MATH WITH DIDACTIC ENGINEERING

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ABSTRACT

This work aims to show an experience of didactic engineering through mathematics education, which was carried out with first-level semester students at a university in the city of Bogotá Colombia in a course called introduction to calculus, in this it will be shown that proposing challenging problems based on the French mathematical school called didactics of mathematics can lead to the student being able to build mathematics.

KEYWORDS: Didactic engineering, mathematics, challenging problems

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INTRODUCTION

At the beginning of the eighties, the notion of didactic engineering emerged in the didactics of mathematics. This term was called a form of didactic work comparable to the work of the engineer, in which Michelle Artigue assures that to carry out a certain project, she is based on the scientific knowledge of her domain and agrees to undergo a scientific type control, it also studies the relationship between research and action on the teaching system (Artigue 1995). However, at the same time, he is forced to work with objects much more complex than the refined objects of science and therefore has to deal with practically, with all available means, problems that science does not want or cannot take over

The French school of mathematics education called didactic engineering, is based on a priori control of the situations that are put into play within the experimental process (Campos 2007).

Such control is carried out through an a priori analysis that seeks to specify the possibilities that have been selected, the values of the didactic variables that occur as a result of this selection and the meaning that the expected behaviors can take taking into account these values. Subsequently, in the a posteriori analysis, this apriori analysis is compared with the actual performance and what rejects or confirms the hypotheses on which it was based is sought.

THEORETICAL ASSUMPTIONS

Next, the phases that must be taken into account in the theoretical development of didactic engineering are taken verbatim:

Figure 1: (Artigue 1995)

Phase 1: Preliminary analysis: It is the epistemological study of the content to be transmitted, that is, that of knowledge.

Phase 2: Conception and a priori analysis of the situations: It must be planned how the class is going to be carried out, I must choose the situations and variables that are supposed to be relevant.

Phase 3: Experimentation: It is the execution of the designed sequences.

Phase 4: A posteriori analysis and validation: It is about carrying out the analysis of the observations made regarding the teaching sequence and the students' productions during the experimentation.

The components and methodological prescriptions are the four phases described above, the fundamental concepts are the methodology based on comparisons of experiments in class, in response to two problems: attending to the complexity of the class in the research methodologies and studying the relationship between research and action on the education system (Godino 2014).

An experimental system based on didactic realizations of the class should be sought, registering cases whose validation arises from the comparison between a priori and a posteriori analysis, and then carrying out a validation of these.

METHODOLOGY

This work was developed under a qualitative approach and the theoretical bases of action research, whose fundamental purpose is focused on improving the quality of an action, in this case of education through theoretical and practical advances.

Sampieri (2010) describes what action research is for several authors. Sandin affirms that "action research essentially aims to promote social change, transform reality and make people aware of their role in this transformation process." Elliot "conceives of action research as the study of a social situation with a view to improving the quality of action within it." Kurt Lewis proposes that "through such research, theoretical and social advances could be achieved." For his part, Stenhouse adapts what is known today as action research to education. In this way, the method to study and explore this problem, which originates in teaching and learning, and more in the second, it aims to improve it; This is the most important hypothesis of this research. The main purpose of the research is to establish and evaluate strategies to enrich the practice.

The study is carried out with all first-semester engineering students at a private university in the city of Bogotá, Colombia. The majority of the students are from strata 3, 4 and 5 and many come from private schools, García (2019)

Study design.

- Description of different investigations related to the objectives and problem statement of the present study.
- Collection and analysis of syllabus from the linear algebra program of national and foreign universities.
- Theoretical foundation of plausible reasoning and others that may be required.
- Design and implementation of the linear algebra course.
- Design and application of diagnostic tests.
- Surveys of teachers and students.
- Design of activities.
- Design of the model and didactic procedure and final test.
- Attitudinal survey design for students.

Study application. In this fundamental stage for the investigation, the proposed activities are applied in order to observe and record their development by the student in the classroom.

Among the tasks at this stage are the following:

- Development and application of the different activities proposed.
- Application of a final test or survey.

Analysis of the learning process. Concerning the analysis of the results obtained, after carrying out the entire application during the study, it was carried out using the case study. The tasks were:

- Organization and analysis of the activities carried out by the students.
- Organization and analysis of the responses of the survey made to the students.

ANALYSIS OF RESULTS

Problem 1. Given the following points on the plane (Own source):

Figure 2: Problem 1

For $x = 3$, what is the value of the ordinate so that the point is on the line?

For any value of x , what is the value of y for the point on the line?

The purpose of the previous question is to explore the knowledge that students have about the equation of a straight line and to introduce linear situations.

Solution presented by the G1 group.

1) $\frac{m-1}{6-2} = \frac{3}{4} = m \rightarrow$ Con esta fórmula encontramos la pendiente de la recta dada por dos puntos

$y - y_1 = m(x - x_1) \rightarrow$ tenemos la ecuación punto pendiente

$y = \frac{3}{4}(3-2) + 1 \rightarrow$ reemplazando los valores dados en la recta y el valor de la pendiente

$y = \frac{9}{4} - \frac{6}{4} + 1$

$y = \frac{3}{4} + 1 = \frac{3+4}{4} = \frac{7}{4}$

$y = \frac{3}{4}x - \frac{1}{2}$

Figure 3: Solution presented by the G1 group.

Reviewing the solutions of all groups, it is observed that the students know the equation of a line in the plane and

that in order to find it they must first calculate the slope, as observed in the solution of this group; then find the equation and plug in the given value.

Problem 2. What geometric figure can be seen in the graph that corresponds to an equation of the form? (Own source):

- $ax = 0$
- $ax + by + c = 0$
- $ax + by + cz + d = 0$

This is a question whose purpose is for the student to explore giving different values to the constants in the given equations, to arrive at a determined equation and to conjecture about it.

Solution presented by the G2 and G3 groups.

Figure 4: Solution of problem 2a of activity 1 by group G2.

Figure 5: Solution presented by the G3 group.

Students explore the equations by giving different values to the constants, to identify the figures that represent the equations, especially the plane, which they also explore in GeoGebra. Then they correctly identify the geometric figures that each equation represents.

Problem 3. Represent by means of an equation all pairs of real numbers (a, b) such that when multiplying a by 4 and b by 5, the sum of these products is 10. Make the graph. Are there only positive solutions? (Own source)

This is a question that is intended for the student to analyze and graph possible solutions that are obtained by graphically representing the linear equation.

Solution presented by the G2.

Figure 6: Problem 3, G2 solution

Students pose the equation and graph it, then use it to answer the questions in the following sections correctly.

Problem 4. A construction company stores three basic mixes A, B, and C. Quantities are measured in grams, and each "unit" of mix weighs 60 grams. Special mortar mixtures can be formulated by combining the basic mixtures. The composition of these is:

Table 1: Conditions of problem 4

Material \ Mix	A	B	C
Cement	20	18	12
Water	10	12	14
Sand	20	21	28
Gravel	10	9	6

- Is it possible to make a mix consisting of 80 grams of cement and 50 grams of water, using only the basic mixes A and B? If possible, are there only positive solutions? Are there only whole solutions?
- Is it possible to make a mix consisting of 50 grams of water and 100 grams of sand using only the basic mixes A and C? If possible, are there only positive solutions? Are there only whole solutions?
- Is it possible to make a mix consisting of 90 grams of cement and 70 grams of gravel, using only basic mixes B and C? Why can it or why not? If possible, are there only positive solutions? Are there only whole

solutions?

- Is it possible to make a mix consisting of 100 grams of cement, 200 grams of water, and 50 grams of gravel using the three basic mixes A, B, and C? State the equation or equations that represent this situation. Why can it or why not? If possible, are there only positive solutions? Are there only whole solutions?
- Propose a system of linear equations in two variables that has one, none, or infinite solutions.
- Repeat the previous part for linear systems with three variables and make a conjecture about the results obtained. (Modified from Grossman's book)

This is a question that, based on a given information, intends for the student to pose equations and find solutions according to the context of the problem.

Solution presented by the G4 group.

" sistema de ecuaciones: $20a + 78b = 80$
 $70a + 72b = 50$

$$R = \left[\begin{array}{cc|c} 20 & 78 & 80 \\ 70 & 72 & 50 \end{array} \right] \xrightarrow{\frac{1}{20}} \left[\begin{array}{cc|c} 1 & 3.9 & 4 \\ 70 & 72 & 50 \end{array} \right] \xrightarrow{70f_1 - f_2} \left[\begin{array}{cc|c} 1 & 3.9 & 4 \\ 0 & -3 & -10 \end{array} \right] \xrightarrow{\frac{1}{-3}} \left[\begin{array}{cc|c} 1 & 3.9 & 4 \\ 0 & 1 & 10/3 \end{array} \right] \xrightarrow{3.9f_2 - f_1} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 10/3 \end{array} \right]$$

$a = 1, b = 10/3$ solo existe una unica solucion.

Figure 7: Solution of problem 4a by the G4 group.

Students pose the system of equations correctly, solve it using the Gauss-Jordan method, and answer the question correctly.

b) ¿Es posible hacer una mezcla que consiste en 50 gramos de agua y 700 gramos de arena utilizando unicamente las mezclas basicas A y C?

" sistema de ecuaciones: ① $70a + 71c = 50$
 ② $20a + 28c = 100$

El sistema tiene infinitas soluciones ya que la ecuacion 2 es multiplo de la primera.

Figure 8: Solution of problem 4b by the G4 group.

Here the students pose the system and give the solution without having to solve the equation, since they observe that one equation is a multiple of another. In this solution it is observed that the students begin to conjecture about the solutions of linear systems, which was one of the objectives of the problem.

As in the previous paragraph, the students pose the system correctly and by observing the equations, they infer without having to solve it, that the system has no sol

c) ¿Es posible hacer una mezcla que consista en 90 gramos de cemento y 70 gramos de gránulo, utilizando únicamente las mezclas básicas B y C?

• Sistema de ecuaciones:

$$\begin{cases} 18b + 12c = 90 \\ 9b + 6c = 70 \end{cases}$$

No es posible hacer una mezcla ya que el sistema no tiene solución debido a que las ecuaciones son múltiples pero 90 no es el doble de 70.

Figure 9. Solution of the problem by the G4 group.

d) ¿Es posible hacer una mezcla que consista en 100 gramos de cemento, 200 gramos de agua y 50 gramos de gránulo utilizando las mezclas básicas A, B y C?

• Sistema de ecuaciones:

$$\begin{cases} 20a + 18b + 12c = 100 \\ 10a + 12b + 9c = 200 \\ 10a + 9b + 6c = 50 \end{cases}$$

$$X = \begin{bmatrix} 20 & 18 & 12 & 100 \\ 10 & 12 & 9 & 200 \\ 10 & 9 & 6 & 50 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 10 & 12 & 9 & 200 \\ 20 & 18 & 12 & 100 \\ 10 & 9 & 6 & 50 \end{bmatrix} \xrightarrow{R_2 - 2R_1, R_3 - R_1} \begin{bmatrix} 10 & 12 & 9 & 200 \\ 0 & -6 & -6 & -300 \\ 0 & -9 & -3 & -150 \end{bmatrix}$$

$$\xrightarrow{R_2 \cdot (-1/6), R_3 \cdot (-1/3)} \begin{bmatrix} 10 & 12 & 9 & 200 \\ 0 & 1 & 1 & 50 \\ 0 & 3 & 1 & 50 \end{bmatrix} \xrightarrow{R_3 - 3R_2} \begin{bmatrix} 10 & 12 & 9 & 200 \\ 0 & 1 & 1 & 50 \\ 0 & 0 & -2 & -100 \end{bmatrix}$$

$$\xrightarrow{R_3 \cdot (-1/2)} \begin{bmatrix} 10 & 12 & 9 & 200 \\ 0 & 1 & 1 & 50 \\ 0 & 0 & 1 & 50 \end{bmatrix} \xrightarrow{R_2 - R_3} \begin{bmatrix} 10 & 12 & 9 & 200 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 50 \end{bmatrix}$$

$$\xrightarrow{R_1 - 10R_2 - 9R_3} \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 50 \end{bmatrix}$$

El sistema tiene infinitas soluciones ya que a y b están en términos de c.

Figure 10: Solution of problem 4d by the G4 group.

Students pose and solve the system correctly.

e) Propongo un sistema de ecuaciones lineales en dos variables que tenga una, ninguna o infinitas soluciones.

• Una solución:

$$\begin{cases} 2a + 3b = 5 \\ a + 2b = 2 \end{cases}$$

• sin solución:

$$\begin{cases} 4x + 6y = 10 \\ 2x + 3y = 9 \end{cases}$$

• infinitas soluciones:

$$\begin{cases} 2x + 2y = 10 \\ x + y = 5 \end{cases}$$

Figure 11: Solution of problem 4e by the G4 group.

CONCLUSIONS

As a consequence of the previous paragraphs, students correctly pose the requested systems without having to solve the systems.

Didactic engineering is an experimental scheme based on "didactic realizations" in class, that is, on the conception, realization, observation and analysis of teaching sequences, whose purpose is the learning of knowledge.

The student must be proactive in the activities assigned by the teacher, their participation must be active throughout the experimental stage.

The teacher must assume this methodology as a scientific researcher, who must develop a project (teaching knowledge) applying the steps that an engineer should take to implement a development, applying the four steps described above.

The teacher must propose hypotheses and contrast them, to verify and evaluate the teaching and learning process developed.

The teacher during implementation should try to minimize the presence of inappropriate interventions. In the context of engineering, an inappropriate intervention by the teacher would be, for example, to offer a response or comment that provides a lot of information or that cancels the exploration by the student in the search for the solution or simply that interferes with the didactic intentionality of certain slogan. In this way, it is usual to make a kind of "script" for the teacher in which they try to foresee different questions, answers or comments from the students and make explicit what intervention the teacher would carry out accordingly

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