larger the rotor diameter the lower the disc loading, induced velocities, and a decrease in induced power requirements [3]. It has been noted that an 8% reduction in the upper rotor radius enhances the performance of the lower rotor due to an increase of exposed clean air. This variable will be controlled only using a select “family” of propellers to determine the performance attributes related to the decrease of the upper and lower rotors[9].

1.1.4 Propeller Blade Configuration

It refers to the number of individual blades on a propeller. The most efficient propeller is actually single-bladed, but due to its imbalance, it is not practical for use in most flying aircraft. Increasing the number of blades on a propeller is a suitable substitute for increasing the size where space is constrained and has very similar characteristics. So 3-blade vs. 2-blade. “Most multi-rotors make use of propellers with two blades and these propellers are also most widely available. Some researchers also offer three-bladed versions of some of their propellers. In the following experiment a comparison is made between a two-blade and a three-blade Graupner 9x5 E prop. The data presented in figure 5, show that the three-bladed propeller is less efficient with an average drop in efficiency of around 4%. However, more blades result in a lower

efficiency rpm for the same thrust” [11].

This is indicated by a graph shown below.

Figure 5: Thrust as a Function of the rpm for a two-bladed propeller and three-Bladed Propeller [11].

This is the reason why small planes use 3 and 4 bladed propellers, they needed more thrust and power but could not fit larger propellers because they would hit the ground, so they added more blades. Originally multirotor largely used 2 bladed propellers like planes, they are more efficient for their size because they only have two blades creating drag in the air. But as motors became more powerful, the higher performance was required and frames became smaller, there was not enough space to increase the size of the propellers to get the required thrust and power needed. Due to complicated physics and fluid dynamics, increasing the number of blades is not as efficient as increasing size. So a propeller with double the number of blades will not perform as well as one twice the size but it does provide more thrust at the cost of slightly more power. Similar to the size section, increasing the number of blades will increase the amount of thrust and grip in the air, at the cost of responsiveness and increased power consumption [11].

1.1.5 Coaxial propellers setup

It is often a topic of discussion whether a counter-rotating pair of propellers, also referred to as coaxial propellers, is more efficient than a single propeller. Since coaxial propellers rotate in the opposite direction, swirl losses can be minimized compared to a single propeller. On the other hand, the downstream propeller operates in an air stream that is disturbed by the upstream propeller and can, therefore, lose efficiency. A carefully designed coaxial setup requires a downstream propeller with a slightly smaller diameter due to contraction of the flow, and a slightly higher pitch angle of the blades due to operation in an already accelerated flow. On most coaxial multi-rotors used nowadays however 2, the upstream and downstream propellers are a counter-rotating pair of identical propellers. Four different setups with coaxial Graupner 9x5 E propellers, as presented in Figure 6, are tested[11].

The performance resulting from each of these setups is presented in Figure 7. The left graph does not reveal any large differences in propulsion system efficiency. However, from the right graph, showing the relative difference in propulsion system efficiency compared to the conventional coaxial setup, it becomes clear that a single propeller propulsion system is significantly more efficient than the coaxial setups, but the difference becomes smaller at higher disk loadings. At the highest disk loading of the single propeller at full throttle, the coaxial setups even become slightly more efficient. The right graph also shows that the conventional coaxial setup is more efficient compared to the other coaxial setups. There is no noticeable difference between 10cm and 15cm of spacing between the propellers, however, for the setup in which the coaxial pair is very close at 3.5cm, there is a clear loss in efficiency. From these experiments, it is

concluded that the propellers require some space in between, and this space is best used to incorporate one arm, supporting both motors and propellers. It is also clear that coaxial propellers can be also interesting in terms of efficiency when high disk loadings are required for example if the UAV has to be compact or wind resistant. The additional weight due to the extra motor, ESC, and propeller for the coaxial setup compared to a single propeller system, was not taken into account in the evaluation of the efficiency. In practice, this extra weight has to be subtracted from the produced thrust of the propulsion system since the system itself becomes heavier[11].

Figure 6: Four different coaxial setups and a single-propeller to compare with [11].

Figure 7: Performance of different Coaxial Propeller Setups [11].

2. Flight Dynamics

The aircraft frame, also called the body-fixed frame, is described by a set of Cartesian axes attached to the aircraft with the center of mass as the origin. Conventionally, the x-axis points forward along with the nose, the y-axis points out along the starboard wing, and the z-axis points down [12].

Figure 8: Definition of Helicopter Body Axes.

The origin of the body-fixed coordinate frame is the center of gravity of the body, and the body is assumed to be rigid, an assumption that eliminates the need to consider the forces acting between individual elements of mass. We will model the helicopter as a rigid body moving in space. As shown in Figure 8, we use the variables (x, y, z) to represent the position of the helicopter in body coordinates [12].

Euler angles are a means of representing the spatial orientation of any frame of the space as a composition of rotations from a reference frame. The body axis system is fixed in the vehicle, with the origin, usually, the center of mass, consisting of the following axes: longitudinal (x-axis), lateral (y-axis), and normal or vertical (z-axis) [12].

2.1 NEWTON-EULER EQUATIONS

The center of mass for a system of particles, S, accelerates in an inertial frame (A) as if it were a single particle with mass m (equal to the total mass of the system) acted upon by a force equal to the net external force.

$$\mathbf{r}_c = \frac{1}{m} \sum_{i=1, N} m_i \mathbf{p}_i$$

Newton's Equation of motion for a single particle of mass m is given by $[\mathbf{F} = m\mathbf{a}]$

For a number of particle becomes

$$\mathbf{F} = \sum_{i=1}^N \mathbf{F}_i = m \frac{{}^A d \mathbf{A}_{\mathbf{v}^C}}{dt}$$

here, A denotes that we are computing all these quantities in inertial frame A.

C denotes that we are computing the velocity of the center of mass.

Since the right-hand side represents the rate of change of momentum

$$\mathbf{F} = {}^A d \mathbf{L} / dt$$

2.2 Rotational Equations of Motion for a Rigid Body

The rate of change of angular momentum of the rigid body B relative to C in A is equal to the resultant moment of all external forces acting on the body relative to C

$${}^A d {}^A \mathbf{H}^S / dt = \mathbf{M}_C^S$$

$\mathbf{M}_C^S$ denotes net moment from all external forces and torques about the reference C

Here ${}^A\mathbf{H}_C^S$ denotes angular momentum which is also defined by

$${}^A\mathbf{H}_C^S = \mathbf{I}_C \cdot {}^A\boldsymbol{\omega}^B$$

Since these computations are done in three dimensions (3-D) therefore, the angular momentum (${}^A\mathbf{H}_C^S$) is a three-dimensional vector, the angular velocity is a three-dimensional vector.

${}^A\boldsymbol{\omega}^B$ represents the angular velocity of B in A, which is a three-dimensional vector,

$\mathbf{I}_C$ represents inertia tensor with C as the origin, three-dimensional be written as 3 X 3 matrix

Since these computations are done in three dimensions (3-D) therefore, the angular momentum is a three-dimensional vector, the angular velocity is a three-dimensional vector.

3. DYNAMIC FORCE ANALYSIS

There are a number of forces acting on the rotor blades. This paper identifies all the forces predominantly acting on the coaxial system. The first force that acts on the rotor blades is its own weight downwards at its center of gravity(COG).

The second force is the lift/thrust force that acts on the blades on account of the cutting of airstream by the curvature of the blades [13].

The Third force, as the blades rotate, cut the airstream and therefore, the airstream also exerts a force on the blades due to the air resistance. This is the drag force that acts on the blades of the rotor. And finally, on account of the rotation of the rotor blades, there acts a centrifugal force on the blades. The next step is to consider the forces separately and find out their values for the rotor blades individually [13].

Forces acting on the subject:

- Weight: Weight is the force of gravity. It acts in a downward direction—toward the center of the Earth.
- Lift Force: Lift is the force that acts at a right angle to the direction of motion through the air. Lift is created by differences in air pressure
- Thrust Force: It is the amount of upward force a drone can produce when at the throttle. To get a drone airborne, we need the right amount of thrust from its motor.
- Drag Force: Drag is the force that acts opposite to the direction of motion. Drag is caused by friction and differences in air pressure.

3.1 Weight

Due to the mass of the drone, there acts a weight force downwards at the center of gravity of the drone. Now, we know that:

$$\mathbf{W} = \mathbf{M}_d \mathbf{g}$$

3.2 Lift/Thrust Force

When the blade cuts across an airstream, there is a flow of air both above and below the blade. The blade profile is such that the air that passes above the blade has to travel more distance in the same time as compared to the air that passes

below the blade. Hence, the air that passes above the blade has a higher velocity than the air that passes below it. Now, according to Bernoulli's principle, the fluid having higher velocity is at a lower pressure. Hence, the air above the blade, due to its higher velocity, is at a lower pressure. That means there is a pressure difference between the air above and below the blade. Thus, there arises a force from the high-pressure region to the low-pressure region. This is the lift force that acts essentially in the upward direction due to the blade profile.

At zero speed, or with the drone at stationary condition, the thrust produced is known as **static thrust**.

$$F = 1.225 \frac{\pi (0.0254 \cdot d)^2}{4} \left[\left(RPM_{prop} \cdot 0.0254 \cdot pitch \cdot \frac{1min}{60sec} \right)^2 - \left(RPM_{prop} \cdot 0.0254 \cdot pitch \cdot \frac{1min}{60sec} \right) V_0 \right] \left(\frac{d}{3.29546 \cdot pitch} \right)^{1.5}$$

The thrust force on a blade is given approximately by:

$$\text{Also, } F = \left(\frac{1}{2}\right) \times \rho \times A \times v_b^2$$

Now, let's lift force we are,

$$\text{Lift force, } F_L = \left(\frac{1}{2}\right) \times \rho \times A \times v_b^2 \times C_L = F \times C_L$$

Now, combining the lift force for the 2 rotor disks, we get the total lift force acting on the helicopter fuselage.

$$\text{Hence, Total Lift Force, } F_{L(t)} = 2 \times F_L$$

3.3 Drag Force

The drag force acts on the rotor blades as they cut through the airstream. As they do so, they exert a force on the airstream, which itself exerts a force opposite to the direction of movement of the rotor blades. This force is called the drag force [13].

$$\text{Drag force is given by: } F_D = \left(\frac{1}{2}\right) \times \rho \times A \times v_b^2 \times C_D = F \times C_D$$

The coefficient of drag C_D is assumed to be 0.5 which is a typical value for helicopters.

3.4 Centrifugal Force

As the blade rotates at a very high-speed rpm, there is a force acting towards the center of the hub along the plane of the blade. This force is the centrifugal force which is an inherent force for any rotating body [13].

The centrifugal force is given by:

$$F_C = M_b \Omega^2 R$$

Here, Ω is rev/s. The weight acts downwards at the center of gravity. The lift force acts at the center of gravity as well but upwards. The drag force acts opposite to the direction of motion of the rotor blade.

4. CONCLUSIONS

Investigation of the coaxial rotor system is done by a theoretical analysis that discusses the five factors responsible for flight efficiency.

It also discusses the effects of rotor separation and blade configuration of coaxial drones for 2 blades vs 3 blades setup. In which 2 blade setup is found out to be more efficient. This paper also examines flight dynamics on coaxial drones. It also enlists out all the forces acting on the drone during flight conditions and at rest.

Furthermore, the body-fixed frame and Newton-Euler equation have also been explained. And lastly, 4 forces that are also being discussed are drone's own weight, thrust force, drag force, and lift force.

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