BENDING STRENGTH DETERMINATION OF INVOLUTE SPUR GEAR USING ANALYTICAL METHOD AND FEM

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ABSTRACT

The study of stresses and methods to eliminate stress in order to improve performance parameters of gears is an important aspect of modern machinery design processes. Bending stress calculation in spur gear teeth for the design purpose proposed in the traditional approach uses Lewis equation and linear theory of elasticity which is based on a uniform load distribution model. However, bending stress produced in the gear has the non-uniform load along the line of action as the primary input. So, the results obtained by a uniform load distribution model are not in agreement with the experimental results. In this paper, A non-uniform model of load distribution along the line of contact has been introduced. This model provides the load distribution on different gear teeth with the help of minimum elastic potential energy method. The Lagrangian variational technique has been used to solve the obtained equations. This approach provides a much accurate result as compared to other traditional models. Finally, the result obtained by the analytical method is verified with the FEM result. Both results are quite comparable to each other.

KEYWORDS: Spur Gear, Load Distribution, Elastic Potential Energy, Finite Element Method

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INTRODUCTION

The modern power transmission industry needs highly efficient gears so as to own high strength and less prone to failure. To achieve this, it is quite necessary to reduce unnecessary stresses on gears which cause gear teeth weaker. The study of stresses and methods to reduce stress in order to improve performance of gears is an important aspect of modern machinery design processes. Much advancement has been achieved in the investigation of various types of stresses and methods to reduce them as depicted in technical literature. Numerous methodologies have been adopted in the past few decades to investigate the stresses in spur gear teeth. These methodologies can be grouped under three broad classes, namely analytical, experimental and finite element methods. Analytical methods involve bi-potential equations and using Airy stress function, finite prism method, or Muskhelishvili method applied to circular elastic rings. Experimental methods involve using electric resistance wire strain gauges or utilizing photo-elastic gauges, a digital photo-elastic system with the real-time imaging process. Numerical methods involve the use of FEM based simulation. Rendering analytical analysis, experimental work and comparing these with numerical work using finite element methods validate the accuracy of the analysis result.

International standards like AGMA [1] and ISO[2] has provided methods to evaluate the values of bending stress on the gear teeth. The relations provided by both standards are based on the Lewis equation for the calculation of bending stress. Both approaches calculate only approximate values and are based on various approximations. They are not sufficient to calculate stresses accurately due to the inclusion of various assumptions. Also, this method of stress calculation in spur gear teeth for the design purpose assumes uniform load distribution which is not suitable, because it depends on the meshing stiffness which is different at different points along the load distribution line and on the different teeth, so it can't be considered uniform. Although several correction factors are applied to the basic formula, yet, they are unable to bring proper accuracy as suggested by experimental results.

In the present work, a non-uniform model of load distribution is introduced which is based on the minimum elastic potential energy criterion. The expression for total potential energy is formulated and with the condition of total force being equal to the sum of forces applied on individual teeth, it is minimized using the Lagrangian multiplier technique. Correspondingly, the concept of inverse unitary potential is introduced with the help of which load sharing ratio is calculated. This load sharing ratio brings the concept of non-uniform load distribution of teeth. Various parameters in the gear teeth are expressed in the form of the ξ parameter which specifies the load point at the involute profile. Load conditions and expressions for the contact and bending stresses are written as the function of the ξ parameter. Finally, the result obtained by the analytical method is verified with the FEM result. Both results are in agreement with each other.

METHODOLOGY

Load Distribution Model

The presented model of load distribution is based on the assumption that the load distribution along the line of contact provides minimum elastic potential energy considering one or more pair of teeth in simultaneous contact with an unknown fraction of load acting on each other. The elastic potential energy U is calculated from the relations given by theory of elasticity and the geometric parameters of the teeth. The elastic potential energy of spur gear U can be expressed as the sum of the compressive component U_n , bending component U_x and shear component U_s . All the components can be extracted from the equations of theory of elasticity. The elastic potential energy depends on the load point which can be expressed in the form of ξ parameter[3] which is the ratio of the curvature radius at the load point and the circular base pitch. ξ parameter can be represented as:

$$\xi = \frac{Z}{2\pi} \sqrt{\left(\frac{r_c}{r_b}\right)^2 - 1} \quad (1)$$

Where Z is the number of teeth. r_b is the base radius. r_c is the radius on the contact point. The potential energy of a pair of teeth in contact U_p will be the sum of those of the pinion and the wheel, which can also be expressed as a function of the ξ parameter of pinion since the sum of the curvature radii of both transverse profiles at the respective contact points is constant along the path of contact and equal to the distance between the tangency points of the operating pressure line and both base circles of pinion and wheel, such that

$$\xi_1 + \xi_2 = \lambda = \frac{Z_1 + Z_2}{2\pi} \tan \alpha_t \quad (2)$$

Where ξ_1 and ξ_2 corresponds to pinion and gear respectively. α_t is the operating transverse pressure angle which is applicable for both spur and helical gear.

In case of spur gear, there is only single pressure angle α so that $\alpha_t = \alpha$

All the components of tooth pair elastic potential energy are proportional to square load F^2 and inversely proportional to the face width b . Thus, if we define the unitary potential energy of the tooth pair which is the potential energy of unitary load and face width, we will have the following relation:

$$U_p(\xi) = \frac{F^2}{b} u_p(\xi) \quad (3)$$

The elastic unitary potential energy of a considered pair of teeth can be evaluated by numerical technique of integration. In every situation, the result is always function of ξ .

If there are n pair of teeth in simultaneous contact, then the total elastic potential energy U_T will be as followed:

$$U_T = \sum_{i=1}^n U_{pi}(\xi_i) = \frac{1}{b} \sum_{i=1}^n F_i^2 u_i(\xi_i) = U_T(\xi_1) \quad (4)$$

Which can be expressed as a function of ξ parameter of the first gear tooth pair according to following relation.

$$\xi_{i+1} = \xi_i + 1 \quad (5)$$

From the relation (5) it is clear that ξ parameter of every gear tooth pair is related to each other.

Summarizing, we have a variational problem which can be represented as follows.

Find the value of F_i which minimize the function:

$$U_T = \sum_{i=1}^n U_{pi}(\xi_i) = \frac{1}{b} \sum_{i=1}^n F_i^2 u_i(\xi_i) \quad (6)$$

Regarding the restriction

$$\sum_{i=1}^n F_i = F \quad (7)$$

By mean of variational technique [6], the above problem yields the following result:

$$F_i(\xi_i) = \frac{\frac{1}{u_i(\xi_i)}}{\sum_{j=1}^n \frac{1}{u_j(\xi_j)}} F = \frac{v_i(\xi_i)}{\sum_{j=1}^n [v_j(\xi_j)]} F \quad (8)$$

Here $v(\xi_i) = u^{-1}(\xi_i)$ is the inverse unitary potential.

It is clear that F_i depends on the contact point. It can be expressed as a function of the ξ parameter of the first tooth pair (ξ_1) or its own parameter (ξ_i).

The fraction of the load taken by the considered pair of teeth is called load sharing ratio [4]. It is expressed as:

$$R_i(\xi_i) = \frac{F_i(\xi_i)}{F} = \frac{v_i(\xi_i)}{\sum_{j=1}^n v_j(\xi_j)} \quad (9)$$

The load per unit length denoted by $f_i(\xi_i)$ for spur gear can be expressed as:

$$f(\xi_i) = \frac{F}{b} R(\xi_i) \quad (10)$$

Load Sharing Ratio

Along with the interval of single pair tooth contact, the load sharing ratio is:

$$R(\xi) = 1$$

This is due to the fact that the total load is taken by a single pair of teeth. So the fraction of the load sharing ratio is 1.

Along with the intervals of two pair tooth contact the variation of $R(\xi)$ is quite linear. It is found that the limit value of the intervals of $R(\xi)$ corresponding to the interval of two pair tooth contacts are always very close to 0.33 and 0.66. This does not change with the number of teeth, pressure angle, rack shift coefficients, operating center distance, etc.

If ξ_{inn} and ξ_α denote the profile parameter corresponding to the inner and outer points of contact of the pinion respectively, then the load sharing ratio [4] for the different interval can be calculated as:

$$R(\xi) = \frac{1}{3} \left(1 + \frac{\xi - \xi_{inn}}{\xi_\alpha - \xi_{inn} - 1} \right) \text{ for } \xi_{inn} \leq \xi \leq \xi_\alpha - 1 \quad (11)$$

$$R(\xi) = 1 \text{ for } \xi_\alpha - 1 \leq \xi \leq \xi_\alpha \quad (12)$$

$$R(\xi) = \frac{1}{3} \left(1 + \frac{\xi - \xi_\alpha}{\xi_{inn} + 1 - \xi_\alpha} \right) \text{ for } \xi_{inn} + 1 \leq \xi \leq \xi_\alpha \quad (13)$$

Where

$$\xi_\alpha = \frac{Z_1}{2\pi} \sqrt{\left(\frac{r_{a1}}{r_{b1}} \right)^2 - 1} = \xi_{inn} + s_\alpha$$

r_{a1} is the outside radius of the pinion and remaining parameter have their usual meaning.

Approximate Equation for the Inverse Unitary Potential

The inverse unitary potential $v(\xi)$ can be computed by numerical integration of the total elastic potential energy. This gives the numerical value of $v(\xi)$ at discrete contact points and meshing positions, which allows to obtain some conclusions regarding the considered gear- pair, but makes it very difficult to extract general conclusion valid for any other gear-pair. In order to solve this problem, an approximate equation for $v(\xi)$ is found. The actual shape of $v(\xi)$ fits quite accurately to a fourth-degree parabola. A typical fourth-degree parabola requires five coefficients, and every coefficient is

expressed as a function of a number of geometrical and operating parameters of the gears such as number of teeth on both the gears, module, pressure angle, helix angle, addendum and dedendum coefficients, tool tip radius, operating center distance, rack shift coefficient. It is very difficult to get accurate correlations for five coefficients, each one as a function of ten variables. The final expression for the load per unit length becomes very complex. In order to avoid such difficulty, a simple, accurate function is developed which can be used to perform various calculations. The function is of the form.

$$v(\xi) = \cos [b_o (\xi - \xi_m)] \quad (14)$$

This cosine function [3] has the similar shape to the fourth degree parabola. Important feature of this cosine function is that it has only two parameters b_o and ξ_m . Amplitude coefficient is not necessary in this equation.

Now our objective is to compute the value of b_o and ξ_m .

Taking the load sharing ratio as follows:

$$R(\xi) = \frac{1}{3}$$

At the inner point of two pair tooth contact i.e. $\xi = \xi_{inn}$

And

$$R(\xi) = \frac{2}{3}$$

At the inner point of single pair tooth contact i.e. $\xi = \xi_{inn} + \varepsilon_\alpha - 1$

We have the following relations:

$$\frac{\cos [b_o (\xi_{inn} + 1 - \xi_m)]}{\cos [b_o (\xi_{inn} - \xi_m)]} = 2 \quad (15)$$

$$\frac{\cos [b_o (\xi_{inn} + \varepsilon_\alpha - 1 - \xi_m)]}{\cos [b_o (\xi_{inn} + \varepsilon_\alpha - \xi_m)]} = 2 \quad (16)$$

Now cosine function may be expended according to Taylor series as follows:

$$\cos \alpha = 1 - \frac{\alpha^2}{2!} + \frac{\alpha^4}{4!} - \frac{\alpha^6}{6!} \dots \quad (17)$$

Taking first two terms, we get

$$\cos \alpha = 1 - \frac{\alpha^2}{2!} \quad (18)$$

Putting the relation from (18) in equation (15) and (16), we get

$$\frac{2 - [b_o (\xi_{inn} + 1 - \xi_m)]^2}{2 - [b_o (\xi_{inn} - \xi_m)]^2} = 2 \quad (19)$$

$$\frac{2 - [b_o (\xi_{inn} + \varepsilon_\alpha - 1 - \xi_m)]^2}{2 - [b_o (\xi_{inn} + \varepsilon_\alpha - \xi_m)]^2} = 2 \quad (20)$$

Let $\xi_{inn} - \xi_m = X$ and $\xi_{inn} + \varepsilon_\alpha - \xi_m = Y$

Thus the equations (19) and (20) transform to

$$\frac{2 - [b_o (X + 1)]^2}{2 - [b_o X]^2} = 2 \quad (21)$$

$$\frac{2 - [b_o (Y - 1)]^2}{2 - [b_o Y]^2} = 2 \quad (22)$$

Solving equations (21) and (22), we get:

$$[X + Y][X - Y - 2] = 0$$

$$\Rightarrow X + Y = 0 \text{ or } X - Y - 2 = 0$$

From $X + Y = 0$, we get

$$\xi_m = \xi_{inn} + \frac{\varepsilon_\alpha}{2} \quad (23)$$

From $-Y - 2 = 0$, we get

$\varepsilon_\alpha = -2$ Which is not valid, as contact ratio can't be negative.

Putting the value of equation (23) in equation (19), we get:

$$\frac{2 - [b_o (\xi_{inn} + 1 - \xi_{inn} - \frac{\varepsilon_\alpha}{2})]^2}{2 - [b_o (\xi_{inn} - \xi_{inn} - \frac{\varepsilon_\alpha}{2})]^2} = 2 \quad (24)$$

$$\Rightarrow b_o^2 \left(\frac{\varepsilon_\alpha}{2}\right)^2 + b_o^2 \varepsilon_\alpha - b_o^2 = 2$$

$$\Rightarrow b_o = \frac{1}{\sqrt{\frac{1}{2} \left(1 + \frac{\varepsilon_\alpha}{2}\right)^2 - 1}} \quad (25)$$

BENDING STRESS IN SPUR GEAR TOOTH

Derivation of Bending Stress According to ISO Standard

In order to find the bending stress in the loaded gear tooth, ISO standard introduced a model. This model introduces the tooth form factor Y_F and stress concentration factor Y_{σ} , which can be expressed as a function of the tooth thickness at the critical section at which the stress is maximum, and the distance between this section and the load one, which is defined by the intersection of the line of action and the tooth centerline. According to the method described by ISO, this distance would be computed for load acting at the highest point of the single tooth contact which is called

HPSTC. Y_F and Y_s depends on many parameters such as tool geometry, addendum, rack shift factor, number of teeth on both the gears and operating central distance. The location and the thickness of the critical section are determined from the critical section parameter which can be obtained from the tool geometry, tooth number and the rack shift factor on the considered gear.

While determining the bending stress as according to ISO standard, following assumptions are taken:

- The compressive stress produced by the radial component of the load can be neglected.
- The critical section of the tooth is defined by the point of tangency with the root fillet of a line rotated 30 degree from the tooth centerline.
- The bending behavior of a helical tooth is equivalent to that of a virtual spur gear.

Adopting these hypotheses, the nominal bending stress [2] at the critical section, for the load acting on any point of the teeth profile is derived as:

$$\sigma_F = \frac{F_t \cos \alpha_F h_F s_F}{\frac{1}{12} b s_F^3} \frac{Y_s}{2} = \frac{6 \frac{F_{it}}{\cos \alpha} \cos \alpha_F h_F}{b s_F^2} Y_s = \frac{F_{it} 6 \cos \alpha_F h_F}{b s_F^2 \cos \alpha} Y_s \quad (26)$$

Where F_t is the normal load acting on the considered gear teeth profile. F_{it} is the component of normal load at the standard pitch circumference in the direction of the normal to the tooth center line. s_F is the tooth thickness at the critical section. h_F is the distance from the critical section to the intersection of the tooth centerline and the line of action for the load at the point on the gear tooth. α_F is the load angle. α is the standard pressure angle. b is the face width and Y_s is the stress concentration factor for the load acting at the point on the tooth.

According to ISO, the above equation can be represented in the modified form as:

$$\sigma_F = \frac{F_{it}}{b m} Y_F Y_s \quad (27)$$

Where m is the normal module, Y_F is the tooth form factor for the load acting on the considered point of the teeth.

From equation (26), Y_F is described by:

$$Y_F = \frac{6 \cos \alpha_F \left(\frac{h_F}{m} \right)}{\cos \alpha \left(\frac{s_F}{m} \right)^2} \quad (28)$$

As suggested by ISO, the expression for the parameters are provided as follows:

Stress concentration factor is given by:

$$Y_s = \left(1.2 + 0.13 \frac{s_F}{h_F} \right) \left(\frac{s_F}{2 \rho_F} \right)^{\frac{1}{1.21 + 2.3 \frac{h_F}{s_F}}} \quad (29)$$

Where ρ_F is the curvature radius of the tooth fillet at the critical section. It is suggested by the AGMA standard as follows:

$$\rho_f = r_f + \frac{(mh_{ao} - mx - r_f)^2}{\sin \phi (r_{vp} \sin^2 \phi + mh_{ao} - mx - r_f)} \quad (30)$$

Where r_f is the tool tip radius. h_{ao} is the tool addendum factor, x is the rack shift factor, r_{vp} is the standard pitch radius. ϕ is the critical section parameter.

The angle between the tooth centerline and the tangent to the root fillet at the point corresponding with the ϕ is $\phi - \gamma$.

From the geometry, the value of γ is derived as:

$$\gamma = \frac{mh_{ao} - mx - r_f}{r_{vp} \tan \phi} + \frac{1}{r_{vp}} \left(\frac{\pi}{4} + \frac{r_f}{\cos \alpha} + (mh_{ao} - r_f) \tan \alpha \right) \quad (31)$$

According to ISO standard, the condition for the critical section is given by:

$$\phi - \gamma = \frac{\pi}{6}$$

$$\phi - \frac{mh_{ao} - mx - r_f}{r_{vp} \tan \phi} - \frac{1}{r_{vp}} \left(\frac{\pi}{4} + \frac{r_f}{\cos \alpha} + (mh_{ao} - r_f) \tan \alpha \right) = \frac{\pi}{6} \quad (32)$$

Iterating the above equation and obtaining much accurate value of ϕ , value of tooth thickness at the critical section s_F and the distance from the critical section to the intersection of the tooth centerline and the line of action for the load at the tooth profile h_F can be obtained by the following expression:

$$s_F = 2 \frac{\sin \gamma}{\sin \phi} (r_{vp} \sin \phi - \delta) - 2 \frac{\sin \frac{\pi}{6}}{\tan \phi} \delta \quad (33)$$

And

$$h_F = \frac{r_{vb}}{\cos \alpha_F} - \frac{r_{vp} \sin \phi - \delta}{\sin \frac{\pi}{6}} + \frac{\sin \gamma (r_{vp} \sin \phi - \delta)}{\tan \frac{\pi}{6} \sin \phi} - \frac{\cos \frac{\pi}{6}}{\tan \phi} \delta \quad (34)$$

Where r_{vb} is the base radius. The value of δ can be obtained as:

$$\delta = r_f + \frac{mh_{ao} - mx - r_f}{\sin \phi} \quad (35)$$

The load angle [5] can be derived as:

$$\alpha_p = \left(\frac{r_L^2}{r_{vb}^2} - 1 \right)^{\frac{1}{2}} - \frac{\gamma_b}{2} = \frac{2\pi}{Z} \xi - \frac{\gamma_b}{2} \quad (36)$$

Where γ_b is the angular thickness of both involute profile at the load point and is given by:

$$\gamma_b = \frac{\pi}{Z} + \frac{4x}{Z} \tan \alpha + 2(\tan \alpha - \alpha) \quad (37)$$

The various parameters used in the above relations can be described in the following figure:

Figure 1: Configuration of Loaded Gear Teeth with Necessary Parameters and Fillet Generation.

Bending Stress in Spur Gear Tooth with Introduction of ξ Parameter

The nominal bending stress at the critical section as provided by ISO standard [2, 7] is given by:

$$\sigma_F = \frac{6F_i \cos \alpha_F h_F}{b s_F^2} Y_s \quad (38)$$

Where F_i the normal load acting on the tooth profile. s_F is the tooth thickness at the critical section, h_F is the distance from the critical section to the intersection of the tooth centerline and the line of action for the load point. α_F is the load angle. α is the standard pressure angle. b is the face width of spur gear. Y_s is the stress concentration factor for load acting on the teeth.

If the equation for bending stress is expressed in the form of ξ parameter, then it will be as follows:

$$\sigma_F = \frac{6f(\xi) \cos \alpha_F(\xi) h_F(\xi)}{s_F^2} Y_s(\xi) \quad (39)$$

Where f is the load per unit length along the line of contact.

As parameters in the equation (39) exclusively depend on the load angle and load angle is the function of ξ parameters. Thus it is obvious that these parameters are also the function of ξ parameters.

Critical Bending Stress and Load Condition for Spur Gear

According to equation (10) and (39), the critical tooth root stress for the spur gear will be given by the maximum of the function:

$$\Pi(\xi) = R(\xi) \cos \alpha_F(\xi) h_F(\xi) Y_s(\xi) \quad (40)$$

From the equation (36), we can see that α_F increases with ξ . consequently $\cos \alpha_F$ decreases

From the equation (34), we can see that h_F increases with ξ .

From the equation (29), we can see that Y_s decreases with ξ .

Consequently, function $\cos \alpha_F(\xi) h_F(\xi) Y_s(\xi)$ may increase or decrease with ξ , depending on the slope of each curve. However, for standard tooth proportions, function $\cos \alpha_F(\xi) h_F(\xi) Y_s(\xi)$ always grows with ξ . This is the result of more than 1800 case, in which wide range of geometrical parameters have been considered:

- Number of teeth on pinion Z_1 , between 23 and 80.
- Gear ratio G between 1 and 4.
- Standard normal pressure angle α_n , between 19° and 28° .
- Rack shift coefficient on pinion x_1 , between -0.1 and 0.1 .
- Rack shift coefficient on wheel x_2 , between -0.1 and 0.1 .
- Tool addendum coefficient h_{a0} , between 1.2 and 1.3 .

It is observed that in all the cases, function $\cos \alpha_F(\xi) h_F(\xi) Y_s(\xi)$ grows up with ξ without exception.

According to this result and taking into account the symmetry of function $R(\xi)$ respect to the mid-point of the interval of contact, the maximum of function $\Pi(\xi)$ should be necessarily located between the outer point of single pair tooth contact and the outer point of double pair tooth contact where the load sharing ratio is unity such that total load is exerted by the single pair gear tooth. However, for standard spur gears, the maximum is always located at the outer point of single pair tooth contact, as obtained for the 1800 cases above, without exception.

Summarizing the above, the critical tooth root stress for standard spur gear should be computed with total load acting at the outer point of single pair tooth contact. Thus critical tooth root stress is given by:

$$\sigma_F = \frac{6F_t}{s_F^3 b} \cos \alpha_F(\xi_{inn} + 1) h_F(\xi_{inn} + 1) Y_s(\xi_{inn} + 1) \quad (41)$$

RESULT AND DISCUSSION

For mathematical analysis of proposed method for the analysis and calculation of bending and contact stress in the spur gear teeth, a standard specimen of gear is taken which have addendum modification coefficient $x_1 = 0$.

Following are the various gear parameters considered for the analysis:

Table 1

Parameters	Pinion	Wheel
Number of teeth	21	58
Normal pressure angle	20°	20°
Module	4	4
Face width	20mm	20mm
Addendum modification coefficient	0	0
Load	1000N	1000N
Modulus of elasticity	200GPa	200GPa
Material	Structural Steel	Structure steel
Standard	ISO	-

Calculation for Pinion with Normal Contact Ratio

Number of teeth on the pinion $Z_p = 21$

Normal module $m = 4$

Pitch radius $r_p = \frac{mZ_p}{2} = 42\text{mm}$

Base radius $r_b = r_p \cos \alpha = 42 \cos 20^\circ = 39.46 \text{ mm}$

ξ parameter at the base radius:

$$r_b = \frac{Z_p}{2\pi} \sqrt{\frac{r_c^2}{r_b^2} - 1}$$

At the base radius, ξ parameter

$$\xi = \frac{21}{2\pi} \sqrt{\frac{39.46^2}{39.46^2} - 1} = 0$$

Thus the actual contact between the teeth of gear and pinion would take place not at the point of the teeth on the base radius. The point below the base radius will give imaginary value which is not appropriate. Contact will take place only if the ξ parameter will give real and positive value. All other values are not accepted for tooth contact.

It is obvious that contact on gear tooth will take place only if the following condition will be satisfied:

$$r_b \leq r_c$$

Let us take a load point on the gear tooth profile for which tooth radius is greater than base radius of the pinion.

Let $r_c = 40 \text{ mm}$

$$\text{Then } \xi = \frac{21}{2\pi} \sqrt{\frac{40^2}{39.46^2} - 1} = 0.55$$

This point is considered as involute parameter of the inner point of contact of the pinion denoted by ξ_{inn} .

$$\text{So } \xi_{inn} = 0.55$$

Calculation of Bending Stress for Spur Gear Teeth

In order to calculate the bending stress at the tooth root of the gear by the minimum elastic potential energy method, the following assumptions are taken into consideration:

- The gear is standard for which implementation of a profile shift is not required. So the profile shift coefficient for the gear is 0

$$x_1 = 0$$

- The gear teeth follow the involute profile and the involute curve starts from the base circle to the tooth tip.
- The compressive stress produced by the radial component of the load is negligible.
- The critical section of the tooth is defined by the point of tangency with the tooth root fillet of a line rotated 30° from the tooth centerline.
- The tooth is considered as the cantilever beam with point load acting on it at a distance from the tooth root.

Following the assumptions adopted by the ISO standard, we can evaluate the value of critical bending stress in the gear teeth. But this is not quite accurate. By introduction of involute parameter in our analysis, we can enhance the accuracy of our results.

From equation (37) the angular thickness of both involute profiles at the load point can be calculated as:

$$\gamma_b = \frac{\pi}{21} + \frac{4 * 0}{21} \tan 20^\circ + 2(\tan 20^\circ - 0.348) = 0.1815 \text{ rad}$$

Now from the intense investigation, it is found that the critical bending stress always exists at the point on the gear teeth profile on which

$$\xi = \xi_{inn} + 1 = 0.55 + 1 = 1.55$$

Load angle may be calculated from equation (36) as follows:

$$\alpha_F = \frac{2\pi}{21} * 1.55 - \frac{0.1815}{2} = 0.373 \text{ rad}$$

Now let us take following values of the parameters used in the relations from the ISO standard:

$$\text{Tool addendum factor } h_{ao} = 1.25$$

Tool tip radius $r_f = 0.25 \text{ mm}$

Addendum modification coefficient for pinion $x_1 = 0$

Putting the value in the equation (31), we get

$$\gamma = \frac{4 * 1.25 - 0 - 0.25}{42 \tan \phi} + \frac{1}{42} \left(\frac{\pi}{4} + \frac{0.25}{\cos 20^\circ} + (4 * 1.25 - 0.25) \tan 20^\circ \right) \\ = \frac{0.113}{\tan \phi} + 0.066 \quad (42)$$

Putting the value of γ in equation (32) we have:

$$\phi - \frac{0.113}{\tan \phi} - 0.066 = \frac{\pi}{6} \\ \Rightarrow \phi - \frac{0.113}{\tan \phi} - 0.457 = 0$$

Now applying the Newton method of iteration in the above equation by taking the seed value $\phi_1 = \frac{\pi}{6}$, we get:

$$\phi_2 = \phi_1 - \frac{\frac{\pi}{6} - \frac{0.113}{\tan \phi_1} - 0.457}{1 + 0.113 \operatorname{cosec}^2 \phi_1} = \frac{\pi}{6} - \frac{\frac{\pi}{6} - \frac{0.113}{\tan \frac{\pi}{6}} - 0.456}{1 + 0.113 \operatorname{cosec}^2 \frac{\pi}{6}} = 0.6113 \text{ rad}$$

Again iterating for $\phi_2 = 0.6113$, we get,

$$\phi_3 = \phi_2 - \frac{0.6113 - \frac{0.113}{\tan \phi_2} - 0.457}{1 + 0.113 \operatorname{cosec}^2 \phi_2} = 0.6113 - \frac{0.6113 - \frac{0.113}{\tan 0.6113} - 0.451}{1 + 0.113 \operatorname{cosec}^2 0.6113} \\ = 0.6119 \text{ rad}$$

Take this value as the required accurate value. Thus we have:

$$\phi = 0.6119 \text{ rad}$$

Putting this value of ϕ in equation (42) we get:

$$\gamma = \frac{0.113}{\tan 0.6119} + 0.066 = 0.2270 \text{ rad}$$

Tooth thickness at the critical section can be calculated as:

$$s_F = 2 \frac{\sin 0.2270}{\sin 0.6119} (42 * \sin 0.6119 - \delta) - 2 \frac{\sin \frac{\pi}{6}}{\tan \phi} \delta$$

Where δ be calculated from the equation (35) as

$$\delta = 0.25 + \frac{4 * 1.25 - 0 - 0.25}{\sin 0.6119} = 8.51 \text{ mm}$$

$$\text{Thus } s_F = 8.31 \text{ mm}$$

Bending moment arm can be calculated from the equation (34) as follows:

$$h_F = \frac{39.46}{\cos 0.373} - \frac{(42 * \sin 0.6119 - 8.51)}{\sin \frac{\pi}{6}} + \frac{\sin 0.2207(42 * \sin 0.6119 - 8.31)}{\tan \frac{\pi}{6} \sin 0.6119} - \frac{\cos \frac{\pi}{6}}{\tan 0.6119} * 8.51 = 11.56 \text{ mm}$$

Now curvature radius of the root of the gear can be calculated from the equation (30) as:

$$\rho_f = 0.25 + \frac{(4 * 1.25 - 0 - 0.25)^2}{\sin 0.6119(42 * \sin^2 0.6119 + 4 * 1.25 - 0 - 0.25)^2} = 0.363 \text{ mm}$$

From the equation (29), stress correction factor can be calculated as:

$$Y_s = \left(1.2 + 0.13 \frac{8.31}{11.56}\right) \left(\frac{8.31}{2 * 0.363}\right)^{\frac{1}{1.21 + 2.5 \frac{11.56}{8.31}}} = 2.27$$

Tooth form factor can be calculated from the equation (28) as:

$$Y_F = \frac{6 \cos 0.373 * \left(\frac{11.56}{4}\right)}{\cos 0.348 * \left(\frac{8.31}{4}\right)^2} = 3.98$$

Thus from the equation critical bending stress can be calculated as:

$$\sigma_F = \frac{6 * 1000 * \cos 0.373 * 11.56 * 10^{-3} * 2.27}{(8.31 * 10^{-3})^2 * 20 * 10^{-3}} = 106.16 \text{ MPa}$$

Calculation of Bending Stress by ANSYS

The bending stress in the gear teeth is calculated by Lewis equation which assumes cantilever beam as bending stress with the load applied at a distance from the tooth root, called bending moment arm. In ANSYS also, we adopt the same procedure. A gear model is created by the CAD software and imported by the ANSYS.

Using ANSYS for the calculation of bending stress, it provided the result as shown in the figure.

Figure 2: Meshing of Spur Gear with Refined Tooth.

Figure C: Critical Bending Stress in Spur Gear Teeth(Deformed+Undeformed).

From the figure, Critical Bending stress obtained by ANSYS=**101.13MPa**

Comparison of Results

In the present analysis, we observe that there is an acceptable difference between the results obtained by the two methods. There is a variation in the results obtained by mathematical analysis and by ANSYS. The result obtained by the two methods can be tabulated as follows:

Table 2

Method of Analysis	Bending Stress
Analytical Method	106.16 MPa
FEM	101.13 MPa

$$\% \text{ error} = \frac{\text{analyticalvalue} - \text{FEMvalue}}{\text{analyticalvalue}} * 100$$

$$\text{For Critical Bending Stress } \sigma_F, \% \text{ error} = \frac{(106.16 - 101.130) \times 100}{106.16} = 4.73 \%$$

CONCLUSIONS

In the present work, calculation of stress in spur gear drive has been made for the normal contact ratio having the value of 1.5. Results obtained from a study of several hundred gear drives with a normal contact ratio yield the conclusion which can be expressed as follows:

- The critical tooth root stress corresponds to the contact at the inner point of one pair tooth contact for which tooth is loaded with the total transmitted load having load sharing ratio 1.

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