RELIABILITY MEASURES OF A 1-OUT-OF-2 SYSTEM WITH STANDBY AND DELAYED SERVICE

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ABSTRACT

In this paper, reliability measures of a stochastic model developed for a 1-out-of-2 system with single standby are obtained by considering delayed service. The system consists of three identical units, two are initially operative and one is in cold standby. The system remains operative if any one of the units operates and system fails at the failure of the entire units. Each unit is in either operative or complete failure mode. Single server carries out the repair of failed unit with delay while reaching the system. However, server visits the system immediately if all the three units fail. Switch devices are faultless and repaired unit works similar to new one. Random variables of failure time, repair time and arrival time (or delay time) of the server are statistically independent. Semi-Markov process and regenerative point technique are used to evaluate mean time to system failure (MTSF), steady state availability, and busy period of the server and profit of the system. The trend of various reliability measures for a particular case are shown graphically.

KEYWORDS: 1-Out-Of-2 System, Identical Units, Reliability, Evaluation and Delayed Service

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1. INTRODUCTION

Redundancy and service discipline play an important role in increasing the availability and thus profit of the system. Keeping this fact in mind scholars including Murari and Goel (1984) compared two-unit cold standby system with three types of repair facilities, Singh (1989) evaluated the profit of a two-unit cold standby system with random appearance and disappearance time of the service facility, Malik and Bhardwaj(2007) analyzed profit and reliability of 2-out-of-3 redundant system with general distribution of repair and waiting time. Malik and Gitanjali (2012) discussed a parallel system with maximum repair time and single server who appears and disappears randomly. Chopra and Ram (2017) analyzed a two non-identical unit parallel system stochastically with waiting time. All these researchers' analyzed parallel systems with waiting time of the server or random appearance and disappearance of the server.

It is a known fact that down time of the system keeps on increasing with the increase of waiting time of the server and thus due to delay in service facility. Therefore, to increase the availability of the system there must be a clause in administration design that server will visit the system immediately at its complete failure. This will decrease the down time of the system and hence optimize the profit. Further, a two-unit parallel system can be made more reliable by providing one additional unit as cold standby. For instance, availability of petrol pumps having two machines working simultaneously all the while might be expanded by providing a third machine as a backup. In such cases one of the working units fails and backup unit will

replace the failed one. Also, system will be working continuously until all the machines quit working. Keeping the above facts in mind we developed a comprehensive mathematical model with following assumptions:

- The system comprises of three indistinguishable units – two are in use and one is in cold standby.
- Each unit has two modes–working or failed.
- The system remains operative till the time at least one of the units is operative.
- There is a single server who takes time (delay time) to visit the failed unit and visits the system immediately if all the units fail.
- The failure time of the units are exponentially distributed while the distributions of repair rates are arbitrary.
- All the random variables are statistically independent.
- Semi-Markov process and regenerative point technique is used for the evaluation of various reliability measures.
- The switch devices are faultless and repaired unit works similar to new.
- For particular case, various reliability measures are shown graphically.

1.1. Notations

- **O**: Unit is in operative mode.
- **λ** : Constant failure rate of a single unit.
- **$g(t)/G(t)$** : Probability density function (pdf)/cumulative distribution function (cdf) of repair time of the unit.
- **$w(t)/W(t)$** : pdf/ cdf of waiting time of the server.
- **fur/FUR**: Unit is completely failed and under repair/under continuous repair.
- **fwr/FWR**: Unit is completely failed and waiting for repair/ continuously waiting for repair.
- **$q_{ij}(t)/Q_{ij}(t)$** : pdf / cdf of first passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in $(0, t]$.
- **m_{ij}** : The unconditional mean time taken by the system to transit from any regenerative state S_i when it (time) is counted from epoch of entrance in to the state S_j . Mathematically, it can be written as $m_{ij} = \int_0^\infty t d[Q_{ij}(t)] = -q_{ij}'(0)$.
- **μ_i** : Mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int P(T_i > t) dt = \sum_j m_{ij}$, where T denotes the time to system failure.
- **$M_i(t)$** : Probability that the system is initially up in the regenerative state S_i at time 't' without passing through any other regenerative state.
- **$W_i(t)$** : Probability that the server is busy in the state S_i upto time 't', without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states.
- **$\otimes / \odot / ' :$** Symbol of Stieltjes Convolution / Laplace Convolution/ derivative of the function.

- $**/*$: Symbol of Laplace Stieltjes Transform (LST)/ Laplace Transform (LT).

The transition states for the model are regenerative. The possible transition between states along with transition rate for the model is shown in Figure 1.

Figure 1: State Transition Diagram.

1.2. Transition probabilities and mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$\begin{aligned}
 p_{ij}(t) &= Q_{ij}(\infty) = \int q_{ij}(t) dt \\
 p_{01} &= p_{64} = p_{74} = 1, \quad p_{12} = w^*(2\lambda), p_{13} = 1 - w^*(2\lambda), p_{20} = g^*(2\lambda), p_{25} = 1 - g^*(2\lambda), \\
 p_{34} &= w^*(\lambda), p_{37} = 1 - w^*(\lambda), p_{42} = p_{52} = g^*(\lambda), p_{46} = p_{56} = 1 - g^*(\lambda)
 \end{aligned} \tag{1}$$

It can be verified that

$$p_{01} = p_{12} + p_{13} = p_{20} + p_{25} = p_{34} + p_{37} = p_{42} + p_{46} = p_{52} + p_{56} = p_{64} = p_{74} = 1 \tag{2}$$

The mean sojourn times μ_i in the state S_i are

$$\mu_0 = \int_0^\infty P(T > t) dt = \frac{1}{2\lambda}, \mu_1 = \frac{1}{2\lambda} \{1 - w^*(2\lambda)\}, \quad \mu_2 = \frac{1}{2\lambda} \{1 - g^*(2\lambda)\} \tag{3}$$

The unconditional mean time taken by the system to transit for any regenerative state S_j when it (time) is counted from the epoch of entrance into the state S_i is mathematically, states as

$$\begin{aligned}
 \int_0^\infty t q_{ij}(t) dt &= -q_{ij}^*(0) \\
 \mu_1 &= m_{12} + \mu_{13}, \mu_2 = m_{20} + \mu_{25}, \mu_1' = m_{12} + \mu_{12:34} + m_{17:3} + \mu_{16:34}, \mu_2' = m_{20} + \mu_{22:5} + m_{26:5}, \\
 \mu_1'' &= m_{12} + \mu_{12:34} + m_{12:3(4,6)^n} + \mu_{12:374} + m_{12:37(4,6)^n}, \mu_2'' = m_{20} + \mu_{22:5} + m_{22:5(6,4)^n}
 \end{aligned} \tag{4}$$

1.3. Reliability and Mean Time to System Failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from the regenerative state i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relation for $\phi_i(t)$:

$$\begin{aligned}
 \phi_0(t) &= Q_{01}(t) \otimes \phi_1(t) \\
 \phi_1(t) &= [Q_{12}(t) + Q_{12:34}(t)] \otimes \phi_2(t) + Q_{17:3}(t) + Q_{16:34}(t)
 \end{aligned}$$

$$\phi_2(t) = Q_{20}(t) \otimes \phi_0(t) + Q_{26.5}(t) + Q_{22.5}(t) \otimes \phi_2(t) \quad (5)$$

Taking LST of above relation (5) and solving for $\phi_0^{**}(s)$, we have

$$R^*(s) = \frac{1 - \phi_0^*(s)}{s} \quad (6)$$

The reliability of the system model can be obtained by taking inverse LT of (6).

The mean time to system failure (MTSF) is given by $\lim_{s \rightarrow 0} R^*(s)$. Thus,

$$\text{MTSF} = \frac{N_0}{D_0} = \frac{(1 - p_{22.5})(\mu_0 + \mu'_1) + \mu'_2(p_{12} + p_{12.34})}{1 - p_{22.5} - p_{20}(p_{12} + p_{12.34})}$$

1.4. Steady State Availability

Let $A_i(t)$ be the probability that the system is in upstate at instant 't' given that the system entered regenerative state i at $t = 0$. The recursive relations for $A_i(t)$ are given as:

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t) \odot A_1(t) \\ A_1(t) &= M_1(t) + [q_{12}(t) + q_{12.34}(t) + q_{12.37(4,6)}(t) + q_{12.374}(t) + q_{12.3(4,6)}(t)] \odot A_2(t) \\ A_2(t) &= M_2(t) + q_{20}(t) \odot A_0(t) + [q_{22.5}(t) + q_{22.5(6,4)}(t)] \odot A_2(t) \end{aligned} \quad (7)$$

where $M_0(t) = e^{-2\lambda t}$, $M_1(t) = e^{-2\lambda t} \bar{W}(t)$, $M_2(t) = e^{-2\lambda t} \bar{G}(t)$

Taking LT of (7) and solving for $A_0^*(s)$, the steady state availability is given by

$$A(\infty) = \lim_{s \rightarrow 0} s A_0^*(s) = \frac{N_1}{D_1} = \frac{\mu_0 p_{20} + \mu_1 p_{20} + \mu_2}{\mu_0 p_{20} + \mu_1 p_{20} + \mu_2}$$

1.5. Busy Period of the Server

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit at an instant 't' given that the system entered regenerative state i at $t = 0$. The recursive relations for $B_i^R(t)$ are:

$$\begin{aligned} B_0(t) &= q_{01}(t) \odot B_1(t) \\ B_1(t) &= M_1(t) + [q_{12}(t) + q_{12.34}(t) + q_{12.37(4,6)}(t) + q_{12.374}(t) + q_{12.3(4,6)}(t)] \odot B_2(t) \\ B_2(t) &= W_2(t) + q_{20}(t) \odot B_0(t) + [q_{22.5}(t) + q_{22.5(6,4)}(t)] \odot B_2(t) \end{aligned} \quad (8)$$

Where

$$W_2(t) = e^{-2\lambda t} \bar{G}(t) + (2\lambda e^{-2\lambda t} \odot 1) \bar{G}(t) + (2\lambda e^{-2\lambda t} \odot \lambda e^{-\lambda t} \odot (g(t) \odot \lambda e^{-\lambda t})^n \odot g(t) \odot 1) \bar{G}(t)$$

Taking LT of relations (8) solving for $B_0^*(s)$. The time for which server is busy is given by

$$B(\infty) = \lim_{s \rightarrow 0} s B_0^*(s) = \frac{N_2}{D_1} = \frac{w_2^*}{\mu_0 p_{20} + \mu_1 p_{20} + \mu_2}$$

1.6. Expected Number of Visits by the Server

Let $N_i(t)$ be the expected number of visits by the server for repair in $(0, t]$ given that the system entered the regenerative state i at $t = 0$. The recursive relations for $N_i(t)$ are given as:

$$N_0(t) = Q_{01} \otimes N_0(t)$$

$$N_1(t) = [Q_{12}(t) + Q_{12:34}(t) + Q_{12:37(4,6)^n}(t) + Q_{12:374}(t) + Q_{12:3(4,6)^n}(t)] \otimes [1 + N_2(t)]$$

$$N_2(t) = Q_{20}(t) \otimes N_0(t) + [Q_{22:5}(t) + Q_{22:5(6,4)^n}(t)] \otimes N_2(t) \quad (9)$$

Taking LT of relations (9) and solving for $N_0^{**}(s)$. The expected no of visits by the server can be obtained as:

$$N_0(\infty) = \lim_{s \rightarrow 0} s N_0^{**}(s) = \frac{N_3}{D_1} = \frac{1 - p_{22:5} - p_{22:5(6,4)^n}}{\mu_0 p_{20} + \mu_1 p_{20} + \mu_2}$$

2. PROFIT ANALYSIS

Profit incurred to the system model in steady state is given by $P = K_1 A - K_2 B - K_3 N$ where, K_1 = Revenue per unit up time of the system, K_2 = Cost per unit time for which server is busy and K_3 = Cost per visit by the server.

3. CONCLUSIONS

The reliability measures of the system model have been evaluated by assuming arrival and repair times as exponentially distributed ($\alpha e^{-\alpha t}$ and $\beta e^{-\beta t}$). The graphs plotted for these measures indicate that the reliability measures go on decreasing with the increase in failure rate. However, there is a sharp decline in MTSF as compared to that of availability and profit. On the other hand the reliability measures will have an upward trend with the increase of arrival rate of the server and repair rate.

Hence it is concluded that better repair rate with less arrival time increase the MTSF and profit of the system.

Figure 2: MTSF Vs Failure Rate.

Figure 3: Availability Vs Failure Rate.

Figure 4: Profit Vs Failure Rate.

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