COST OPTIMIZATION STUDY FOR TWO-STEP HELICAL GEARBOXES

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ABSTRACT

The present work deals with the optimization problem, for finding the optimum gear ratios to get the minimum cost of a two-step helical gearbox. To do that, a simulation experiment was designed and conducted by a computer program. Also, nine input factors including the total gearbox ratio u_b , the wheel face width coefficients of both steps (X_{ba1} and X_{ba2}), the permissible contact stresses of the first and the second steps (AS_1 and AS_2), the output torque T_{out} , the gearbox house cost C_{gh} , the gear cost C_g , and the shaft cost C_s were selected to explore their effect on the optimum gear ratio of the first step u_1 . Besides, a regression model to calculate u_1 was found with highly suited to the experimental results.

KEYWORDS: Gearbox, Gear ratio, Optimum gear ratio, Two-step helical gearbox

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1. INTRODUCTION

In optimal gearbox design, many studies have focused on finding optimum partial gear ratios. The reason is that the gearbox mass and size is strongly influenced by the partial gear ratio. So far, many studies have been done to find the optimum gear ratios for several gearbox types including helical gearboxes, bevel-helical gearboxes, worm gearboxes and gearboxes with a step double gear-set. For the helical gearbox, the optimum partial gear ratios have been found for two steps [1-5], three steps [6-10] and four steps [11,12]. For the bevel helical gearbox, several studies on finding optimum gear ratios have been done for two-step gearboxes [13, 14] and three step gearboxes [15-18]. There have also been many pieces of researches on worm gearboxes such as worm-helical gearboxes [19-22] or two step worm gearboxes [23]. Besides, the optimum partial gear ratios were determined for gearboxes with a step double gear set [24-27]. In addition, several studies have been carried out to find the optimum gear ratios for a mechanical system with the use of a reducer and a V-belt drive [28-31] or a chain drive [32-34]. The optimum gear ratios were defined for different target functions such as the minimum gearbox length [5, 14, 19, 35, 36], the minimum gearbox cross section [4, 9, 21, 24, 28, 29], or the minimum gear mass [7, 11, 37]. Recently, the minimum gearbox cost was first examined for three step helical gearboxes [38].

From the above studies, it can be seen that so far, several works have been done on determining the optimum gear ratios. Nonetheless, until now, there has not been any work for calculation of the optimum gear ratios for two-step helical gearboxes to achieve the lowest gear cost target. This work will analyze the effect of the input design factors on the optimum gear ratios. Also, regression models to find the optimum gear ratios for two-step gearboxes will be proposed.

2. METHODOLOGY

2.1. Calculation of Gearbox Cost

The cost of a two-step helical gearbox C_{gb} is calculated as:

$$C_{gb} = C_g + C_{gh} + C_s \quad (1)$$

Where, C_g , C_{gh} , and C_s are the gear cost, the gearbox house cost and the shaft cost, which are determined by:

$$C_g = c_{g.m} \cdot m_g \quad (2)$$

$$C_{gh} = c_{gh.m} \cdot m_{gh} \quad (3)$$

$$C_s = c_{s.m} \cdot m_s \quad (4)$$

In the above equations, $c_{g.m}$, $c_{gh.m}$ and $c_{s.m}$ are the cost per kilogram of gears, gearbox house and shafts (USD/kg); m_g , m_{gh} and m_s are the mass of gears, gearbox house, and of all shafts (kg).

From the above equations, to calculate the gearbox cost C_{gb} , it is needed to determine m_g , m_{gh} and m_s .

2.2 Determining the Mass of Gearbox House

Figure 1: Schema for Calculating Gearbox Mass [2].

The gearbox house mass m_{gh} (kg) is calculated by:

$$m_{gh} = \rho_{gh} \cdot V_{gh} \quad (5)$$

In which, ρ_{gh} is the specific weight of gearbox house (kg/m³); In practice, the gearbox house is usually made by cast iron, $\rho_{gh} = 7.2$ (kg/m³) [39]; V_{gh} is the gearbox house volume (m³) which can be determined as:

$$V_{gh} = 2 \cdot V_b + 2 \cdot V_{A1} + 2 \cdot V_{A2} \quad (6)$$

In Equation (6), V_b , V_{A1} , and V_{A2} are calculated by:

$$V_b = L \cdot B_1 \cdot 1.5 \cdot S_G \quad (7)$$

$$V_{A1} = L \cdot H \cdot S_G \quad (8)$$

$$V_{A2} = B_2 \cdot H \cdot S_G = (B_1 - 2 \cdot S_G) \cdot H \cdot S_G \quad (9)$$

Replacing three above equations into (6) have:

$$V_{gh} = 3 \cdot L \cdot B_1 \cdot S_G + 2 \cdot L \cdot H \cdot S_G + 2 \cdot (B_1 - 2 \cdot S_G) \cdot H \cdot S_G \quad (10)$$

In the above equations, L , H , B_1 and S_G are calculated as [2]:

$$L = (d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22} + 22.5)/0.975 \quad (11)$$

$$H = d_{w22} + 6.5 \cdot S_G \quad (12)$$

$$B_1 = b_{w1} + b_{w2} + 6 \cdot S_G \quad (13)$$

$$S_G = 0.005 \cdot L + 4.5 \quad (14)$$

In which, d_{w11} , d_{w12} , d_{w21} , and d_{w22} are the pinion and the gear diameters which can be found by [40]:

$$d_{w11} = 2 \cdot a_{w1}/(u_1 + 1) \quad (15)$$

$$d_{w12} = 2 \cdot a_{w2}/(u_2 + 1) \quad (16)$$

$$d_{w21} = 2 \cdot a_{w1} \cdot u_1/(u_1 + 1) \quad (17)$$

$$d_{w22} = 2 \cdot a_{w2} \cdot u_2/(u_2 + 1) \quad (18)$$

In Equations (15) to (17), u_1 and u_2 are the partial gear ratios; a_{w1} and a_{w2} are the center distances of both steps which are found by:

$$a_{w1} = k_a \cdot (u_1 + 1) \cdot \sqrt[3]{T_{11} \cdot k_{H\beta}/([\sigma_{H1}]^2 \cdot u_1 \cdot X_{ba1})} \quad (19)$$

$$a_{w2} = k_a \cdot (u_2 + 1) \cdot \sqrt[3]{T_{12} \cdot k_{H\beta}/([\sigma_{H2}]^2 \cdot u_2 \cdot X_{ba2})} \quad (20)$$

Where, $k_{H\beta}$ is the contacting load ratio for pitting resistance; with this gearbox, $k_{H\beta} = 1.02 \div 1.28$ and it can be chosen as $k_{H\beta} = 1.15$; $[\sigma_{H1}]$ and $[\sigma_{H2}]$ are the permissible contact stress of both steps (MPa); X_{ba1} and X_{ba2} are wheel face width coefficients of both steps; T_{11} and T_{12} are the torques on the pinions of both steps (Nmm) which can be determined by:

$$T_{11} = T_r/(u_t \cdot \eta_{hg}^3 \cdot \eta_{be}^4) \quad (21)$$

$$T_{12} = T_r/(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2) \quad (22)$$

Where, u_t is the total gearbox ratio; η_{hg} is the gear efficiency; $\eta_{hg} = 0.96 \div 0.98$ [40]; η_{be} is the bearing efficiency; $\eta_{be} = 0.99 \div 0.995$ [40].

2.3. Determining the Mass of Gears

The gear mass m_g is calculated as:

$$m_g = m_{g1} + m_{g2} \quad (23)$$

Wherein, m_{g1} and m_{g2} is gear mass of steps 1 and 2(kg) which is found by:

$$m_{g1} = \rho_g \cdot (\pi \cdot e_1 \cdot d_{w11}^2 \cdot b_{w1}/4 + \pi \cdot e_2 \cdot d_{w21}^2 \cdot b_{w1}/4) \quad (24)$$

$$m_{g2} = \rho_g \cdot (\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2}/4 + \pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2}/4) \quad (25)$$

In which, ρ_g is the gear material specific weight(kg/m³); With steelgears, $\rho_g = 7.82 \text{ (kg/m}^3\text{)}$ [39]; e_1 and e_2 are volume coefficients of the drive and driven gears; In practice, $e_1 = 1$ and $e_2 = 0.6$ [40]; b_{w1} and b_{w2} are the gear width of both steps (mm): $b_{w1} = X_{ba1} \cdot a_{w1}$ and $b_{w2} = X_{ba2} \cdot a_{w2}$ [40] with X_{ba1} and X_{ba2} are the wheel face coefficients of both steps.

2.4. Determining the Mass of Shafts

The mass of shafts m_s can be determined by:

$$m_s = m_{s1} + m_{s2} + m_{s3} \quad (26)$$

Where,

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s1}^2 \cdot l_{s1}/4 \quad (27)$$

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s2}^2 \cdot l_{s2}/4 \quad (28)$$

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s3}^2 \cdot l_{s3}/4 \quad (29)$$

In which, m_{s1} , m_{s2} , and m_{s3} are the mass of shafts 1, 2 and 3 (kg); ρ_s is the shaft material specific weight (kg/m³); As the shaft material is steel, $\rho_s = 7.82 \text{ (kg/m}^3\text{)}$ [39]; l_{s1} , l_{s2} and l_{s3} are the length of shafts 1, 2 and 3 which are calculated by (see Figure 1):

$$l_{s1} = B_1 + 1.2 \cdot d_{s1} \quad (30)$$

$$l_{s2} = B_1 \quad (31)$$

$$l_{s3} = B_1 + 1.2 \cdot d_{s3} \quad (32)$$

With

$$d_{s1} = [T_{11}/(0.2 \cdot [\tau])]^{1/3} \quad (33)$$

$$d_{s2} = [T_{12}/(0.2 \cdot [\tau])]^{1/3} \quad (34)$$

$$d_{s3} = [T_{13}/(0.2 \cdot [\tau])]^{1/3} \quad (35)$$

Where $[\tau] = 15 \div 20 \text{ (Mpa)}$ is the permissible shear stress [40].

2.6. Optimization Problem

The optimization problem to get the minimal gearbox cost is defined by:

$$\text{minimize } C_{gb} \quad (36)$$

With the constraints as: $1 \leq u_1 \leq 9$ (37)

$$1 \leq u_2 \leq 9$$

3. SIMULATION EXPERIMENT

To study the effect of design parameters on the optimal gear ratios, a simulation experiment was carried out. For this experiment, nine design parameters were selected (Table 1), and a 2-level factorial experiment with 1/4 fraction was considered. Accordingly, $2^{9-2} = 128$ experimental runs were planned. Also, based on Equations (36) and (37), a program was done to perform the experiment. Table 2 presents various levels of the input parameters and the optimum gear ratio of step 1 u_1 .

Table 1: Design factors

Factor	Code	Low	High
Total ratio	u_t	10	50
Wheel face width coefficient of step 1	X_{ba1}	0.3	0.35
Wheel face width coefficient of step 2	X_{ba2}	0.35	0.4
Permissible contact stress of step 1 (Mpa)	AS_1	350	420
Permissible contact stress of step 2 (Mpa)	AS_2	350	420
Output torque (Nmm)	T_{out}	102	104
Gearbox house cost (USD/kg)	C_{gh}	1	5
Gear cost (USD/kg)	C_g	2	9
Shaft cost (USD/kg)	C_s	1.5	5

4. RESULTS AND DISCUSSIONS

The influence of the design factors on u_1 is displayed in Figure 2. It can be seen that u_1 depends significantly on u_t . When u_t increases, u_1 grows significantly. Also, u_1 is affected by the following parameters in descending order C_s , C_g , C_{gh} , AS_2 , AS_1 , T_{out} , X_{ba1} and X_{ba2} .

The Pareto chart for the standardized effects for u_1 is shown in Figure 3. From this chart, it is noted that u_t (factor A), C_s (factor J), C_g (factor H), AS_2 (factor E), C_{gh} (factor G), AS_1 (factor D), T_{out} (factor F) and the interactions HJ, AH, AJ, GH, FG, AE, FH, AD, and DE are significant at the 0.05 level with u_1 because the bars representing these parameters cross the reference line. Also, u_1 is not affected by factors B (X_{ba1}), C (X_{ba2}), and the interactions AG, EH, GJ, EG, EF, CD, DG, AB, DF, BH, and CH as the bars representing them do not cut the reference line.

Table 2: Plan of Experiment and Optimum Gear Ratio u_1

RunOrder	u_t	X_{ba1}	X_{ba2}	AS_1	AS_2	T_{out}	C_{gh}	C_g	C_s	u_1
1	10	0.3	0.4	350	420	10000	5	9	1.5	2.66
2	50	0.3	0.35	350	420	10000	1	2	1.5	7.65
3	10	0.35	0.35	350	420	10000	1	9	5	2.25
4	10	0.3	0.4	350	350	10000	5	9	5	2.60
5	10	0.35	0.4	420	420	100	1	2	5	1.70
6	10	0.35	0.35	420	350	100	1	9	5	2.64
127	50	0.3	0.35	420	350	100	5	9	5	9.00
128	50	0.35	0.4	350	420	10000	5	2	5	6.33

Figure 2: Graph of Main Effects for u_1 . Figure 3: Pareto Chart for u_1 . Figure 4: Normal Plot for u_1

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		5.4090	0.0197	274.96	0.000	
ut	5.8105	2.9052	0.0197	147.69	0.000	1.00
AS1	0.2627	0.1313	0.0197	6.68	0.000	1.00
AS2	-0.4123	-0.2062	0.0197	-10.48	0.000	1.00
Tout	-0.1542	-0.0771	0.0197	-3.92	0.000	1.00
Cgh	0.3077	0.1538	0.0197	7.82	0.000	1.00
Cg	0.6580	0.3290	0.0197	16.72	0.000	1.00
Cs	-0.7261	-0.3630	0.0197	-18.46	0.000	1.00
ut*AS1	0.0939	0.0470	0.0197	2.39	0.019	1.00
ut*AS2	-0.1498	-0.0749	0.0197	-3.81	0.000	1.00
ut*Cg	0.2117	0.1059	0.0197	5.38	0.000	1.00
ut*Cs	-0.1836	-0.0918	0.0197	-4.67	0.000	1.00
AS1*AS2	0.0930	0.0465	0.0197	2.36	0.020	1.00
AS2*Cs	-0.0902	-0.0451	0.0197	-2.29	0.024	1.00
Tout*Cgh	-0.1552	-0.0776	0.0197	-3.94	0.000	1.00
Tout*Cg	0.1077	0.0538	0.0197	2.74	0.007	1.00
Cgh*Cg	-0.1823	-0.0912	0.0197	-4.63	0.000	1.00
Cg*Cs	0.3633	0.1816	0.0197	9.23	0.000	1.00

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.222559	99.52%	99.45%	99.35%

Figure 5: Estimated Effects and Coefficients for u_1

Figure 4 illustrates the Normal Plot of the standardized effects. From the plot, u_t (factor A) is the most significant factor which affects u_1 . In addition, C_g (factor H), C_{gh} (factor G), and the interactions HJ, AH, AD, and DE have a positive influence. When these factors surge, u_1 will rise. Besides, C_s (factor J), AS_2 (factor E), T_{out} (factor F), and the interactions AJ, GH, FG, AE, and EJ has a negative standardized effect. That means u_1 reduces, when these factors grow.

The estimated effects and the coefficients for u_1 are designated in Figure 5. It was found from the figure that all of the P-values of u_t , AS_1 , AS_2 , T_{out} , C_{gh} , C_g , C_s , and the interactions $u_t \cdot AS_1$, $u_t \cdot AS_2$, $u_t \cdot C_g$, $u_t \cdot C_s$ are zero. Therefore, these factors are significant to u_1 , and the model to predict u_1 is given as:

$$u_1 = 6.4 + 0.1609 \cdot u_t - 0.01287 \cdot AS_1 - 0.0149 \cdot AS_2 - 0.000009 \cdot T_{out} + 0.1881 \cdot C_{gh} - 0.0244 \cdot C_g - 0.009 \cdot C_s + 0.000067 \cdot u_t \cdot AS_1 - 0.000107 \cdot u_t \cdot AS_2 + 0.001512 \cdot u_t \cdot C_g - 0.002623 \cdot u_t \cdot C_s + 0.000038 \cdot AS_1 \cdot AS_2 - 0.000736 \cdot AS_2 \cdot C_s - 0.000008 \cdot T_{out} \cdot C_{gh} + 0.000003 \cdot T_{out} \cdot C_g - 0.01302 \cdot C_{gh} \cdot C_g + 0.02966 \cdot C_g \cdot C_s \quad (38)$$

From Figure 5, it is reported that model (38) fits the data very well (all of the R_{sq} are more than 99.35%). Hence, this model can be used to calculate u_1 . The optimum gear ratio of step 2 u_2 after that can be easily found by $u_2 = u_t/u_1$.

4. CONCLUSIONS

The present work deals with finding the optimum gear ratios for two step helical gearboxes to obtain the minimum gearbox cost. To do that, an optimization problem was carried out with the target is the minimum gearbox cost. Also, nine design parameters containing the total ratio, the coefficients of wheel face width and the permissible stresses of two steps, the output torque, the cost of the gearbox house, the gears and the shafts were selected for the investigation of their impact on the response. As well, proposed models to determine the optimum ratios were presented.

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